Exercise 1.1

1. Find the order of the following matrices.

$$A=\begin{bmatrix}2 & 3\\ -5 & 6\end{bmatrix},\quad B=\begin{bmatrix}2 & 0\\ 3 & 5\end{bmatrix},\quad C=\begin{bmatrix}2 & 4\end{bmatrix}$$

$$D=\begin{bmatrix}4\\ 0\\ 6\end{bmatrix},\quad E=\begin{bmatrix}a & d\\ b & e\\ c & f\end{bmatrix},\quad F=[2]$$

$$G=\begin{bmatrix}2 & 3 & 0\\ 1 & 2 & 3\\ 2 & 4 & 5\end{bmatrix},\quad H=\begin{bmatrix}2 & 3 & 4\\ 1 & 0 & 6\end{bmatrix}$$

Solution: $A=\begin{bmatrix}2 & 3\\ -5 & 6\end{bmatrix}$

Number of Rows (R) = 2
Number of Columns = 2
Order of the matrix = R by C
Order of the matrix = 2 by 2

$$B=\begin{bmatrix}2 & 0\\ 3 & 5\end{bmatrix}$$

Order = 2 by 2

$$C=\begin{bmatrix}2 & 4\end{bmatrix}$$

Order = 1 by 2

$$D=\begin{bmatrix}4\\ 0\\ 6\end{bmatrix}$$

Order = 3 by 1

$$E=\begin{bmatrix}a & d\\ b & e\\ c & f\end{bmatrix}$$

Order = 3 by 2

$$F=[2]$$

Order = 1 by 1

$$G=\begin{bmatrix}2 & 3 & 0\\1 & 2 & 3\\2 & 4 & 5\end{bmatrix}$$

Order = 3 by 3

$$H=\begin{bmatrix}2 & 3 & 4\\1 & 0 & 6\end{bmatrix}$$

Order = 2 by 3

2. Which of the following matrices are equal?

$A=[3]$, $B=[3\quad 5]$, $C=[5-2]$

$D=[5\quad 3]$, $E=\begin{bmatrix}4 & 0\\6 & 2\end{bmatrix}$, $F=\begin{bmatrix}2\\6\end{bmatrix}$

$G=\begin{bmatrix}3-1\\3+3\end{bmatrix}$, $H=\begin{bmatrix}4 & 0\\6 & 2\end{bmatrix}$, $I=[3\quad 3+2]$

$J=\begin{bmatrix}2+2 & 2-2\\2+4 & 2+0\end{bmatrix}$

Solution:

(i) A = C (ii) B = I (iii) E = H = J

(iv) F = G

Order of one is equal to the order of the other and their corresponding elements are equal.

Q.3. Find the values of a, b, c and d which satisfy the matrix equation.

$$\begin{bmatrix}a+c & a+2b\\c-1 & 4d-6\end{bmatrix}=\begin{bmatrix}0 & -7\\3 & 2d\end{bmatrix}$$

Solution: a + c = 0 (i)

$$
\begin{aligned}
c - 1 &= 3 \quad \text{(ii)}\\
a + 2b &= -7 \quad \text{(iii)}\\
4d - 6 &= 2d \quad \text{(iv)}\\
c - 1 &= 3 \quad \text{From (ii)}\\
c &= 3 + 1 = 4\\
a + c &= 0 \quad \text{(i)}\\
a + 4 &= 0 \quad \text{From (ii)}\\
a &= -4\\
4d - 6 &= 2d \quad \text{From (iv)}\\
4d - 2d &= 6\\
2d &= 6\\
\therefore \quad d &= 3\\
a + 2b &= -7 \quad \text{From (iii)}
\end{aligned}
$$

Putting value of 'a'.

$$
\begin{aligned}
-4 + 2b &= -7\\
2b &= -7 + 4\\
2b &= -3\\
b &= -\frac{3}{2}
\end{aligned}
$$

Thus $a = -4$, $b = -\frac{3}{2}$, $c = 4$, $d = 3$

Types of Matrices:

Row Matrix: A row matrix has only ONE row. e.g.

$A = [3 \ -2 \ \ 0]$ is a row matrix.

Column Matrix: A column matrix has only ONE column e.g.

$B = \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}$ is a column matrix.

Rectangular Matrix: If the number of rows is not equal to the number of columns in a matrix then it is a rectangular matrix.

Square Matrix: A matrix is called a square matrix if its number of rows is equal to its number of columns.

Null or Zero Matrix: A matrix is called a null or zero matrix if each of its entries is zero.

Transpose of a Matrix: A matrix obtained by interchanging rows into columns or columns into rows is called a transpose of the given matrix.

Negative of a Matrix: A matrix obtained by changing the signs of all the entries of a matrix is called negative of the given matrix.

Symmetric Matrix: A square matrix is symmetric if it is equal to its transpose. If A is a square matrix then its symmetrix matric is written as A^t and $A^t = A$.

Skew-Symmetric Matrix: A square matrix A is said to be skew-symmetric if $A^t = -A$

Diagonal Matrix: A square matrix is called a diagonal matrix if atleast any one of the entries of its diagonal is not zero and non-diagonal entries must all be zero. e.g.

Scalar Matix. A diagonal matrix is called a scalar matrix, if all the diagonal entries are same and non-zero.

for example $\begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix}$ where k is a constant.

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

Identity Matrix: A diagonal matrix is called identity (unit) matrix if all diagonal entries are '1'. It is denoted by I.

Exercise 1.2

1. Find the following matrices, identify unit matrices, row matrices, column matrices and null matrices.

$$A=\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix},\ B=\begin{bmatrix}2 & 3 & 4\end{bmatrix},\ C=\begin{bmatrix}4\\0\\6\end{bmatrix}$$

$$D=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix},\ E=\begin{bmatrix}0\end{bmatrix},\ F=\begin{bmatrix}5\\6\\7\end{bmatrix}$$

Solution: $A=\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$

A is a zero matrix.

$$B=\begin{bmatrix}2 & 3 & 4\end{bmatrix}$$

B is a row matrix.

$$C=\begin{bmatrix}4\\0\\6\end{bmatrix}$$

C is a column matrix.

$$D=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

D is an identity (unit) matrix.

$$E=\begin{bmatrix}0\end{bmatrix}$$

E is a zero matrix.

$$F=\begin{bmatrix}5\\6\\7\end{bmatrix}$$

F is a column matrix.

Q.2. From the following matrices, identify.

(a) Square matrices (b) Rectangular matrices

(c) Row matrices (d) Column matrices

(e) Identity matrices (f) Null matrices

$(i)\begin{bmatrix}-8 & 2 & 7\\ 12 & 0 & 4\end{bmatrix}$ $(ii)\begin{bmatrix}3\\0\\1\end{bmatrix}$ $(iii)\begin{bmatrix}6 & -4\\ 3 & -2\end{bmatrix}$

$(iv)\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ $(v)\begin{bmatrix}1 & 2\\ 3 & 4\\ 5 & 6\end{bmatrix}$ $(vi)\begin{bmatrix}3 & 10 & -1\end{bmatrix}$

$(vii)\begin{bmatrix}1\\0\\0\end{bmatrix}$ $(viii)\begin{bmatrix}1 & 2 & 3\\ -1 & 2 & 0\\ 0 & 0 & 1\end{bmatrix}$ $(ix)\begin{bmatrix}0 & 0\\ 0 & 0\\ 0 & 0\end{bmatrix}$

Solution:

$$\begin{bmatrix}6 & -4\\ 3 & -2\end{bmatrix}, \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}, \begin{bmatrix}1 & 2 & 3\\ -1 & 2 & 0\\ 0 & 0 & 1\end{bmatrix} \text{... } \textit{Square matrices}$$

$$\begin{bmatrix}-8 & 2 & 7\\ 12 & 0 & 4\end{bmatrix}, \begin{bmatrix}3\\0\\1\end{bmatrix}, \begin{bmatrix}1 & 2\\ 3 & 4\\ 5 & 6\end{bmatrix} \text{... } \textit{Rectangular matrices}$$

$$\begin{bmatrix}3 & 10 & -1\end{bmatrix}, \begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0 & 0\\ 0 & 0\\ 0 & 0\end{bmatrix}$$

$$\begin{bmatrix}3 & 10 & -1\end{bmatrix} \text{... } \textit{Row matrix.}$$

$$\begin{bmatrix}3\\0\\1\end{bmatrix}, \begin{bmatrix}1\\0\\0\end{bmatrix} \text{... } \textit{Column matrix}$$

$$\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \text{... } \textit{Identity matrix}$$

$$\begin{bmatrix}0 & 0\\ 0 & 0\\ 0 & 0\end{bmatrix} \text{... } \textit{Zero matrix or null}$$

Q.3. From the following matrices, identify diagonal, scalar and unit (identity).

$$A=\begin{bmatrix}4 & 0\\0 & 4\end{bmatrix},\ B=\begin{bmatrix}2 & 0\\0 & -1\end{bmatrix},\ C=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

$$D=\begin{bmatrix}3 & 0\\0 & 0\end{bmatrix},\ and\ \ E=\begin{bmatrix}5-3 & 0\\0 & 1+1\end{bmatrix}$$

Solution:

$$A=\begin{bmatrix}4 & 0\\0 & 4\end{bmatrix},\ B=\begin{bmatrix}2 & 0\\0 & -1\end{bmatrix} \text{... Diagonal matrices}$$

$$C=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}, D=\begin{bmatrix}3 & 0\\0 & 0\end{bmatrix}, E=\begin{bmatrix}5-3 & 0\\0 & 1+1\end{bmatrix}$$

$$A=\begin{bmatrix}4 & 0\\0 & 4\end{bmatrix},\ \ E=\begin{bmatrix}5-3 & 0\\0 & 1+1\end{bmatrix} \text{... Scalar matrices}$$

$$C=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} \text{... Identity matrix}$$

Q.4. Find negative of matrices A,B,C,D and F when:

$$A=\begin{bmatrix}1\\0\\-1\end{bmatrix},\ B=\begin{bmatrix}3 & -1\\2 & 1\end{bmatrix}, C=\begin{bmatrix}2 & 6\\3 & 2\end{bmatrix}$$

$$D=\begin{bmatrix}-3 & 2\\-4 & 5\end{bmatrix},\ E=\begin{bmatrix}1 & -5\\2 & 3\end{bmatrix}$$

Solution:

$$A=\begin{bmatrix}1\\0\\-1\end{bmatrix} \text{ sign of every element is changed.}$$

$$-A = -\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}$$

$$-B = -\begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} -3 & 1 \\ -2 & -1 \end{bmatrix}$$

$$C = \begin{bmatrix} 2 & 6 \\ 3 & -2 \end{bmatrix}$$

$$-C = -\begin{bmatrix} 2 & 6 \\ 3 & -2 \end{bmatrix} = \begin{bmatrix} -2 & -6 \\ -3 & 2 \end{bmatrix}$$

$$D = \begin{bmatrix} -3 & 2 \\ -4 & 5 \end{bmatrix}$$

$$-D = -\begin{bmatrix} -3 & 2 \\ -4 & 5 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ 4 & -5 \end{bmatrix}$$

$$E = \begin{bmatrix} 1 & -5 \\ 2 & 3 \end{bmatrix}$$

$$-E = -\begin{bmatrix} 1 & -5 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ -2 & -3 \end{bmatrix}$$

Q.5. Find the transpose of each of the following matrices.

$$A = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix},\ B = \begin{bmatrix} 5 & 1 & -6 \end{bmatrix},\ C = \begin{bmatrix} 1 & 2 \\ 2 & -1 \\ 3 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix},\ E = \begin{bmatrix} 2 & 3 \\ -4 & 5 \end{bmatrix},\ F = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$A=\begin{bmatrix}0\\1\\-2\end{bmatrix}$$ columns are changed into rows

$$A^t=\begin{bmatrix}0 & 1 & -2\end{bmatrix}$$

$$B=\begin{bmatrix}5 & 1 & -6\end{bmatrix}\ ;\ B^t=\begin{bmatrix}5\\1\\-6\end{bmatrix}$$

$$C=\begin{bmatrix}1 & 2\\2 & -1\\3 & 0\end{bmatrix}\ ;\ D^t=\begin{bmatrix}1 & 2 & 3\\2 & -1 & 0\end{bmatrix}$$

$$D=\begin{bmatrix}2 & 3\\0 & 5\end{bmatrix}\ ;\ D^t=\begin{bmatrix}2 & 0\\3 & 5\end{bmatrix}$$

$$E=\begin{bmatrix}2 & 3\\-4 & 5\end{bmatrix}\ ;\ E^t=\begin{bmatrix}2 & -4\\3 & 5\end{bmatrix}$$

$$F=\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}\ ;\ F^t=\begin{bmatrix}1 & 3\\2 & 4\end{bmatrix}$$

Q.6. Verify that if $A=\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}, B=\begin{bmatrix}1 & 1\\2 & 0\end{bmatrix}$ **then**

(i) $(A^t)^t=A$ **(ii)** $(B^t)^t=B$

Solution:

6(*i*) $$A=\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}$$

$$A^t=\begin{bmatrix}1 & 0\\2 & 1\end{bmatrix}$$

$$(A^t)^t = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = A$$

6(ii) $$B = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$$

$$B^t = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$$

$$(B^t)^t = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix} = B$$

Addition and subtration of Matrices

Remember:

(i) Two matrices are conformable for addition if they have the same order.

(ii) In case of addition, corresponding elements are added.

(iii) In case of subtraction, corresponding elements are subtracted in an order.

Commutative and Associative Laws of Addition of Matrices:

(a) Commutative law under Addition.

$$A + B = B + A$$

(b) Associative law under addition.

$$(A + B) + C = A + (B + C)$$

Additive identity of a matrix

For any matrix A and zero matrix O of the same order. O is called additive identity of A as

$$A + O = O + A$$

Additive Inverse of a matrix

If A and B are two matrices of the same order such that

$$A + B = O = B + A$$

then A and B are called additive inverse of each other.

We change the sign of all the elements of a matrix to get its additive inverse

Exercise 1.3

1. Which of the following matrices are conformable for addition?

$$A=\begin{bmatrix}2 & 1\\ -1 & 3\end{bmatrix},\ B=\begin{bmatrix}3\\ 1\end{bmatrix},\ C=\begin{bmatrix}1 & 0\\ 2 & -1\\ 1 & -2\end{bmatrix}$$

$$D=\begin{bmatrix}2+1\\ 3\end{bmatrix},\ E=\begin{bmatrix}-1 & 0\\ 1 & 2\end{bmatrix},\ F=\begin{bmatrix}3 & 2\\ 1+1 & -4\\ 3+2 & 2+1\end{bmatrix}$$

Solution:

order of A = 2 by 2 order of B = 2 by 1
order of C = 3 by 2 order of D = 2 by 1
order of E = 2 by 2 order of F = 3 by 2

Matrices of the same order are conformable for addition. A and E, B and D, C and F are conformable for addition.

2. Find the additive inverse of the following matrices.

$$A=\begin{bmatrix}2 & 4\\ -2 & 1\end{bmatrix},\ B=\begin{bmatrix}1 & 0 & -1\\ 2 & -1 & 3\\ 3 & -2 & 1\end{bmatrix},\ C=\begin{bmatrix}4\\ -2\end{bmatrix}$$

$$D=\begin{bmatrix}1 & 0\\ -3 & -2\\ 2 & 1\end{bmatrix},\ E=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},\ F=\begin{bmatrix}\sqrt{3} & 1\\ -1 & \sqrt{2}\end{bmatrix}$$

Solution:

$$A=\begin{bmatrix}2 & 4\\ -2 & 1\end{bmatrix}$$

$$-A=-\begin{bmatrix}2 & 4\\ -2 & 1\end{bmatrix}=\begin{bmatrix}-2 & -4\\ 2 & -1\end{bmatrix}$$ (Additive inverse of A)

$$B=\begin{bmatrix}1 & 0 & -1\\ 2 & -1 & 3\\ 3 & -2 & 1\end{bmatrix}$$

$$-B=-\begin{bmatrix}1 & 0 & -1\\ 2 & -1 & 3\\ 3 & -2 & 1\end{bmatrix}=\begin{bmatrix}-1 & 0 & 1\\ -2 & 1 & -3\\ -3 & 2 & -1\end{bmatrix}\text{(Additive inverse of B)}$$

$$C=\begin{bmatrix}4\\ -2\end{bmatrix}$$

$$-C=-\begin{bmatrix}4\\ -2\end{bmatrix}=\begin{bmatrix}-4\\ 2\end{bmatrix}\text{(additive inverse of C)}$$

$$D=\begin{bmatrix}1 & 0\\ -3 & -2\\ 2 & 1\end{bmatrix}$$

$$-D=-\begin{bmatrix}1 & 0\\ -3 & -2\\ 2 & 1\end{bmatrix}=\begin{bmatrix}-1 & 0\\ 3 & 2\\ -2 & -1\end{bmatrix}\text{(additive inverse of D)}$$

$$E=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$-E=-\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}\text{(additive inverse of E)}$$

$$F=\begin{bmatrix}\sqrt{3} & 1\\ -1 & \sqrt{2}\end{bmatrix}$$

$$-F=-\begin{bmatrix}\sqrt{3} & 1\\ -1 & \sqrt{2}\end{bmatrix}=\begin{bmatrix}-\sqrt{3} & -1\\ 1 & -\sqrt{2}\end{bmatrix}\text{(additive inverse of F)}$$

3. If

$$A=\begin{bmatrix}-1 & 2\\ 2 & 1\end{bmatrix},\ B=\begin{bmatrix}1\\ -1\end{bmatrix},\ C=\begin{bmatrix}1 & -1 & 2\end{bmatrix},$$

$$D=\begin{bmatrix}1 & 2 & 3\\ -1 & 0 & 2\end{bmatrix} \text{ then find}$$

(i) $A+\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$ (ii) $B+\begin{bmatrix}-2\\ 3\end{bmatrix}$

(iii) $C+\begin{bmatrix}-2 & 1 & 3\end{bmatrix}$ (iv) $D+\begin{bmatrix}0 & 1 & 0\\ 2 & 0 & 1\end{bmatrix}$

(v) 2 A (vi) (−1) B

(vii) (−2) C (viii) 3D (ix) 3C

Solution:

(i) $A+\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$

$$=\begin{bmatrix}-1 & 2\\ 2 & 1\end{bmatrix}+\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix} \quad \textit{Putting } A=\begin{bmatrix}-1 & 2\\ 2 & 1\end{bmatrix}$$

$$=\begin{bmatrix}-1+1 & 2+1\\ 2+1 & 1+1\end{bmatrix}=\begin{bmatrix}0 & 3\\ 3 & 2\end{bmatrix}$$

(ii) $B+\begin{bmatrix}-2\\ 3\end{bmatrix}$

$$=\begin{bmatrix}1\\ -1\end{bmatrix}+\begin{bmatrix}-2\\ 3\end{bmatrix} \quad \textit{Putting } B=\begin{bmatrix}1\\ -1\end{bmatrix}$$

$$=\begin{bmatrix}1 & -2\\ -1 & +3\end{bmatrix}=\begin{bmatrix}-1\\ 2\end{bmatrix}$$

(iii) $C+[-2 \quad 1 \quad 3]$

$=[1 \quad -1 \quad 2]+[-2 \quad 1 \quad 3]$ *Putting* $C=[1 \quad -1 \quad 2]$

$=[1-2 \quad -1+1 \quad 2+3]=[-1 \quad 0 \quad 5]$

(iv) $D+\begin{bmatrix}0 & 1 & 0\\ 2 & 0 & 1\end{bmatrix}$

$$=\begin{bmatrix}1 & 2 & 3\\ -1 & 0 & 2\end{bmatrix}+\begin{bmatrix}0 & 1 & 0\\ 2 & 0 & 1\end{bmatrix} \quad \textit{Putting } D=\begin{bmatrix}1 & 2 & 3\\ -1 & 0 & 2\end{bmatrix}$$

$$=\begin{bmatrix}1+0 & 2+1 & 3+0\\ -1+2 & 0+0 & 2+1\end{bmatrix}=\begin{bmatrix}1 & 3 & 3\\ 1 & 0 & 3\end{bmatrix}$$

(v) **2A**

$$=2\begin{bmatrix}-1 & 2\\ 2 & 1\end{bmatrix} \quad \textit{Putting } A=\begin{bmatrix}-1 & 2\\ 2 & 1\end{bmatrix}$$

$$=\begin{bmatrix}2(-1) & 2(2)\\ 2(2) & 2(1)\end{bmatrix}=\begin{bmatrix}-2 & 4\\ 4 & 2\end{bmatrix}$$

(vi) **(−1) B**

$$=(-1)\begin{bmatrix}1\\ -1\end{bmatrix} \quad \textit{Putting } B=\begin{bmatrix}1\\ -1\end{bmatrix}$$

$$=\begin{bmatrix}(-1)(1)\\ (-1)(-1)\end{bmatrix}=\begin{bmatrix}-1\\ 1\end{bmatrix}$$

(vii) **(−2) C**

$=(-2)[1 \quad -1 \quad 2]$ *Putting* $C=[1 \quad -1 \quad 2]$

$$=\begin{bmatrix}(-2)(1) & (-2)(-1) & (-2)(2)\end{bmatrix}=\begin{bmatrix}-2 & 2 & -4\end{bmatrix}$$

(viii) **3 D**

$$=3\begin{bmatrix}1 & 2 & 3\\ -1 & 0 & 2\end{bmatrix} \quad \textit{Putting } D=\begin{bmatrix}1 & 2 & 3\\ -1 & 0 & 2\end{bmatrix}$$

$$=\begin{bmatrix}3(1) & 3(2) & 3(3)\\ 3(-1) & 3(0) & 3(2)\end{bmatrix}=\begin{bmatrix}3 & 6 & 9\\ -3 & 0 & 6\end{bmatrix}$$

(ix) **3 C**

$$=3\begin{bmatrix}1 & -1 & 2\end{bmatrix} \quad \textit{Putting } C=\begin{bmatrix}1 & -1 & 2\end{bmatrix}$$

$$=\begin{bmatrix}3(1) & 3(-1) & 3(2)\end{bmatrix}=\begin{bmatrix}3 & -3 & 6\end{bmatrix}$$

4. **Perform the indicated operations and simplify the followers.**

(i) $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}+\begin{bmatrix}0 & 2\\ 3 & 0\end{bmatrix}+\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}$

(ii) $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}+\begin{bmatrix}0 & 2\\ 3 & 0\end{bmatrix}-\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}$

(iii) $\begin{bmatrix}2 & 3 & 1\end{bmatrix}+(\begin{bmatrix}1 & 0 & 2\end{bmatrix})-\begin{bmatrix}2 & 2 & 2\end{bmatrix})$

(iv) $\begin{bmatrix}1 & 2 & 3\\ -1 & -1 & -1\\ 0 & 1 & 2\end{bmatrix}+\begin{bmatrix}1 & 1 & 1\\ 2 & 2 & 2\\ 3 & 3 & 3\end{bmatrix}$

(v) $\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 3 & 1 & 2\end{bmatrix}+\begin{bmatrix}1 & 0 & -2\\ -2 & -1 & 0\\ 0 & 2 & -1\end{bmatrix}$

(vi) $\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}+\begin{bmatrix}2 & 1\\1 & 0\end{bmatrix}+\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}$

Solution:

4 (i) $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}+\begin{bmatrix}0 & 2\\3 & 0\end{bmatrix}+\begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}$

$$=\left(\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}+\begin{bmatrix}0 & 2\\3 & 0\end{bmatrix}\right)+\begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}1+0 & 0+2\\0+3 & 1+0\end{bmatrix}+\begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}1 & 2\\3 & 1\end{bmatrix}+\begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}=\begin{bmatrix}1+1 & 2+1\\3+1 & 1+0\end{bmatrix}=\begin{bmatrix}2 & 3\\4 & 1\end{bmatrix}$$

4 (ii) $=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}+\begin{bmatrix}0 & 2\\3 & 0\end{bmatrix}-\begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}$

$$=\left(\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}+\begin{bmatrix}0 & 2\\3 & 0\end{bmatrix}\right)-\begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}1+0 & 0+2\\0+3 & 1+0\end{bmatrix}-\begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}1 & 2\\3 & 1\end{bmatrix}-\begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}=\begin{bmatrix}1-1 & 2-1\\3-1 & 1-0\end{bmatrix}=\begin{bmatrix}0 & 1\\2 & 1\end{bmatrix}$$

4 (iii) $\begin{bmatrix}2 & 3 & 1\end{bmatrix}+\left(\begin{bmatrix}1 & 0 & 2\end{bmatrix}-\begin{bmatrix}2 & 2 & 2\end{bmatrix}\right)$

$$=\begin{bmatrix}2 & 3 & 1\end{bmatrix}+\left(\begin{bmatrix}1-2 & 0-2 & 2-2\end{bmatrix}\right)$$

$$=\begin{bmatrix}2 & 3 & 1\end{bmatrix}+\begin{bmatrix}-1 & -2 & 0\end{bmatrix}$$

$$=\begin{bmatrix}2-1 & 3-2 & 1+0\end{bmatrix}=\begin{bmatrix}1 & 1 & 1\end{bmatrix}$$

4 (iv) $\begin{bmatrix} 1 & 2 & 3 \\ -1 & -1 & -1 \\ 0 & 1 & 2 \end{bmatrix}+\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$

$$=\begin{bmatrix} 1+1 & 2+1 & 3+1 \\ -1+2 & -1+2 & -1+2 \\ 0+3 & 1+3 & 2+3 \end{bmatrix}=\begin{bmatrix} 2 & 3 & 4 \\ 1 & 1 & 1 \\ 3 & 4 & 5 \end{bmatrix}$$

4 (v) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}+\begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 0 \\ 0 & 2 & -1 \end{bmatrix}$

$$=\begin{bmatrix} 1+1 & 2+0 & 3-2 \\ 2-2 & 3-1 & 1+0 \\ 3+0 & 1+2 & 2-1 \end{bmatrix}=\begin{bmatrix} 2 & 2 & 1 \\ 0 & 2 & 1 \\ 3 & 3 & 1 \end{bmatrix}$$

4 (vi) $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}+\begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}+\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$$=\left(\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}+\begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}\right)+\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$=\begin{bmatrix} 1+2 & 2+1 \\ 0+1 & 1+0 \end{bmatrix}+\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$=\begin{bmatrix} 3 & 3 \\ 1 & 1 \end{bmatrix}+\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$=\begin{bmatrix} 3+1 & 3+1 \\ 1+1 & 1+1 \end{bmatrix}=\begin{bmatrix} 4 & 4 \\ 2 & 2 \end{bmatrix}$$

5. **For the matrices**

$$A=\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix}, B=\begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix} \text{ and } C=\begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$$

verify the following rules:

(i) $A + C = C + A$ **(ii)** $A + B = B + A$

(iii) $B + C = C + B$ **(iv)** $A + (B + A) = 2A + B$

(v) $(C - B) + A = C + (A - B)$

(vi) $2A + B = A + (A + B)$

(vii) $(C - B) - A = (C - A) - B$

(viii) $(A + B) + C = A + (B + C)$

(ix) $A + (B - C) = (A - C) + B$

(x) $2A + 2B = 2(A + B)$

Solution:

5 (i) $A + C = C + A$

L.H.S $= A + C$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1-1 & 2+0 & 3+0 \\ 2+0 & 3-2 & 1+3 \\ 1+1 & -1+1 & 0+2 \end{bmatrix} = \begin{bmatrix} 0 & 2 & 3 \\ 2 & 1 & 4 \\ 2 & 0 & 2 \end{bmatrix} \quad (i)$$

R.H.S $= C + A$

$$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1+1 & 0+2 & 0+3 \\ 0+2 & -2+3 & 3+1 \\ 1+1 & 1-1 & 2+0 \end{bmatrix} = \begin{bmatrix} 0 & 2 & 3 \\ 2 & 1 & 4 \\ 2 & 0 & 2 \end{bmatrix} \quad (ii)$$

$A + C = C + A$ From (i) and (ii)

5 (ii) $A + B = B + A$

L.H.S $= A + B$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1 & 2-1 & 3+1 \\ 2+2 & 3-2 & 1+2 \\ 1+3 & -1+1 & 0+3 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 4 \\ 4 & 1 & 3 \\ 4 & 0 & 3 \end{bmatrix} \quad (i)$$

$$\text{R.H.S} = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1 & -1+2 & 1+3 \\ 2+2 & -2+3 & 2+1 \\ 3+1 & 1-1 & 3+0 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 4 \\ 4 & 1 & 3 \\ 4 & 0 & 3 \end{bmatrix} \quad (ii)$$

$A + B = B + A$ *From (i) and (ii)*

5(iii) $\mathbf{B + C = C + B}$

$\text{L.H.S} = B + C$

$$= \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1-1 & -1+0 & 1+0 \\ 2+0 & -2-2 & 2+3 \\ 3+1 & 1+1 & 3+2 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 1 \\ 2 & -4 & 5 \\ 4 & 2 & 5 \end{bmatrix} \quad (i)$$

$\text{R.H.S} = C + B$

$$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -1+1 & 0-1 & 0+1 \\ 0+2 & -2-2 & 3+2 \\ 1+3 & 1+1 & 2+3 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 1 \\ 2 & -4 & 5 \\ 4 & 2 & 5 \end{bmatrix} \quad (ii)$$

$B + C = C + B$ *From (i) and (ii)*

5(iv) $\mathbf{A + (B + A) = 2A + B}$

$\text{L.H.S} = A + (B + A)$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \left(\begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 1+1 & -1+2 & 1+3 \\ 2+2 & -2+3 & 2+1 \\ 3+1 & 1-1 & 3+0 \end{bmatrix}$$

$$=\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}+\begin{bmatrix}2 & 1 & 4\\ 4 & 1 & 3\\ 4 & 0 & 3\end{bmatrix}$$

$$=\begin{bmatrix}1+2 & 2+1 & 3+4\\ 2+4 & 3+1 & 1+3\\ 1+4 & -1+0 & 0+3\end{bmatrix}$$

$$=\begin{bmatrix}3 & 3 & 7\\ 6 & 4 & 4\\ 5 & -1 & 3\end{bmatrix} \quad (i)$$

R.H.S $= 2A + B$

$$=2\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}+\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}2(1) & 2(2) & 2(3)\\ 2(2) & 2(3) & 2(1)\\ 2(1) & 2(-1) & 2(0)\end{bmatrix}+\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}2 & 4 & 6\\ 4 & 6 & 2\\ 2 & -2 & 0\end{bmatrix}+\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}2+1 & 4-1 & 6+1\\ 4+2 & 6-2 & 2+2\\ 2+3 & -2+1 & 0+3\end{bmatrix}$$

$$=\begin{bmatrix}3 & 3 & 7\\ 6 & 4 & 4\\ 5 & -1 & 3\end{bmatrix} \quad (ii)$$

$A + (B + A) = 2A + B$

5 (v) $\quad \mathbf{(C - B) + A = C + (A - B)}$

L.H.S $= (C - B) + A$

$$=\left(\begin{bmatrix}-1 & 0 & 0\\ 0 & -2 & 3\\ 1 & 1 & 2\end{bmatrix}-\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}\right)+\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}$$

$$=\begin{bmatrix} -1-1 & 0+1 & 0-1 \\ 0-2 & -2+2 & 3-2 \\ 1-3 & 1-1 & 2-3 \end{bmatrix}+\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$=\begin{bmatrix} -2 & 1 & -1 \\ -2 & 0 & 1 \\ -2 & 0 & -1 \end{bmatrix}+\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$=\begin{bmatrix} -2+1 & 1+2 & -1+3 \\ -2+2 & 0+3 & 1+1 \\ -2+1 & 0-1 & -1+0 \end{bmatrix}=\begin{bmatrix} -1 & 3 & 2 \\ 0 & 3 & 2 \\ -1 & -1 & -1 \end{bmatrix} \quad (i)$$

$$\text{R.H.S} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix}+\left(\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix}-\begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}\right)$$

$$=\begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix}+\begin{bmatrix} 1-1 & 2+1 & 3-1 \\ 2-2 & 3+2 & 1-2 \\ 1-3 & -1-1 & 0-3 \end{bmatrix}$$

$$=\begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix}+\begin{bmatrix} 0 & 3 & 2 \\ 0 & 5 & -1 \\ -2 & -2 & -3 \end{bmatrix}$$

$$=\begin{bmatrix} -1+0 & 0+3 & 0+2 \\ 0+0 & -2+5 & 3-1 \\ 1-2 & 1-2 & 2-3 \end{bmatrix}$$

$$=\begin{bmatrix} -1 & 3 & 2 \\ 0 & 3 & 2 \\ -1 & -1 & -1 \end{bmatrix} \quad (ii)$$

$(C-B)+A=C+(A-B)$ *From (i) and (ii)*

5 (vi) $2A+B=A+(A+B)$

L.H.S $=2A+B$

$$=2\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix}+\begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$=\begin{bmatrix}2(1) & 2(2) & 2(3)\\ 2(2) & 2(3) & 2(1)\\ 2(1) & 2(-1) & 2(0)\end{bmatrix}+\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}2 & 4 & 6\\ 4 & 6 & 2\\ 2 & -2 & 0\end{bmatrix}+\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}2+1 & 4-1 & 6+1\\ 4+2 & 6-2 & 2+2\\ 2+3 & -2+1 & 0+3\end{bmatrix}=\begin{bmatrix}3 & 3 & 7\\ 6 & 4 & 4\\ 5 & -1 & 3\end{bmatrix} \quad (i)$$

R.H.S. $= A + (A + B)$

$$=\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}+\left(\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}+\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}\right)$$

$$=\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}+\begin{bmatrix}1+1 & 2-1 & 3+1\\ 2+2 & 3-2 & 1+2\\ 1+3 & -1+1 & 0+3\end{bmatrix}$$

$$=\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}+\begin{bmatrix}2 & 1 & 4\\ 4 & 1 & 3\\ 4 & 0 & 3\end{bmatrix}$$

$$=\begin{bmatrix}1+2 & 2+1 & 3+4\\ 2+4 & 3+1 & 1+3\\ 1+4 & -1+0 & 0+3\end{bmatrix}=\begin{bmatrix}3 & 3 & 7\\ 6 & 4 & 4\\ 5 & -1 & 3\end{bmatrix} \quad (ii)$$

$2A + B = A + (A + B)$ *From (i) and (ii)*

5 (vii) $(C - B) - A = (C - A) - B$

L.H.S. $= (C - B) - A$

$$=\left(\begin{bmatrix}-1 & 0 & 0\\ 0 & -2 & 3\\ 1 & 1 & 2\end{bmatrix}-\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}\right)-\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}-1-1 & 0+1 & 0-1\\ 0-2 & -2+2 & 3-2\\ 1-3 & 1-1 & 2-3\end{bmatrix}-\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}-2 & 1 & -1\\ -2 & 0 & 1\\ -2 & 0 & -1\end{bmatrix}-\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}-2-1 & 1-2 & -1-3\\ -2-2 & 0-3 & 1-1\\ -2-1 & 0+1 & -1-0\end{bmatrix}=\begin{bmatrix}-3 & -1 & -4\\ -4 & -3 & 0\\ -3 & 1 & -1\end{bmatrix} \quad (i)$$

R.H.S. = (C – A) – B

$$=\left(\begin{bmatrix}-1 & 0 & 0\\ 0 & -2 & 3\\ 1 & 1 & 2\end{bmatrix}-\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}\right)-\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}-1-1 & 0-2 & 0-3\\ 0-2 & -2-3 & 3-1\\ 1-1 & 1+1 & 2-0\end{bmatrix}-\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}-2 & -2 & -3\\ -2 & -5 & 2\\ 0 & 2 & 2\end{bmatrix}-\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}-2-1 & -2+1 & -3-1\\ -2-2 & -5+2 & 2-2\\ 0-3 & 2-1 & 2-3\end{bmatrix}=\begin{bmatrix}-3 & -1 & -4\\ -4 & -3 & 0\\ -3 & 1 & -1\end{bmatrix} \quad (ii)$$

(C – B) – A = (C – A) – B *From (i) and (ii)*

5 (viii) (A + B) + C = A + (B + C)

L.H.S. = (A + B) + C

$$=\left(\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}+\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}\right)+\begin{bmatrix}-1 & 0 & 0\\ 0 & -2 & 3\\ 1 & 1 & 2\end{bmatrix}$$

$$=\begin{bmatrix}1+1 & 2-1 & 3+1\\ 2+2 & 3-2 & 1+2\\ 1+3 & -1+1 & 0+3\end{bmatrix}+\begin{bmatrix}-1 & 0 & 0\\ 0 & -2 & 3\\ 1 & 1 & 2\end{bmatrix}$$

$$=\begin{bmatrix}2 & 1 & 4\\ 4 & 1 & 3\\ 4 & 0 & 3\end{bmatrix}+\begin{bmatrix}-1 & 0 & 0\\ 0 & -2 & 3\\ 1 & 1 & 2\end{bmatrix}$$

$$=\begin{bmatrix} 2-1 & 1+0 & 4+0 \\ 4+0 & 1-2 & 3+3 \\ 4+1 & 0+1 & 3+2 \end{bmatrix}=\begin{bmatrix} 1 & 1 & 4 \\ 4 & -1 & 6 \\ 5 & 1 & 5 \end{bmatrix} \quad (i)$$

R.H.S. = A + (B + C)

$$=\left(\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix}+\begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}\right)+\begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$$

$$=\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix}+\begin{bmatrix} 1-1 & -1+0 & 1+0 \\ 2+0 & -2-2 & 2+3 \\ 3+1 & 1+1 & 3+2 \end{bmatrix}$$

$$=\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix}+\begin{bmatrix} 0 & -1 & 1 \\ 2 & -4 & 5 \\ 4 & 2 & 5 \end{bmatrix}$$

$$=\begin{bmatrix} 1+0 & 2-1 & 3+1 \\ 2+2 & 3-4 & 1+5 \\ 1+4 & -1+2 & 0+5 \end{bmatrix}=\begin{bmatrix} 1 & 1 & 4 \\ 4 & -1 & 6 \\ 5 & 1 & 5 \end{bmatrix} \quad (ii)$$

(A + B) + C = A + (B + C) *From (i) and (ii)*

5 (ix) A + (B − C) = (A − C) + B

L.H.S. = A + (B − C)

$$=\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix}+\left(\begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}-\begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix}\right)$$

$$=\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix}+\begin{bmatrix} 1+1 & -1-0 & 1-0 \\ 2-0 & -2+2 & 2-3 \\ 3-1 & 1-1 & 3-2 \end{bmatrix}$$

$$=\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix}+\begin{bmatrix} 2 & -1 & 1 \\ 2 & 0 & -1 \\ 2 & 0 & 1 \end{bmatrix}$$

$$=\begin{bmatrix} 1+2 & 2-1 & 3+1 \\ 2+2 & 3+0 & 1-1 \\ 1+2 & -1+0 & 0+1 \end{bmatrix}=\begin{bmatrix} 3 & 1 & 4 \\ 4 & 3 & 0 \\ 3 & -1 & 1 \end{bmatrix} \quad (i)$$

R.H.S. = (A − C) + B

$$= \left(\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} - \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix}\right) + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1 & 2-0 & 3-0 \\ 2-0 & 3+2 & 1-3 \\ 1-1 & -1-1 & 0-2 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 & 3 \\ 2 & 5 & -2 \\ 0 & -2 & -2 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2+1 & 2-1 & 3+1 \\ 2+2 & 5-2 & -2+2 \\ 0+3 & -2+1 & -2+3 \end{bmatrix} = \begin{bmatrix} 3 & 1 & 4 \\ 4 & 3 & 0 \\ 3 & -1 & 1 \end{bmatrix} \quad (ii)$$

$A + (B - C) = (A - C) + B$ *From (i) and (ii)*

5 (x) $\mathbf{2A + 2B = 2\ (A + B)}$

L.H.S. $= 2A + 2B$

$$= 2\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + 2\begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2(1) & 2(2) & 2(3) \\ 2(2) & 2(3) & 2(1) \\ 2(1) & 2(-1) & 2(0) \end{bmatrix} + \begin{bmatrix} 2(1) & 2(-1) & 2(1) \\ 2(2) & 2(-2) & 2(2) \\ 2(3) & 2(1) & 2(3) \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 4 & 6 \\ 4 & 6 & 2 \\ 2 & -2 & 0 \end{bmatrix} + \begin{bmatrix} 2 & -2 & 2 \\ 4 & -4 & 4 \\ 6 & 2 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 2+2 & 4-2 & 6+2 \\ 4+4 & 6-4 & 2+4 \\ 2+6 & -2+2 & 0+6 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 8 \\ 8 & 2 & 6 \\ 8 & 0 & 6 \end{bmatrix} \quad (i)$$

R.H.S. $= 2\ (A + B)$

$$= 2\left(\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}\right)$$

$$= 2\begin{bmatrix} 1+1 & 2-1 & 3+1 \\ 2+2 & 3-2 & 1+2 \\ 1+3 & -1+1 & 0+3 \end{bmatrix}$$

$$= 2\begin{bmatrix} 2 & 1 & 4 \\ 4 & 1 & 3 \\ 4 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2(2) & 2(1) & 2(4) \\ 2(4) & 2(1) & 2(3) \\ 2(4) & 2(0) & 2(3) \end{bmatrix} = \begin{bmatrix} 4 & 2 & 8 \\ 8 & 2 & 6 \\ 8 & 0 & 6 \end{bmatrix} \quad (ii)$$

$2A + 2B = 2(A + B)$ *From (i) and (ii)*

6. If $A = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 7 \\ -3 & 8 \end{bmatrix}$ *then find :*

(i) $3A - 2B$ (ii) $2A^t - 3B^t$

6 (i) $3A - 2B$

$$= 3\begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix} - 2\begin{bmatrix} 0 & 7 \\ -3 & 8 \end{bmatrix} \text{ Putting values of } A \text{ and } B$$

$$= \begin{bmatrix} 3(1) & 3(-2) \\ 3(3) & 3(4) \end{bmatrix} - \begin{bmatrix} 2(0) & 2(7) \\ 2(-3) & 2(8) \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -6 \\ 9 & 12 \end{bmatrix} - \begin{bmatrix} 0 & 14 \\ -6 & 16 \end{bmatrix}$$

$$= \begin{bmatrix} 3-0 & -6-14 \\ 9+6 & 12-16 \end{bmatrix} = \begin{bmatrix} 3 & -20 \\ 15 & -4 \end{bmatrix}$$

6 (ii) $2A^t - 3B^t$

$$A = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix} \text{ We find } A^t \text{ and } B^t$$

$$A^t = \begin{bmatrix} 1 & 3 \\ -2 & 4 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 7 \\ -3 & 8 \end{bmatrix}$$

$$B^t = \begin{bmatrix} 0 & -3 \\ 7 & 8 \end{bmatrix}$$

$$2A^t - 3B^t$$

$$= 2\begin{bmatrix} 1 & 3 \\ -2 & 4 \end{bmatrix} - 3\begin{bmatrix} 0 & -3 \\ 7 & 8 \end{bmatrix} \text{ (Putting values } A^t \text{ and } B^t\text{)}$$

$$= \begin{bmatrix} 2(1) & 2(3) \\ 2(-2) & 2(4) \end{bmatrix} - \begin{bmatrix} 3(0) & 3(-3) \\ 3(7) & 3(8) \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 6 \\ -4 & 8 \end{bmatrix} - \begin{bmatrix} 0 & -9 \\ 21 & 24 \end{bmatrix}$$

$$= \begin{bmatrix} 2-0 & 6+9 \\ -4-21 & 8-24 \end{bmatrix} = \begin{bmatrix} 2 & 15 \\ -25 & -16 \end{bmatrix}$$

7. *If* $2\begin{bmatrix} 2 & 4 \\ -3 & a \end{bmatrix} + 3\begin{bmatrix} 1 & b \\ 8 & -4 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 18 & 1 \end{bmatrix}$, *then find a and b.*

$$\begin{bmatrix} 7 & 10 \\ 18 & 1 \end{bmatrix} = 2\begin{bmatrix} 2 & 4 \\ -3 & a \end{bmatrix} + 3\begin{bmatrix} 1 & b \\ 8 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 2(2) & 2(4) \\ 2(-3) & 2a \end{bmatrix} + \begin{bmatrix} 3(1) & 3b \\ 3(8) & 3(-4) \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 8 \\ -6 & 2a \end{bmatrix} + \begin{bmatrix} 3 & 3b \\ 24 & -12 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 10 \\ 18 & 1 \end{bmatrix} = \begin{bmatrix} 4+3 & 8+3b \\ -6+24 & 2a-12 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 10 \\ 18 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 8+3b \\ 18 & 2a-12 \end{bmatrix}$$

Therefore $8 + 3b = 10$ (Corresponding elements are same)

$$3b = 2$$

$$b = \frac{2}{3}$$

and $1 = 2a - 12$

$$13 = 2a$$

$\therefore$ $a = \frac{13}{2}$

8. *If* $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$ *then verify that*

(i) $(A + B)^t = A^t + B^t$ (ii) $(A - B)^t = A^t - B^t$

(iii) $A + A^t$ is symmetric (iv) $A - A^t$ is skew symmetric

(v) $B + B^t$ is symmetric (vi) $B - B^t$ is skew symmetric

8 (i) $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$

Solution:

$(A + B)^t = A^t + B^t$

$(A + B)^t = ?$ (Taking L.H.S)

$A + B = ?$ (Taking L.H.S)

$$A+B=\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}+\begin{bmatrix}1 & 1\\2 & 0\end{bmatrix}$$

$$=\begin{bmatrix}1+1 & 2+1\\0+2 & 1+0\end{bmatrix}$$

$$A+B=\begin{bmatrix}2 & 3\\2 & 1\end{bmatrix}$$

$$\therefore \quad (A+B)^t=\begin{bmatrix}2 & 2\\3 & 1\end{bmatrix} \qquad \text{(i)}$$

$$A^t+B^t = ? \qquad \text{(Taking R.H.S)}$$

$$A^t=\begin{bmatrix}1 & 0\\2 & 1\end{bmatrix} \text{ and}$$

$$B^t=\begin{bmatrix}1 & 2\\1 & 0\end{bmatrix}$$

$$A^t+B^t=\begin{bmatrix}1 & 0\\2 & 1\end{bmatrix}+\begin{bmatrix}1 & 2\\1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}1+1 & 0+2\\2+1 & 1+0\end{bmatrix}=\begin{bmatrix}2 & 2\\3 & 1\end{bmatrix} \quad (ii)$$

$$(A+B)^t=A^t+B^t \qquad \text{From (i) and (ii)}$$

8 (ii) If $A=\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}$, $B=\begin{bmatrix}1 & 1\\2 & 0\end{bmatrix}$

Prove that $(A-B)^t=A^t-B^t$

$$\text{L.H.S}=A-B=\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}-\begin{bmatrix}1 & 1\\2 & 0\end{bmatrix}$$

$$=\begin{bmatrix}1-1 & 2-1\\0-2 & 1-0\end{bmatrix}=\begin{bmatrix}0 & 1\\-2 & 1\end{bmatrix}$$

$$(A-B)^t=\begin{bmatrix}0 & -2\\1 & 1\end{bmatrix}$$

R.H.S $A=\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}$

$$A^t=\begin{bmatrix}1 & 0\\2 & 1\end{bmatrix};\ B=\begin{bmatrix}1 & 1\\2 & 0\end{bmatrix},\ B^t=\begin{bmatrix}1 & 2\\1 & 0\end{bmatrix}$$

$$A^t-B^t=\begin{bmatrix}1 & 0\\2 & 1\end{bmatrix}-\begin{bmatrix}1 & 2\\1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}1-1 & 0-2\\2-1 & 1-0\end{bmatrix}=\begin{bmatrix}0 & -2\\1 & 1\end{bmatrix}\ (ii)$$

$(A-B)^t=A^t-B^t$ From (i) and (ii)

8 (iii) If $A=\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}$

Show $A+A^t$ is a symmetric matrix.

$$A^t=\begin{bmatrix}1 & 0\\2 & 1\end{bmatrix}$$

$$A+A^t=\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}+\begin{bmatrix}1 & 0\\2 & 1\end{bmatrix}$$

$$= \begin{bmatrix} 1+1 & 2+0 \\ 0+2 & 1+1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$$

$$(A+A^t)^t = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$$

So $A + A^t$ is a symmetric matrix.

Note: If $M = M^t$ then M is a symmetric matrix.

8 (iv) If $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

Show that $A - A^t$ is symmetric matrix.

$$A^t = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$A - A^t = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1-1 & 2-0 \\ 0-2 & 1-1 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

Now $(A-A^t)^t = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix} \neq A - A^t$

$\therefore$ $A - A^t$ is Not a symmetric matrix.

8 (v) If $B = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$

Show $B + B^t$ is a symmetric matrix.

$$B^t = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$$

$$B + B^t = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1 & 1+2 \\ 2+1 & 0+0 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 3 & 0 \end{bmatrix}$$

$$(B + B^t)^t = \begin{bmatrix} 2 & 3 \\ 3 & 0 \end{bmatrix}$$

Now $B + B^t = (B + B^t)^t$

$\therefore$ $B + B^t$ is a symmetric matrix.

8 (vi) $B = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$

$$B^t = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$$

$$B - B^t = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1-1 & 1-2 \\ 2-1 & 0-0 \end{bmatrix}$$

$$=\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$(B-B^t)^t=\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\neq B-B^t$$

$B-B^t$ is Not a symmetric matrix.

Multiplication of Matrices

(i) Two matrices are conformable for multiplication if the number of columns of the first matrix is equal to the number of rows of the second matrix.

(ii) **Association Law of multiplication**

If A, B and C are three matrices conformable for multiplication then associative law of multiplication is:

(AB) C = A (BC)

(iii) **Distribution Laws of Multiplication over Addition and Subtraction.**

i. A (B + C) = AB + AC
ii. (A + B) C = AC + BC
iii. A (B – C) = AB – AC
iv. (A – B) C = AC – BC

(iv) **Multiplication Identity of a matrix. If A is a matrix and B another matrix. B is called the identity matrix of A under multiplication if:**

AB = A = BA

(v) **Singular and Non – singular Matrix**

A square matrix A called singular

if |A| = 0 and if |A| ≠ 0 then

A is called non – singular matrix

(vi) Adjoint of a Matrix

Adjoint of a square matrix is obtained by interchanging the diagonal entries and changing the sign of other entries. Adjoint of matrix A is denoted as Adj A.

(vii) Multiplication inverse of a non – singular square matrix if A and B will multiplication of each other if:

$$AB = BA = I$$

Multiplication inverse of A is written as A^{-1}

(viii) Inverse of matrix using Adjoint.

Let M be a square matrix

Then $$M^{-1} = \frac{Adj\ M}{|M|}$$

Exercise 1.4

1. Which of the following product of matrices is conformable for multiplication?

(i) $\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} -2 \\ 3 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$

(iii) $\begin{bmatrix} 1 \\ -1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}$ (iv) $\begin{bmatrix} 1 & 2 \\ 0 & -1 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \end{bmatrix}$

(v) $\begin{bmatrix} 3 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 0 & 2 \\ -2 & 3 \end{bmatrix}$

1 (i) Let $A = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}, B = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

Number of columns of A = 2
Number of rows of B = 2
Multiplication is possible.

1 (ii) Let $A = \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$

Number of columns in A = 2
Number of rows in B = 2
Multiplication is possible.

1 (iii) Let $A = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}$

Number of columns in A = 1
Number of rows in B = 2
Multiplication is NOT possible.

1 (iv) Let $A=\begin{bmatrix}1 & 2\\ 0 & -1\\ -1 & -2\end{bmatrix}, B=\begin{bmatrix}1 & 0 & -1\\ 0 & 1 & 2\end{bmatrix}$

Number of columns in A = 2
Number of rows in B = 2
Multiplication is possible.

1 (v) Let $A=\begin{bmatrix}3 & 2 & 1\\ 0 & 1 & -1\end{bmatrix}, B=\begin{bmatrix}1 & -1\\ 0 & 2\\ -2 & 3\end{bmatrix}$

Number of columns in A = 3
Number of rows in B = 3
Multiplication AB is possible.

2. If $A=\begin{bmatrix}3 & 0\\ -1 & 2\end{bmatrix}, B=\begin{bmatrix}6\\ 5\end{bmatrix}$ find (i)AB (ii) BA (if possible)

2(i) $AB=\begin{bmatrix}3 & 0\\ -1 & 2\end{bmatrix}\begin{bmatrix}6\\ 5\end{bmatrix}$

$$=\begin{bmatrix}3(6) & +0(5)\\ -1(6) & +2(5)\end{bmatrix}=\begin{bmatrix}18+0\\ -6+10\end{bmatrix}=\begin{bmatrix}18\\ 4\end{bmatrix}$$

2(ii) BA is NOT possible.

3. **Find the following products.**

(i) $\begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}4\\ 0\end{bmatrix}$ (ii) $\begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}5\\ -4\end{bmatrix}$

(iii) $\begin{bmatrix}-3 & 0\end{bmatrix}\begin{bmatrix}4\\ 0\end{bmatrix}$ (iv) $\begin{bmatrix}6 & -0\end{bmatrix}\begin{bmatrix}4\\ 0\end{bmatrix}$

(v) $\begin{bmatrix}1 & 2\\ -3 & 0\\ 6 & -1\end{bmatrix}\begin{bmatrix}4 & 5\\ 0 & -4\end{bmatrix}$

Solution:

3 (i) $\begin{bmatrix} 1 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \end{bmatrix}$

$= \begin{bmatrix} 1(4)+2(0) \end{bmatrix} = \begin{bmatrix} 4+0 \end{bmatrix} = \begin{bmatrix} 4 \end{bmatrix}$

3 (ii) $\begin{bmatrix} 1 & 2 \end{bmatrix}\begin{bmatrix} 5 \\ -4 \end{bmatrix}$

$= \begin{bmatrix} 1(5)+2(-4) \end{bmatrix} = \begin{bmatrix} 5-8 \end{bmatrix} = \begin{bmatrix} -3 \end{bmatrix}$

3 (iii) $\begin{bmatrix} -3 & 0 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \end{bmatrix}$

$\begin{bmatrix} -3(4)+0(0) \end{bmatrix} = \begin{bmatrix} -12+0 \end{bmatrix} = \begin{bmatrix} -12 \end{bmatrix}$

3 (iv) $\begin{bmatrix} 6 & 0 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \end{bmatrix}$

$AB = \begin{bmatrix} 6(4)+0(0) \end{bmatrix} = \begin{bmatrix} 24+0 \end{bmatrix} = \begin{bmatrix} 24 \end{bmatrix}$

3 (v) $\begin{bmatrix} 1 & 2 \\ -3 & 0 \\ 6 & -1 \end{bmatrix}\begin{bmatrix} 4 & 5 \\ 0 & -4 \end{bmatrix}$

$$= \begin{bmatrix} 1(4)+2(0) & 1(5)+2(-4) \\ -3(4)+0(0) & -3(5)+0(-4) \\ 6(4)+(-1)(0) & 6(5)+(-1)(-4) \end{bmatrix}$$

$$= \begin{bmatrix} 4+0 & 5-8 \\ -12+0 & -15+0 \\ 24+0 & 30+8 \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ -12 & -15 \\ 24 & 38 \end{bmatrix}$$

4. **Multiply the following matrices.**

(*a*) $\begin{bmatrix} 2 & 3 \\ 1 & 1 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 3 & 0 \end{bmatrix}$ (*b*) $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ -1 & 1 \end{bmatrix}$

(c) $\begin{bmatrix}1 & 2\\3 & 4\\-1 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}$ (d) $\begin{bmatrix}8 & 5\\6 & 4\end{bmatrix}\begin{bmatrix}2 & -\frac{5}{2}\\-4 & 4\end{bmatrix}$

(e) $\begin{bmatrix}-1 & 2\\1 & 3\end{bmatrix}\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$

4 (a) Solution:

$$\begin{bmatrix}2 & 3\\1 & 1\\0 & -2\end{bmatrix}\begin{bmatrix}2 & -1\\3 & 0\end{bmatrix}$$

$$=\begin{bmatrix}2(2)+3(3) & 2(-1)+3(0)\\1(2)+1(3) & 1(-1)+1(0)\\0(2)+(-2)(3) & 0(-1)+(-2)(0)\end{bmatrix}$$

$$=\begin{bmatrix}4+9 & -2+0\\2+3 & -1+0\\0-6 & 0+0\end{bmatrix}=\begin{bmatrix}13 & -2\\5 & -1\\-6 & 0\end{bmatrix}$$

4 (b) $\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}\begin{bmatrix}1 & 2\\3 & 4\\-1 & 1\end{bmatrix}$

$$=\begin{bmatrix}1(1)+2(3)+3(-1) & 1(2)+2(4)+3(1)\\4(1)+5(3)+6(-1) & 4(2)+5(4)+6(1)\end{bmatrix}$$

$$=\begin{bmatrix}1+6-3 & 2+8+3\\4+15-6 & 8+20+6\end{bmatrix}$$

$$=\begin{bmatrix}4 & 13\\13 & 34\end{bmatrix}$$

4 (c) $\begin{bmatrix}1 & 2\\3 & 4\\-1 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}$

$$=\begin{bmatrix} 1(1)+2(4) & 1(2)+2(5) & 1(3)+2(6) \\ 3(1)+4(4) & 3(2)+4(5) & 3(3)+4(6) \\ -1(1)+1(4) & -1(2)+1(5) & -1(3)+1(6) \end{bmatrix}$$

$$=\begin{bmatrix} 1+8 & 2+10 & 3+12 \\ 3+16 & 6+20 & 9+24 \\ -1+4 & -2+5 & -3+6 \end{bmatrix}$$

$$=\begin{bmatrix} 9 & 12 & 15 \\ 19 & 26 & 33 \\ 3 & 3 & 3 \end{bmatrix}$$

4 (d) $\begin{bmatrix} 8 & 5 \\ 6 & 4 \end{bmatrix} \begin{bmatrix} 2 & -\frac{5}{2} \\ -4 & 4 \end{bmatrix}$

$$=\begin{bmatrix} 8(2)+5(-4) & 8\left(-\frac{5}{2}\right)+5(4) \\ 6(2)+4(-4) & 6\left(-\frac{5}{2}\right)+4(4) \end{bmatrix}$$

$$=\begin{bmatrix} 16-20 & -20+20 \\ 12-16 & -15+16 \end{bmatrix}$$

$$=\begin{bmatrix} -4 & 0 \\ -4 & 1 \end{bmatrix}$$

4 (e) $\begin{bmatrix} -1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$$=\begin{bmatrix} -1(0)+2(0) & -1(0)+2(0) \\ 1(0)+3(0) & 1(0)+3(0) \end{bmatrix}$$

$$=\begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix}=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

5. Let $A=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}$, $B=\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}$ and $C=\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}$

verify whether:

(i) AB = BA (ii) A(BC) = (AB) C

(iii) A(B + C) = AB + AC (iv) A(B – C) = AB – AC

5(i) $A=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}, B=\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}$

$$AB=\begin{bmatrix}-1(1)+3(-3) & -1(2)+3(-5)\\ 2(1)+0(-3) & 2(2)+0(-5)\end{bmatrix}$$

$$=\begin{bmatrix}-1-9 & -2-15\\ 2+0 & 4+0\end{bmatrix}=\begin{bmatrix}-10 & -17\\ 2 & 4\end{bmatrix} \quad (i)$$

$$and \quad BA=\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}$$

$$=\begin{bmatrix}1(-1)+2(2) & 1(3)+2(0)\\ -3(-1)+(-5)(2) & -3(3)+(-5)(0)\end{bmatrix}$$

$$=\begin{bmatrix}-1+4 & 3+0\\ 3-10 & -9+0\end{bmatrix}=\begin{bmatrix}3 & 3\\ -7 & -9\end{bmatrix} \quad (ii)$$

$$AB \neq BA \quad From\,(i)\,and\,(ii)$$

5(ii) **Prove that A (BC) = (AB) C**

$$A=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}, B=\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}, C=\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}$$

A(BC) = ?

BC (First we find BC)

$$=\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}1(2)+2(1) & 1(1)+2(3)\\ -3(2)+(-5)(1) & -3(1)+(-5)(3)\end{bmatrix}$$

$$BC = \begin{bmatrix} 2+2 & 1+6 \\ -6-5 & -3-15 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 7 \\ -11 & -18 \end{bmatrix}$$

Now $A(BC) = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 4 & 7 \\ -11 & -18 \end{bmatrix}$

$$= \begin{bmatrix} -1(4)+3(-11) & -1(7)+3(-18) \\ 2(4)+0(-11) & 2(7)+0(-18) \end{bmatrix}$$

$$= \begin{bmatrix} -4-33 & -7-54 \\ 8+0 & 14+0 \end{bmatrix}$$

$$= \begin{bmatrix} -37 & -61 \\ 8 & 14 \end{bmatrix} \quad (i)$$

$(AB)C = ?$

$$AB = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix} \quad \textit{First we find } AB$$

$$= \begin{bmatrix} -1(1)+3(-3) & -1(2)+3(-5) \\ 2(1)+0(-3) & 2(2)+0(-5) \end{bmatrix}$$

$$= \begin{bmatrix} -1-9 & -2-15 \\ 2+0 & 4+0 \end{bmatrix}$$

$$= \begin{bmatrix} -10 & -17 \\ 2 & 4 \end{bmatrix}$$

Now, we find (AB)C

$$(AB)C = \begin{bmatrix} -10 & -17 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -10(2)+(-17)(1) & -10(1)+(-17)(3) \\ 2(2)+4(1) & 2(1)+4(3) \end{bmatrix}$$

$$=\begin{bmatrix} -20-17 & -10-51 \\ 4+4 & 2+12 \end{bmatrix}$$

$$=\begin{bmatrix} -37 & -61 \\ 8 & 14 \end{bmatrix} \quad (ii)$$

A(BC) = (AB) C From (i) and (ii)

5(iii) Prove that A(B +C) = AB + AC

A(B + C) = ?

(B + C) … First we find (B + C)

$$=\begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}+\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$

$$=\begin{bmatrix} 1+2 & 2+1 \\ -3+1 & -5+3 \end{bmatrix}$$

$$=\begin{bmatrix} 3 & 3 \\ -2 & -2 \end{bmatrix}$$

$$A(B+C) = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 3 & 3 \\ -2 & -2 \end{bmatrix}$$

$$=\begin{bmatrix} -1(3)+3(-2) & -1(3)+3(-2) \\ 2(3)+0(-2) & 2(3)+0(-2) \end{bmatrix}$$

$$=\begin{bmatrix} -3-6 & -3-6 \\ 6+0 & 6+0 \end{bmatrix}$$

$$=\begin{bmatrix} -9 & -9 \\ 6 & 6 \end{bmatrix} \quad (i)$$

AB + AC = ? Taking L.H.S.

$$AB = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}$$

$$=\begin{bmatrix} -1(1)+3(-3) & -1(2)+3(-5) \\ 2(1)+0(-3) & 2(2)+0(-5) \end{bmatrix}$$

$$A^{-1} = \frac{\text{Adj } A}{|A|}$$

$$= \frac{\begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}}{8} = \begin{bmatrix} \frac{2}{8} & \frac{0}{8} \\ \frac{1}{8} & \frac{4}{8} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{4} & 0 \\ \frac{1}{8} & \frac{1}{2} \end{bmatrix}$$

$$B^{-1}A^{-1} = \begin{bmatrix} -\frac{1}{6} & \frac{1}{3} \\ -\frac{1}{6} & -\frac{2}{3} \end{bmatrix} \begin{bmatrix} \frac{1}{4} & 0 \\ \frac{1}{8} & \frac{1}{2} \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{1}{6}(\frac{1}{4}) + \frac{1}{3}(\frac{1}{8}) & -\frac{1}{6}(0) + (\frac{1}{3})(\frac{1}{2}) \\ -\frac{1}{6}(\frac{1}{4}) + (-\frac{2}{3})(\frac{1}{8}) & -\frac{1}{6}(0) + (-\frac{2}{3})(\frac{1}{2}) \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{1}{24} + \frac{1}{24} & 0 + \frac{1}{6} \\ -\frac{1}{24} - \frac{1}{12} & 0 - \frac{1}{3} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & \frac{1}{6} \\ -\frac{1}{8} & -\frac{1}{3} \end{bmatrix} \qquad \text{(ii)}$$

$$(AB)^{-1} = B^{-1}A^{-1} \qquad \text{From (i) and (ii)}$$

6(ii) $A = \begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix}, D = \begin{bmatrix} 3 & 1 \\ -2 & 2 \end{bmatrix}$, **verify that:**

$(DA)^{-1} = A^{-1}D^{-1}$

$$(DA) = \begin{bmatrix} 3 & 1 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3(4) + 1(-1) & 3(0) + 1(2) \\ -2(4) + 2(-1) & -2(0) + 2(2) \end{bmatrix}$$

$$= \begin{bmatrix} 12 - 1 & 0 + 2 \\ -8 - 2 & 0 + 4 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 2 \\ -10 & 4 \end{bmatrix}$$

$$|DA| = \begin{vmatrix} 11 & 2 \\ -10 & 4 \end{vmatrix}$$

$$= (11)(4) - (2)(-10)$$

$$= 44 + 20 = 64$$

$$\text{Adj}(DA) = \begin{bmatrix} 4 & -2 \\ 10 & 11 \end{bmatrix}$$

$$(DA)^{-1} = \frac{\text{Adj}(DA)}{|DA|}$$

$$= \frac{\begin{bmatrix} 4 & -2 \\ 10 & 11 \end{bmatrix}}{64} = \begin{bmatrix} \frac{4}{64} & -\frac{2}{64} \\ \frac{10}{64} & \frac{11}{64} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{16} & -\frac{1}{32} \\ \frac{5}{32} & \frac{11}{64} \end{bmatrix} \qquad \text{(i)}$$

$$A = \begin{bmatrix} 4 & 0 \\ -10 & 2 \end{bmatrix}$$

$$|A| = (4)(2) - (-1)(0) = 8 - 0 = 8$$

$$\text{Adj } A = \begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$$

$$A^{-1} = \frac{\text{Adj } A}{|A|} = \frac{\begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}}{8} = \begin{bmatrix} \frac{2}{8} & \frac{0}{8} \\ \frac{1}{8} & \frac{4}{8} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{4} & 0 \\ \frac{1}{8} & \frac{1}{2} \end{bmatrix}$$

$$D = \begin{bmatrix} 3 & 1 \\ -2 & 2 \end{bmatrix}$$

$$|D| = \begin{vmatrix} 3 & 1 \\ -2 & 2 \end{vmatrix}$$

$$\text{Adj } D = \begin{bmatrix} 2 & -1 \\ 2 & 3 \end{bmatrix}$$

$$D^{-1} = \frac{\text{Adj } D}{|D|} = \frac{\begin{bmatrix} 2 & -1 \\ 2 & 3 \end{bmatrix}}{8}$$

$$= \begin{bmatrix} \frac{2}{8} & \frac{-1}{8} \\ \frac{2}{8} & \frac{3}{8} \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & -\frac{1}{8} \\ \frac{1}{4} & \frac{3}{8} \end{bmatrix}$$

$$A^{-1}D^{-1} = \begin{bmatrix} \frac{1}{4} & 0 \\ \frac{1}{8} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} \frac{1}{4} & -\frac{1}{8} \\ \frac{1}{4} & \frac{3}{8} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{4}\left(\frac{1}{4}\right)+0\left(\frac{1}{4}\right) & \frac{1}{4}\left(-\frac{1}{8}\right)+0\left(\frac{3}{8}\right) \\ \frac{1}{8}\left(\frac{1}{4}\right)+\frac{1}{2}\left(\frac{1}{4}\right) & \frac{1}{8}\left(-\frac{1}{8}\right)+\frac{1}{2}\left(\frac{3}{8}\right) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{16}+0 & -\frac{1}{32}+0 \\ \frac{1}{32}+\frac{1}{8} & -\frac{1}{64}+\frac{3}{16} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{16} & -\frac{1}{32} \\ \frac{5}{32} & \frac{11}{64} \end{bmatrix} \qquad \text{(ii)}$$

$(DA)^{-1} = A^{-1}D^{-1}$ From(i), (ii)

SOLUTION OF SIMULTANEOUS LINEAR EQUATIONS

System of two linear equations in two variable in general form are:

$ax + by = m;\ cx + dy = n$ where a, b, c, d, m, n are real numbers.

This system is also called simultaneous linear equations.

Cramer's Rule

Let $\quad ax + by = m$

$\quad cx + dy = n$

writing the equations in matrix form

$$\begin{bmatrix} ax+by \\ cx+dy \end{bmatrix} = \begin{bmatrix} m \\ n \end{bmatrix}$$

or $$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} m \\ n \end{bmatrix}$$

$$AX = B$$

When $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} m \\ n \end{bmatrix}$

$$AX = B$$

$$X = A^{-1}B$$

$$X = \frac{\text{Adj } A}{|A|} \times B \quad [|A| \neq 0]$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}\begin{bmatrix} m \\ n \end{bmatrix}}{|A|}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{\begin{bmatrix} dm & -bn \\ -cm & an \end{bmatrix}}{|A|}$$

Thus $\quad x = \dfrac{dm - bn}{|A|} = \dfrac{|A_x|}{|A|}$

And $\quad y = \dfrac{an - cm}{|A|} = \dfrac{A_y}{|A|}$

When $|A_x| = \begin{vmatrix} m & b \\ n & d \end{vmatrix}$

and $\quad |A_y| = \begin{vmatrix} a & m \\ c & n \end{vmatrix}$

Exercise 1.6

Q.1. Use matrices if possible to solve the following system of linear equations by

(i) The matrix inverse method

(ii) The Cramer's Rule

(i) $2x - 2y = 4$, $3x + 2y = 6$

(ii) $2x + y = 3$, $6x + 5y = 1$

(iii) $4x + 2y = 8$, $3x - y = -1$

(iv) $3x - 2y = -6$, $5x - 2y = -10$

(v) $3x - 2y = 4$, $-6x + 4y = 7$

(vi) $4x + y = 9$, $-3x - y = -5$

(vii) $2x - 2y = 4$, $-5x - 2y = -10$

(viii) $3x - 4y = 4$, $x + 2y = 8$

Solution:-

1(i)

$$2x - 2y = 4$$
$$3x + 2y = 6$$

Writing the equations in matrix form.

$$\begin{bmatrix} 2x & -2y \\ 3x & +2y \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

or
$$\begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

Let $AX = B$

Where $A = \begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$

$$|A| = \begin{vmatrix} 2 & -2 \\ 3 & 2 \end{vmatrix}$$
$$= (2)(2) - (-2)(3)$$
$$= 4 + 6 = 10 \neq 0$$

A is a non-singular matrix. Its multiplicative inverse is A^{-1}

Now $AX = B$

or $X = A^{-1}B$

$$\text{Adj } A = \begin{bmatrix} -2 & -2 \\ -3 & 2 \end{bmatrix}, \quad |A| = -10$$

$$A^{-1} = \frac{\text{Adj } A}{|A|}$$

$$= \frac{\begin{bmatrix} -2 & -2 \\ -3 & 2 \end{bmatrix}}{-10}$$

$$= \begin{bmatrix} \frac{-2}{-10} & \frac{-2}{-10} \\ \frac{-3}{-10} & \frac{2}{-10} \end{bmatrix} = \begin{bmatrix} \frac{1}{5} & \frac{1}{5} \\ \frac{3}{10} & -\frac{1}{5} \end{bmatrix}$$

Now $X = A^{-1}B$

$$\therefore \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{5} & \frac{1}{5} \\ \frac{3}{10} & -\frac{1}{5} \end{bmatrix} \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{5}(4) + \frac{1}{5}(6) \\ \frac{3}{10}(4) - \frac{1}{5}(6) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{4}{5} + \frac{6}{5} \\ \frac{6}{5} - \frac{6}{5} \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

So, $x = 2, y = 0$

Using Cramer's Rule:

$$2x + 2y = 4$$
$$3x - 2y = 6$$

$$A = \begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}$$

$$A_x = \begin{bmatrix} 4 & -2 \\ 6 & 2 \end{bmatrix}$$

$$A_y = \begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & -2 \\ 3 & 2 \end{vmatrix}$$

$$x = \frac{|A_x|}{|A|} = \frac{\begin{vmatrix} 4 & -2 \\ 6 & 2 \end{vmatrix}}{10}$$

$$= \frac{(4)(2)-(-2)(6)}{10}$$

$$= \frac{8+12}{10} = 2$$

$$y = \frac{|A_y|}{|A|} = \frac{\begin{vmatrix} 2 & 4 \\ 3 & 6 \end{vmatrix}}{10}$$

$$= \frac{(2)(6)-(3)(4)}{10}$$

$$= \frac{12-12}{10} = 0$$

So. $x = 2, y = 0$

1(ii) $2x + y = 3$

$6x + 5y = 1$

Writing the equations in the matrix form.

$$\begin{bmatrix} 2x & +y \\ 6x & +5y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

or $$\begin{bmatrix} 2 & 1 \\ 6 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

Let $AX = B$

Where $A = \begin{bmatrix} 2 & 1 \\ 6 & 5 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

$$|A| = \begin{bmatrix} 2 & 1 \\ 6 & 5 \end{bmatrix}$$

$$= (2)(5) - (1)(6) = 10 - 6 = 4$$

A is a non-singular matrix.

$$AX = B$$

$$X = A^{-1}B$$

$$\text{Adj } A = \begin{bmatrix} 5 & -1 \\ -6 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{\text{Adj } A}{|A|} = \frac{\begin{bmatrix} 5 & -1 \\ -6 & 2 \end{bmatrix}}{4}$$

$$\begin{bmatrix} \frac{5}{4} & \frac{-1}{4} \\ -\frac{6}{4} & \frac{2}{4} \end{bmatrix} = \begin{bmatrix} \frac{5}{4} & -\frac{1}{4} \\ -\frac{3}{2} & \frac{1}{2} \end{bmatrix}$$

Now $X = A^{-1}B$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{5}{4} & -\frac{1}{4} \\ -\frac{3}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{5}{4}(3) + \left(-\frac{1}{4}\right)(1) \\ -\frac{3}{2}(3) + \left(\frac{1}{2}\right)(1) \end{bmatrix}$$

$$\begin{bmatrix} \frac{15}{4} & \frac{1}{4} \\ \frac{9}{2} + \frac{1}{2} \end{bmatrix}$$

$$\frac{11}{4}$$

$$x = \frac{7}{2}, y = -4$$

Using Cramer's Rule

$$2x + y = 3$$
$$6x + 5y = 1$$

$$A = \begin{bmatrix} 2 & 1 \\ 6 & 5 \end{bmatrix}$$

$$A_x = \begin{bmatrix} 3 & 1 \\ 1 & 5 \end{bmatrix}$$

$$A_y = \begin{bmatrix} 2 & 3 \\ 6 & 1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & 1 \\ 6 & 5 \end{vmatrix}$$

$$= (2)(5) - (1)(6) = 10 - 6 = 4$$

$$x = \frac{|A_x|}{|A|} = \frac{\begin{vmatrix} 3 & 1 \\ 1 & 5 \end{vmatrix}}{4}$$

$$= \frac{(3)(5) - (1)(1)}{4} = \frac{15 - 1}{4}$$

$$= \frac{14}{4} = \frac{7}{2}$$

$$y = \frac{|A_y|}{|A|} = \frac{\begin{vmatrix} 2 & 3 \\ 6 & 1 \end{vmatrix}}{4}$$

$$= \frac{(2)(1) - (3)(6)}{4} = \frac{2 - 18}{4}$$

$$= \frac{-16}{4} = -4$$

So. $x = \frac{7}{2}, y = -4$

1(iii) $4x + 2y = 8$

$3x - y = -1$

Writing the equations in matrix form

$$\begin{bmatrix} 4x & +2y \\ 3x & -y \end{bmatrix} = \begin{bmatrix} 8 \\ -1 \end{bmatrix}$$

or $$\begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ -1 \end{bmatrix}$$

Let $AX = B$

where $A = \begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 8 \\ -1 \end{bmatrix}$

$$|A| = \begin{vmatrix} 4 & 2 \\ 3 & -1 \end{vmatrix}$$

$$= (4)(-1) - (2)(3)$$

$$= -4 - 6 = -10$$

$$AX = B$$

$$X = A^{-1}B$$

$$\text{Adj } A = \begin{bmatrix} -1 & -2 \\ -3 & 4 \end{bmatrix}$$

$$A^{-1} = \frac{\text{Adj } A}{|A|} = \frac{\begin{bmatrix} -1 & -2 \\ -3 & 4 \end{bmatrix}}{-10}$$

$$= \begin{bmatrix} \frac{-1}{-10} & \frac{-2}{-10} \\ \frac{-3}{-10} & \frac{4}{-10} \end{bmatrix} = \begin{bmatrix} \frac{1}{10} & \frac{1}{5} \\ \frac{3}{10} & \frac{-2}{5} \end{bmatrix}$$

$$X = A^{-1}B$$

$$= \begin{bmatrix} \frac{1}{10} & \frac{1}{5} \\ \frac{3}{10} & \frac{-2}{5} \end{bmatrix} \begin{bmatrix} 8 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{10}(8) + \frac{1}{5}(-1) \\ \frac{3}{10}(8) + \frac{-2}{5}(-1) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{4}{5} - \frac{1}{5} \\ \frac{12}{5} + \frac{2}{5} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{3}{5} \\ \frac{14}{5} \end{bmatrix}$$

So.
$$x = \frac{3}{5}$$
$$y = \frac{14}{5}$$

Using Cramer's Rule:

$$4x + 2y = 8$$
$$3x - y = -1$$

$$A = \begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}$$

$$A_x = \begin{bmatrix} 8 & 2 \\ -1 & -1 \end{bmatrix}$$

$$A_y = \begin{bmatrix} 4 & 8 \\ 3 & -1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 4 & 2 \\ 3 & -1 \end{vmatrix}$$

$$= (4)(-1) - (2)(3)$$

$$= -4 - 6 = -10$$

$$x = \frac{|A_x|}{|A|} = \frac{\begin{vmatrix} 8 & 2 \\ -1 & -1 \end{vmatrix}}{-10}$$

$$= \frac{(8)(-1) - (2)(-1)}{-10}$$

$$= \frac{-8+2}{-10} = \frac{-6}{-10} = \frac{3}{5}$$

$$y = \frac{|A_y|}{|A|} = \frac{\begin{vmatrix} 4 & 8 \\ 3 & -1 \end{vmatrix}}{-10}$$

$$= \frac{(4)(-1) - (8)(3)}{-10} = \frac{-4-24}{-10}$$

$$= \frac{-28}{-10} = \frac{14}{5}$$

So, $x = \frac{3}{5}, y = \frac{14}{5}$

1(iv) $3x - 2y = -6$

$5x - 2y = -10$

Writing the equations in matrix form

$$\begin{bmatrix} 3x & -2y \\ 5x & -2y \end{bmatrix} = \begin{bmatrix} -6 \\ -10 \end{bmatrix}$$

or $$\begin{bmatrix} 3 & -2 \\ 5 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -6 \\ -10 \end{bmatrix}$$

Let AX = B

Where $A = \begin{bmatrix} 3 & -2 \\ 5 & -2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} -6 \\ -10 \end{bmatrix}$

$$|A| = \begin{vmatrix} 3 & -2 \\ 5 & -2 \end{vmatrix}$$

$$= (3)(-2) - (-2)(5)$$

$$= -6 + 10 = 4$$

$$AX = B$$

$$X = A^{-1}B$$

$$\text{Adj } A = \begin{bmatrix} -2 & 2 \\ -5 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{\text{Adj } A}{|A|} = \frac{\begin{bmatrix} -2 & 2 \\ -5 & 3 \end{bmatrix}}{4}$$

$$= \begin{bmatrix} \frac{-2}{4} & \frac{2}{4} \\ \frac{-5}{4} & \frac{3}{4} \end{bmatrix}$$

$$X = A^{-1}B$$

$$= \begin{bmatrix} -\frac{2}{4} & \frac{2}{4} \\ -\frac{5}{4} & \frac{3}{4} \end{bmatrix}\begin{bmatrix} -6 \\ -10 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{2}{4}(-6) & +\frac{2}{4}(-10) \\ -\frac{5}{4}(-6) & +\frac{3}{4}(-10) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{12}{4} - \frac{20}{4} \\ \frac{30}{4} - \frac{30}{4} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -\frac{8}{4} \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$$

So, $x = -2, y = 0$

Using Cramer's Rule:

$$3x - 2y = -6$$
$$5x - 2y = -10$$

$$A = \begin{bmatrix} 3 & -2 \\ 5 & -2 \end{bmatrix}$$

$$A_x = \begin{bmatrix} -6 & -2 \\ -10 & -2 \end{bmatrix}$$

$$A_y = \begin{bmatrix} 3 & -6 \\ 5 & -10 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 3 & -2 \\ 5 & -2 \end{vmatrix}$$

$$= (3)(-2) - (5)(-2)$$
$$= -6 + 10 = 4$$

$$x = \frac{|A_x|}{|A|} = \frac{\begin{vmatrix} -6 & -2 \\ -10 & -2 \end{vmatrix}}{4}$$

$$= \frac{(-6)(-2) - (-2)(-10)}{4}$$

$$= \frac{+12 - 20}{4} = \frac{-8}{4} = -2$$

$$y = \frac{|A_y|}{|A|} = \frac{\begin{vmatrix} 3 & -6 \\ 5 & -10 \end{vmatrix}}{4}$$

$$= \frac{(3)(-10)-(-6)(5)}{4}$$

$$= \frac{-30+30}{4} = 0$$

So, $x = -2,\ y = 0$

1(v) $3x - 2y = 4$

$-6x + 4y = 7$

Writing the equations in matrix form.

$$\begin{bmatrix} 3x & -2y \\ -6x & +4y \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -2 \\ -6 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \end{bmatrix}$$

Let $AX = B$

where $A = \begin{bmatrix} 3 & -2 \\ -6 & 4 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 4 \\ 7 \end{bmatrix}$

$$|A| = \begin{vmatrix} 3 & -2 \\ -6 & 4 \end{vmatrix}$$

$$= (3)(4) - (-2)(-6)$$

$$= 12 - 12 = 0$$

A is a singular matrix, therefore, solution is not possible.

1(vi) $4x + y = 9$

$-3x - y = -5$

Writing the equations in matrix form.

$$\begin{bmatrix} 4x & +y \\ -3x & -y \end{bmatrix} = \begin{bmatrix} 9 \\ -5 \end{bmatrix}$$

or $\begin{bmatrix} 4 & 1 \\ -3 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ -5 \end{bmatrix}$

Let $\quad AX = B$

Where $A = \begin{bmatrix} 4 & 1 \\ -3 & -1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 9 \\ -5 \end{bmatrix}$

$$|A| = \begin{vmatrix} 4 & 1 \\ -3 & -1 \end{vmatrix}$$

$$= (4)(-1) - (1)(-3)$$

$$= -4 + 3 = -1$$

$$AX = B$$

$$X = A^{-1}B$$

$$\text{Adj } A = \begin{bmatrix} -1 & -1 \\ 3 & 4 \end{bmatrix}$$

$$A^{-1} = \frac{\text{Adj } A}{|A|} = \frac{\begin{bmatrix} -1 & -1 \\ 3 & 4 \end{bmatrix}}{-1}$$

$$= \begin{bmatrix} \frac{-1}{-1} & \frac{-1}{-1} \\ \frac{3}{-1} & \frac{4}{-1} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -3 & -4 \end{bmatrix}$$

$$X = A^{-1}B$$

$$= \begin{bmatrix} 1 & 1 \\ -3 & -4 \end{bmatrix} \begin{bmatrix} 9 \\ -5 \end{bmatrix}$$

$$= \begin{bmatrix} 1(9) & +(1)(-5) \\ -3(9) & +(-4)(-5) \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9-5 \\ -27+20 \end{bmatrix} = \begin{bmatrix} 4 \\ -7 \end{bmatrix}$$

So, $\quad x = 4, y = -7$

Using Cramer's Rule:

$$4x + y = 9$$

$$-3x - y = -5$$

$$A = \begin{bmatrix} 4 & 1 \\ -3 & -1 \end{bmatrix}$$

$$A_x = \begin{bmatrix} 9 & 1 \\ -5 & -1 \end{bmatrix}$$

$$A_y = \begin{bmatrix} 4 & 9 \\ -3 & -5 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 4 & 1 \\ -3 & -1 \end{vmatrix}$$

$$= (4)(-1) - (1)(-3)$$

$$= -4 + 3 = -1$$

$$x = \frac{|A_x|}{|A|} = \frac{\begin{vmatrix} 9 & 1 \\ -5 & -1 \end{vmatrix}}{-1}$$

$$= \frac{(9)(-1) - (1)(-5)}{-1}$$

$$= \frac{-9+5}{-1} = \frac{-4}{-1} = 4$$

$$y = \frac{|A_y|}{|A|} = \frac{\begin{vmatrix} 4 & -9 \\ -3 & -5 \end{vmatrix}}{-1}$$

$$= \frac{(4)(-5) - (9)(-3)}{-1}$$

$$= \frac{-20+27}{-1} = \frac{7}{-1} = -7$$

So, $x = 4, y = -7$

1(vii) $2x - 2y = 4$

$-5x - 2y = -10$

Writing the equations in matrix form.

$$\begin{bmatrix} 2x & -2y \\ -5x & -2y \end{bmatrix} = \begin{bmatrix} 4 \\ -10 \end{bmatrix}$$

or $$\begin{bmatrix} 2 & -2 \\ -5 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 10 \end{bmatrix}$$

Let $AX = B$

Where $A = \begin{bmatrix} 2 & -2 \\ -5 & -2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 4 \\ 10 \end{bmatrix}$

$$|A| = \begin{vmatrix} 2 & -2 \\ -5 & -2 \end{vmatrix}$$

$$= (2)(-2) - (-2)(-5)$$

$$= -4 - 10 = -14$$

$$AX = B$$

$$X = A^{-1}B$$

$$Adj\ A = \begin{bmatrix} -2 & 2 \\ 5 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{\text{Adj } A}{|A|} = \frac{\begin{bmatrix} -2 & 2 \\ 5 & 2 \end{bmatrix}}{-14}$$

$$= \begin{bmatrix} \frac{-2}{-14} & \frac{2}{-14} \\ \frac{5}{-14} & \frac{2}{-14} \end{bmatrix} = \begin{bmatrix} \frac{1}{7} & -\frac{1}{7} \\ -\frac{5}{14} & -\frac{1}{7} \end{bmatrix}$$

$$X = A^{-1}B$$

$$= \begin{bmatrix} \frac{1}{7} & -\frac{1}{7} \\ -\frac{5}{14} & -\frac{1}{7} \end{bmatrix}\begin{bmatrix} 4 \\ -10 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{7}(4) & +\left(-\frac{1}{7}\right)(-10) \\ -\frac{5}{14}(4) & +\left(-\frac{1}{7}\right)(-10) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{4}{7} & +\frac{10}{7} \\ \frac{-10}{7} & +\frac{10}{7} \end{bmatrix} = \begin{bmatrix} \frac{14}{7} \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

So, $x = 2, y = 0$

Using Cramer's Rule:

$$2x - 2y = 4$$
$$-5x - 2y = -10$$

$$A = \begin{bmatrix} 2 & -2 \\ -10 & -2 \end{bmatrix}$$

$$A_x = \begin{bmatrix} 4 & -2 \\ -10 & -2 \end{bmatrix}$$

$$A_y = \begin{bmatrix} 2 & 4 \\ -5 & -10 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & -2 \\ -5 & -2 \end{vmatrix}$$

$$= (2)(-2) - (-2)(-5)$$
$$= -4 - 10 = -14$$

$$x = \frac{|A_x|}{|A|} = \frac{\begin{vmatrix} 4 & -2 \\ -10 & -2 \end{vmatrix}}{-14}$$

$$= \frac{4(-2) - (-2)(-10)}{-14}$$

$$= \frac{-8-20}{-14} = \frac{28}{14} = 2$$

$$y = \frac{|A_y|}{|A|} = \frac{\begin{vmatrix} 2 & 4 \\ -5 & -10 \end{vmatrix}}{-14}$$

$$= \frac{(2)(-10)-(4)(-5)}{-14}$$

$$= \frac{-20+20}{-14} = 0$$

So, $x = 2, y = 0$

1(viii) $3x - 4y = 4$

$x + 2y = 8$

writing the equations in matrix form.

$$\begin{bmatrix} 3x & -4y \\ x & +2y \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$$

or $$\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$$

Let $AX = B$

Where $A = \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$

$$|A| = \begin{vmatrix} 2 & -4 \\ 1 & 2 \end{vmatrix} = 3(2) - (-4)(1)$$

$$-6 + 4 = 10$$

$$AX = B$$

$$X = A^{-1}B$$

$$\text{Adj } A = \begin{bmatrix} 2 & 4 \\ -1 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{\text{Adj } A}{|A|} = \frac{\begin{bmatrix} 2 & 4 \\ -1 & 3 \end{bmatrix}}{10}$$

$$= \begin{bmatrix} \frac{2}{10} & \frac{4}{10} \\ \frac{-1}{10} & \frac{3}{10} \end{bmatrix}$$

$$X = A^{-1}B$$

$$= \begin{bmatrix} \frac{2}{10} & \frac{4}{10} \\ -\frac{1}{10} & \frac{3}{10} \end{bmatrix} \begin{bmatrix} 4 \\ 8 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2}{10}(4) & +\frac{4}{10}(8) \\ -\frac{1}{10}(4) & +\frac{3}{10}(8) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{8}{10} & +\frac{32}{10} \\ -\frac{4}{10} & +\frac{24}{10} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{40}{10} \\ \frac{20}{10} \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

So. $x = 4, y = 2$

Using Cramer's Rule:

$$3x - 4y = 4$$
$$x + 2y = 8$$

$$A = \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}$$

$$A_x = \begin{bmatrix} 4 & -4 \\ 8 & 2 \end{bmatrix}$$

$$A_y = \begin{bmatrix} 3 & 4 \\ 1 & 8 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 3 & -4 \\ 1 & 2 \end{vmatrix}$$

$$x = \frac{|A_x|}{|A|} = \frac{\begin{vmatrix} 4 & -4 \\ 8 & 2 \end{vmatrix}}{10}$$

$$= \frac{(4)(2) - (-4)(8)}{10}$$

$$= \frac{8 + 32}{10} = \frac{40}{10} = 4$$

$$y = \frac{|A_y|}{|A|} = \frac{\begin{vmatrix} 3 & 4 \\ 1 & 8 \end{vmatrix}}{10}$$

$$= \frac{(3)(8) - (4)(1)}{10}$$

$$= \frac{24 - 4}{10} = \frac{20}{10} = 2$$

So. $x = 4, y = 2$

Solve the following word problems by using

(i) matrix inversion method

(ii) Cramer's rule.

2. The length of a rectangle is 4 times its width. The perimeter of the rectangle is 150cm. Find the dimensions of the rectangle.

3. Two sides of a rectangle differ by 3.5cm. Find the dimensions of the rectangle its perimeter is 67cm.
4. The third angle of an isosceles triangle is 16° less than the sum of the two equal angles. Find three angles of the triangle.
5. One acute angle of a right triangle is 12° more than twice the other acute angle. Find the acute angles of the right triangle.
6. Two cars that are 600 km apart are moving towards each other. Their speeds differ by 6 km per hour and the cars are 123 km apart after $4\frac{1}{2}$ hours. Find the speed of each car.

Q.2. The length of a rectangle is 4 times its width. The perimeter of the rectangle is 150 cm. find the dimensions of the rectangle.

Suppose length of rectangle

$= x$ cm

$= y$ cm

According to the given conditions

$$4x = y$$

and $$2x + 2y = 150$$

Thus $$4x - y = 0 \quad (i)$$

$$2x + 2y = 150 \quad (ii)$$

Writing the equations in matrix form.

$$\begin{bmatrix} 4x & -y \\ 2x & +2y \end{bmatrix} = \begin{bmatrix} 0 \\ 150 \end{bmatrix}$$

or $$\begin{bmatrix} 4 & -1 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 150 \end{bmatrix}$$

Let $$AX = B$$

Where $A = \begin{bmatrix} 4 & -1 \\ 2 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 0 \\ 150 \end{bmatrix}$

$$|A| = \begin{vmatrix} 4 & -1 \\ 2 & 2 \end{vmatrix}$$

$$= (4)(2) - (-1)(2)$$

$$= 8 + 2 = 10$$

$$AX = B$$

$$X = A^{-1}B$$

$$\text{Adj } A = \begin{bmatrix} 2 & 1 \\ -2 & 4 \end{bmatrix}$$

$$A^{-1} = \frac{\text{Adj } A}{|A|} = \frac{\begin{bmatrix} 2 & 1 \\ -2 & 4 \end{bmatrix}}{10}$$

$$= \begin{bmatrix} \frac{2}{10} & \frac{1}{10} \\ \frac{-2}{10} & \frac{4}{10} \end{bmatrix}$$

$$X = A^{-1}B$$

$$= \begin{bmatrix} \frac{2}{10} & \frac{1}{10} \\ \frac{-2}{10} & \frac{4}{10} \end{bmatrix} \begin{bmatrix} 0 \\ 150 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2}{10}(0) & +\frac{1}{10}(150) \\ \frac{-2}{10}(0) & +\frac{4}{10}(150) \end{bmatrix}$$

$$= \begin{bmatrix} 0 & +15 \\ 0 & +60 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 15 \\ 60 \end{bmatrix}$$

So. $x = 15, y = 60$

Thus width $= 15$ cm

Length = 6 cm

Using Cramer's Rule:

$$4x - y = 0$$
$$2x + 2y = 150$$

$$A = \begin{bmatrix} 4 & -1 \\ 2 & 2 \end{bmatrix}$$

$$A_x = \begin{bmatrix} 0 & -1 \\ 150 & 2 \end{bmatrix}$$

$$A_y = \begin{bmatrix} 4 & 0 \\ 2 & 150 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 4 & -1 \\ 2 & 2 \end{vmatrix}$$
$$= (4)(2) - (-1)(2)$$
$$= 8 + 2 = 10$$

$$x = \frac{|A_x|}{|A|} = \frac{\begin{vmatrix} 0 & -1 \\ 150 & 2 \end{vmatrix}}{10}$$
$$= \frac{(0)(2) - (-1)(150)}{10}$$
$$= \frac{0 + 150}{10} = \frac{150}{10} = 15$$

$$y = \frac{|A_y|}{|A|} = \frac{\begin{vmatrix} 4 & 0 \\ 2 & 150 \end{vmatrix}}{10}$$
$$= \frac{(4)(150) - (0)(2)}{10} = \frac{600}{10} = 60$$

So. $x = 15, y = 60$

Q.3. Two sides of a rectangle differ by 3.5 cm. Find the dimensions of the rectangle if its perimeter is 67 cm.

Suppose:

Length of a rectangle = x cm

Width of a rectangle = y cm

According to the conditions

$$x - y = 3.5$$

and $$2x + 2y = 67$$

Writing the equations in matrix form.

$$\begin{bmatrix} x - y \\ 2x + 2y \end{bmatrix} = \begin{bmatrix} 3.5 \\ 67 \end{bmatrix}$$

or $$\begin{bmatrix} 1 & -1 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3.5 \\ 67 \end{bmatrix}$$

Let $$AX = B$$

Where $A = \begin{bmatrix} 1 & -1 \\ 2 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 3.5 \\ 67 \end{bmatrix}$

$$|A| = \begin{vmatrix} 1 & -1 \\ 2 & 2 \end{vmatrix}$$

$$= (1)(2) - (-1)(2)$$

$$= 2 + 2 = 4$$

$$AX = B$$

$$X = A^{-1}B$$

$$\text{Adj } A = \begin{bmatrix} 2 & 1 \\ -2 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{\text{Adj } A}{|A|} = \frac{\begin{bmatrix} 2 & 1 \\ -2 & 1 \end{bmatrix}}{4} = \begin{bmatrix} \frac{2}{4} & \frac{1}{4} \\ \frac{-2}{4} & \frac{1}{4} \end{bmatrix}$$

$$X = A^{-1}B$$

$$= \begin{bmatrix} \frac{2}{4} & \frac{1}{4} \\ \frac{-2}{4} & \frac{1}{4} \end{bmatrix}\begin{bmatrix} 3.5 \\ 67 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2}{4}(3.5) & +\frac{1}{4}(67) \\ \frac{-2}{4}(3.5) & +\frac{1}{4}(67) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{7.0}{4} & +\frac{67}{4} \\ \frac{-7.0}{4} & +\frac{67}{4} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{74}{4} \\ \frac{60}{4} \end{bmatrix} = \begin{bmatrix} 18.5 \\ 15 \end{bmatrix}$$

So
$$x = 18.5$$
$$y = 15$$

Length = 18.5 cm
Width = 15 cm

Using Cramer's Rule:

$$x - y = 3.5$$
$$2x + 2y = 67$$

$$A = \begin{bmatrix} 1 & -1 \\ 2 & 2 \end{bmatrix}$$

$$A_x = \begin{bmatrix} 3.5 & -1 \\ 67 & 2 \end{bmatrix}$$

$$A_y = \begin{bmatrix} 1 & 3.5 \\ 2 & 67 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & -1 \\ 2 & 2 \end{vmatrix}$$

$$= (1)(2) - (-1)(2)$$
$$= 2 + 2 = 4$$

$$x = \frac{|A_x|}{|A|} = \frac{\begin{vmatrix} 3.5 & -1 \\ 67 & 2 \end{vmatrix}}{4}$$

$$= \frac{(3.5)(2) - (-1)(67)}{4} = \frac{7 + 67}{4}$$

$$= \frac{74}{4} = 18.5$$

$$y = \frac{|A_y|}{|A|} = \frac{\begin{vmatrix} 1 & 3.5 \\ 2 & 67 \end{vmatrix}}{4}$$

$$= \frac{(1)(67) - (2)(3.5)}{4} = \frac{67 - 7}{4}$$

$$= \frac{60}{4} = 15$$

So, $x = 18.5, y = 15$

Length $= 18.5$ cm

Width $= 15$ cm

Q.4. The third angle of an isosceles is 16° less than the sum of the two equal angles. Find three angles of the triangle.

Solution:-

Suppose measure of each equal angle

$= x^\circ$

Let third angle $= y^\circ$

According to the conditions

$$2x - y = 16$$

$$2x + y = 180^\circ \quad \text{(sum of the angles)}$$

Writing the equations in matrix form.

$$\begin{bmatrix} 2x - y \\ 2x + y \end{bmatrix} = \begin{bmatrix} 16 \\ 180 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 16 \\ 180 \end{bmatrix}$$

Let $AX = B$

Where $A = \begin{bmatrix} 2 & -1 \\ 2 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 16 \\ 180 \end{bmatrix}$

$$|A| = \begin{vmatrix} 2 & -1 \\ 2 & 1 \end{vmatrix}$$

$$= (2)(1) - (-1)(2)$$

$$= 2 + 2 = 4$$

$$AX = B$$

$$X = A^{-1}B$$

$$\text{Adj } A = \begin{bmatrix} 1 & 1 \\ -2 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{\text{Adj } A}{|A|} = \frac{\begin{bmatrix} 1 & 1 \\ -2 & 2 \end{bmatrix}}{4}$$

$$= \begin{bmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{-2}{4} & \frac{2}{4} \end{bmatrix}$$

$$X = A^{-1}B$$

$$= \begin{bmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{-2}{4} & \frac{2}{4} \end{bmatrix}\begin{bmatrix} 16 \\ 180 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{4}(16) & +\frac{1}{4}(180) \\ \frac{-2}{4}(16) & +\frac{2}{4}(180) \end{bmatrix}$$

$$= \begin{bmatrix} 4+45 \\ -8+90 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 49 \\ 82 \end{bmatrix}$$

So. $x = 49^\circ$

$y = 82^\circ$

Measure of each equal angle = 49°

Measure of third angle = 82°

Using Cramer's Rule:

$$2x - y = 16$$
$$2x + y = 180$$

$$A = \begin{bmatrix} 2 & -1 \\ 2 & 1 \end{bmatrix}$$

$$A_x = \begin{bmatrix} 16 & -1 \\ 180 & 1 \end{bmatrix}$$

$$A_y = \begin{bmatrix} 2 & 16 \\ 2 & 180 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & -1 \\ 2 & 1 \end{vmatrix}$$

$$= (2)(1) - (-1)(2)$$
$$= 2 + 2 = 4$$

$$x = \frac{|A_x|}{|A|} = \frac{\begin{vmatrix} 16 & -1 \\ 180 & 1 \end{vmatrix}}{4}$$

$$= \frac{(16)(1) - (-1)(180)}{4}$$

$$= \frac{16 + 180}{4}$$

$$= \frac{196}{4} = 49$$

$$y = \frac{|A_y|}{|A|} = \frac{\begin{vmatrix} 2 & 16 \\ 2 & 180 \end{vmatrix}}{4}$$

$$= \frac{(2)(180) - (16)(2)}{4}$$

$$= \frac{360 - 32}{4} = \frac{328}{4} = 82$$

So, $x = 49, y = 82^\circ$

Measure of each equal angle $= 49^\circ$

Measure of third angle $= 82^\circ$

Q.5. One acute angle of a right triangle is 12° more than twice the other acute angle. Find the acute angles of the right triangle.

Solution:-

Let one acute angle $= x^\circ$

and the other acute angle $= y^\circ$

according to the conditions

$$x + y = 90^\circ$$

$$2x = y - 12$$

or $2x - y = -12$

and $x + y = 90^\circ$

Writing the equations in matrix form.

$$\begin{bmatrix} 2x - y \\ x + y \end{bmatrix} = \begin{bmatrix} -12 \\ 90 \end{bmatrix}$$

or $\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -12 \\ 90 \end{bmatrix}$

Let $A\ X = B$

Where $A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} -12 \\ 90 \end{bmatrix}$

$$|A| = \begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix}$$

$$= (2)(1) - (-1)(1)$$

$$= 2 + 1 = 3$$

$$AX = B$$

$$X = A^{-1}B$$

$$\text{Adj } A = \begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{\text{Adj } A}{|A|} = \frac{\begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix}}{3}$$

$$= \begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{-1}{3} & \frac{2}{3} \end{bmatrix}$$

$$X = A^{-1}B$$

$$= \begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{-1}{3} & \frac{2}{3} \end{bmatrix} \begin{bmatrix} -12 \\ 90 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{3}(-12) + \frac{1}{3}(90) \\ \frac{-1}{3}(-12) + \frac{2}{3}(90) \end{bmatrix}$$

$$= \begin{bmatrix} -4 + 30 \\ 4 + 60 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 26 \\ 64 \end{bmatrix}$$

So. $x = 26, y = 64$

One of the acute angle $= 26°$

Other acute angle = 64°

Using Cramer's Rule:

$$2x - y = 12$$
$$x + y = 90$$

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$$

$$A_x = \begin{bmatrix} -12 & 1 \\ 90 & 1 \end{bmatrix}$$

e

$$A_y = \begin{bmatrix} 2 & -12 \\ 1 & 90 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} = (2)(1) - (-1)(1)$$

$$= 2 + 1 = 3$$

$$x = \frac{|A_x|}{|A|} = \frac{\begin{vmatrix} -12 & -1 \\ 90 & 1 \end{vmatrix}}{3}$$

$$= \frac{(-12)(1) - (-1)(90)}{3}$$

$$= \frac{-12 + 90}{3} = \frac{78}{3} = 26$$

$$y = \frac{|A_y|}{|A|} = \frac{\begin{bmatrix} 2 & -12 \\ 1 & 90 \end{bmatrix}}{3}$$

$$= \frac{(2)(90) - (-12)(1)}{3}$$

$$\frac{180-12}{3}$$

So, $x = 26, y = 64$

One acute angle $= 26^\circ$

Other acute angle $= 64^\circ$

Q.6 Two cars that are 600km apart are moving towards each other. Their speeds differ by 6km per hour and cars are 123km apart after $4\frac{1}{2}$ hours. Find the speed of each car.

Solution:

Let speed of one car $= x$ km/hr
Speed of the other car $= y$ km/hr

Distance covered by first car in $4\frac{1}{2}$ hours $= \frac{9}{2}x\,km$

Distance covered by the other car in

$4\frac{1}{2}$ hours $= \frac{9}{2}y\,km$

$x - y = 6$ (First condition)

$\frac{9}{2}x + \frac{9}{2}y = 600 - 123$ (Second condition)

$\frac{9}{2}x + \frac{9}{2}y = 477$

$x + y = 477 \times \frac{2}{9}$

$x + y = 106$

and

$x - y = 6$

Writing the equations in matrix form

$x + y = 106$

$x - y = 6$

Or $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 106 \\ 6 \end{bmatrix}$

Let $AX = B$

Where $A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 106 \\ 6 \end{bmatrix}$

$|A| = \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = (1)(-1) - (1)(1)$

$= -1 - 1 = -2$

$AX = B$

$X = A^{-1}B$

$Adj\,A = \begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix}$

$A^{-1} = \dfrac{Adj\,A}{|A|}$

$= \dfrac{\begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix}}{-2}$

$= \begin{bmatrix} \frac{-1}{-2} & \frac{-1}{-2} \\ \frac{-1}{-2} & \frac{1}{-2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$

$X = A^{-1}B$

$$\begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix}\begin{bmatrix} 106 \\ 6 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2}(106) + \frac{1}{2}(6) \\ \frac{1}{2}(106) + \left(-\frac{1}{2}\right)(6) \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 53+3 \\ 53-3 \end{bmatrix} = \begin{bmatrix} 56 \\ 50 \end{bmatrix}$$

Thus $x = 56$

$y = 50$

speed of the first car = 56km/hr
speed of the other car = 50km/hr
using Cramer's rule

$x + y = 106$

$x - y = 6$

$$A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$A_x = \begin{bmatrix} 106 & 1 \\ 6 & -1 \end{bmatrix}$$

$$A_y = \begin{bmatrix} 1 & 106 \\ 1 & 6 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = (1)(-1) - (1)(1)$$

-1 -1 -2

106 1

1 6 1

1 2

(106) 1 (1)(6)

2

106 6 112 112 56

2 2 2

1 106

1

(1)(6) (1)(106)

2

6 106 100

2 2

so x 56, y 50

speed of the first car 56km hr

speed of other car 50km hr

Unit 2 REAL AND COMPLEX NUMBERS

Real Numbers:

Set of Natural Numbers

$N = \{1, 2, 3, \ldots\}$ Natural Numbers

Set of whole Numbers

$W = \{0, 1, 2, 3, \ldots\}$

Set of Integers

$Z = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

Rational Numbers

$Q = \{\frac{p}{q} \mid p, q \in Z \wedge q \neq 0\}$

Irrational Numbers

$Q' = \{x \mid x \neq \frac{p}{q}, p, q \in Z \wedge q \neq 0\}$

For example $\pi, \sqrt{5}, \sqrt{3}, \sqrt{2}$

Real Numbers:

The union of Q and Q' is a set of real numbers.

It is denoted by R.

$R = Q \cup Q'$

Remember:

(i) $N \subset W \subset Z \subset Q$

(ii) For each prime number p, $\sqrt{q}$ is an irrational number.

(iii) Square root of all positive non-square integers are irrational.

(a) Types of Rational Numbers

The decimal representation of rational numbers are of two types, terminating and recurring.

(i) Terminating Decimal Fractions

The decimal fraction in which there are finite number of digits in its decimal part is called a terminating decimal fraction. For example $\frac{4}{5}$ 0.8 and $\frac{3}{4}$ 0.75

(ii) Recurring and Non-terminating Decimal Fraction

The decimal fraction (non-terminating) in which some digits in the same repeated again and again in the same order in its decimal part is called a recurring decimal fraction. For example $\frac{2}{9}$ 0.2222

(b) Irrational Numbers:

Decimal representation for irrational numbers are neither terminating nor repeating in blocks. For example: $\sqrt{2}$ 1.414213562

π and e

Exercise 2.1

Q.1. ***Identify which of the following are rational and irrational numbers.***

(i) $\sqrt{3}$ (ii) $\frac{1}{6}$ (iii) π

(iv) $\frac{15}{2}$ (v) 7.25 (vi) $\sqrt{29}$

Solutions:

$\frac{1}{6}$, $\frac{15}{2}$, 7.25 (Rational)

$\sqrt{3}$, π, $\sqrt{29}$ (Irrational)

Q.2. ***Convert the following fractions into decimal fractions.***

(i) $\frac{17}{25}$ (ii) $\frac{19}{4}$ (iii) $\frac{57}{8}$

(iv) $\frac{205}{18}$ (v) $\frac{5}{8}$ (vi) $\frac{25}{38}$

Solution:-

2(i) $\frac{17}{25}$

0.68

$$\begin{array}{r|l} & .68 \\ 25 & 17.00 \\ & 150 \\ & 200 \\ & 200 \\ & \times \end{array}$$

2(ii) $\frac{19}{4}$

$= 4.75$

$$\begin{array}{r|l} & 4.75 \\ 4 & 19.00 \\ & 16 \\ & 30 \\ & 28 \\ & 20 \\ & 20 \\ & \times \end{array}$$

2(iii) $\frac{57}{8}$

$= 7.125$

$$\begin{array}{r} 7.125 \\ 8 \overline{)\,57.000} \\ 56 \\ \hline 10 \\ 8 \\ \hline 20 \\ 16 \\ \hline 40 \\ 40 \\ \hline \times \end{array}$$

2(iv) $\frac{205}{18}$

$= 11.3889$

$$\begin{array}{r} 11.3888 \\ 18 \overline{)\,205.0000} \\ 18 \\ \hline 25 \\ 18 \\ \hline 70 \\ 54 \\ \hline 160 \\ 144 \\ \hline 160 \\ 144 \\ \hline 16 \end{array}$$

2(v) $\frac{5}{8}$

$= 0.625$

$$\begin{array}{r} .625 \\ 8 \overline{)\,5.000} \\ 48 \\ \hline 20 \\ 16 \\ \hline 40 \\ 40 \\ \hline \times \end{array}$$

2(vi) $\frac{25}{38}$

$= .65789$

$$\begin{array}{r} .65789 \\ 38 \overline{)\,25.00000} \\ 228 \\ \hline 220 \\ 190 \\ \hline 300 \\ 266 \\ \hline 340 \\ 304 \\ \hline 360 \\ 342 \\ \hline 18 \end{array}$$

Q.3. Which of the statements are true and which are false?

(i) $\frac{2}{3}$ is an irrational number.

(ii) π is an irrational number.

(iii) $\frac{1}{9}$ is a terminating fraction.

(iv) $\frac{3}{4}$ is a terminating fraction.

Solution:-

(i) $\frac{2}{3}$ is an irrational number. **False**

(ii) π is an irrational number. **True**

(iii) $\frac{1}{9}$ is a terminating fraction. **False**

Explanation:

$$
\begin{array}{r|l}
 & .111 \\
9 & 1.000 \\
 & 9 \\
 & 10 \\
 & 9 \\
 & -10
\end{array}
$$

$\frac{1}{9} = 0.1$ non-terminating.

(iv) $\frac{3}{4}$ is a terminating fraction **True**

Explanation:

$$
\begin{array}{r|l}
 & 75 \\
4 & 3.00 \\
 & 28 \\
 & 20 \\
 & 20 \\
 & x
\end{array}
$$

(v) $\frac{4}{5}$ is a recurring fraction. **False**

Q.4. Represent the following numbers on the number line.

(i) $\frac{2}{3}$ *(ii)* $-\frac{4}{5}$ *(iii)* $1\frac{3}{4}$

(iv) $-2\frac{5}{8}$ *(v)* $2\frac{3}{4}$ *(vi)* $\sqrt{5}$

4(i) To represent the rational number $\frac{2}{3}$, divide unit length between 0 and 1 into 3 equal parts.

(ii) Take 2 parts on right of 0.

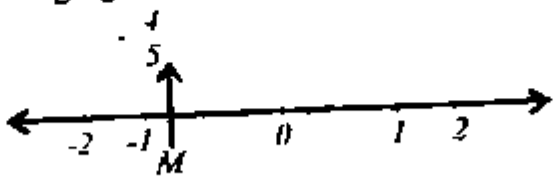
(iii) Point M represents $\frac{2}{3}$ on the number line in the following figure.

4(ii) Represent $-\frac{4}{5}$ on the number line.

(i) To represent the rational number $-\frac{4}{5}$ divide unit length between 0 and -1 into 5 equal parts.

(ii) Take 4 parts on left of '0'.

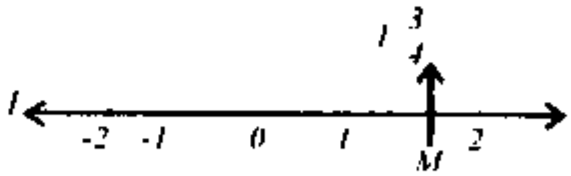
(iii) Point M represents $-\frac{4}{5}$ on the number line in the following figure.



4(iii) Represent $1\frac{3}{4}$ on the number line.

Solution:-

$1\frac{3}{4} = 1 + \frac{3}{4}$, therefore, $1\frac{3}{4}$ lies between 1 and 2 on the number line.



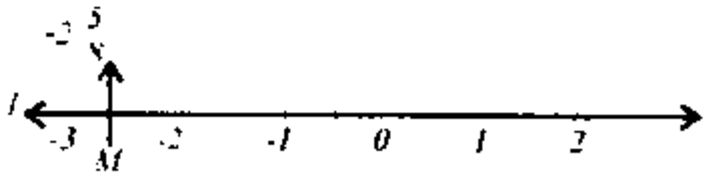
The distance between 1 and 2 is divided into four equal parts. From 1, we take 3 parts.

Point M represent $1\frac{3}{4}$ on the number line.

4(iv) Represent $-2\frac{5}{8}$ on the number line.

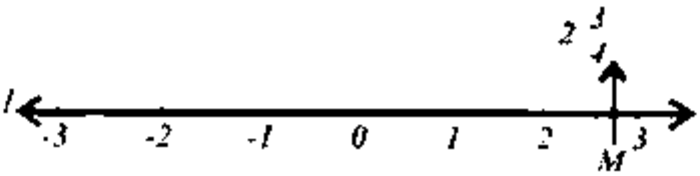
Rational number $-2\frac{5}{8}$ lies between -2 and -3.

We divide the distance between -2 and -3 into eight equal part and take 5 parts from -2.



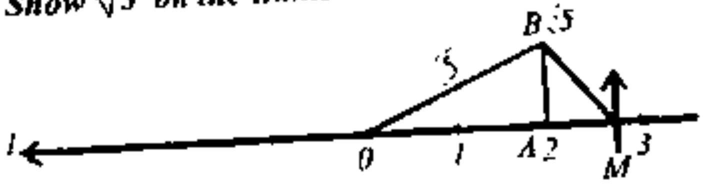
4(v) Represent $2\frac{3}{4}$ on the number line.

Number $2\frac{3}{4}$ lies between 2 and 3. Divided the distance between 2 and 3 into 4 equal parts. Take 3 parts from 2.



Point M shows $2\frac{3}{4}$ on the number line in the above figure.

4(vi) ***Show $\sqrt{5}$ on the number line.***



(i) Construct a ΔOAB which $m\overline{OA} = 2$ units and $\perp BA = 1$

(ii) Take 0 as centre and draw an arc of radius $\overline{OB}$. It cuts the number line at M. $m\overline{OM} = \sqrt{5}$.

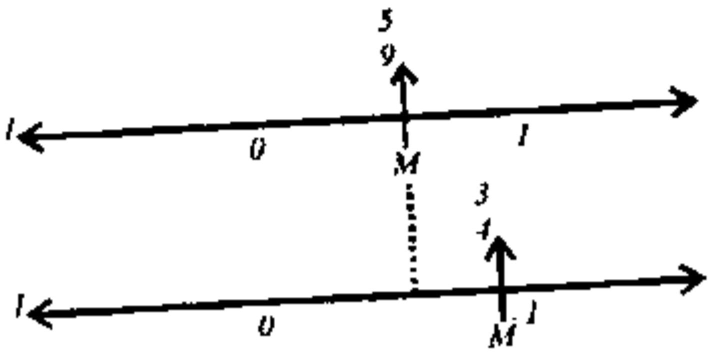
$(m\,OB)^2 = (mOA)^2 + (m\,AB)^2$ (Pythagorean Theorem)

$(m\overline{OB})^2 = 4 + 1 = 5$

$\therefore m\,OB = \sqrt{5}$

Thus $m\overline{OB} = m\overline{OM} = \sqrt{5}$

Q.5. ***Give a rational number between $\frac{3}{4}$ and $\frac{5}{9}$***



(i) on line l a number $\frac{5}{9}$ has been shown between 0 and 1 by M.

(ii) on line l a number $\frac{3}{4}$ has been shown between 0 and 1 by M'. (the same interval)

Result

There are a number of rational numbers between $\frac{5}{9}$ and $\frac{3}{4}$. For example: $\frac{57}{100}, \frac{58}{100}, \frac{59}{100}, \frac{60}{100}, \cdots\cdots \frac{74}{100}$ etc.

Q.6. Express the following recurring decimals as the rational number $\frac{p}{q}$ where p, q are integers and $q \neq 0$.

(i) $0.\overline{5}$ *(ii)* $0.\overline{13}$ *(iii)* $0.\overline{67}$

Solution:-

6(i) $0.\overline{5}$

Let $x = 0.\overline{5}$

That $x = 0.5555...$

Only one digit 5 is being repeated, we multiply by 10 on both sides.

$x = 0.5555...$ *(i)*

and $10x = (0.5555...) \times 10$

That $x = 5.5555...$ *(ii)*

Subtracting *(i)* from *(ii)*

$9x = 5$

$\therefore \quad x = \frac{5}{9}$

6(ii) $0.\overline{13}$

Let $x = 0.\overline{13}$

That $x = 0.13131313...$

Here, a block of 13 is being repeated, we multiply both sides by 100.

$$x = 0.\overline{13}\,\overline{13}\,\overline{13}\,\overline{13}\ldots\ldots(i)$$

$$100x = 13.\overline{13}\,\overline{13}\,\overline{13}\,\overline{13}\ldots\ldots(ii)$$

Subtracting *(i)* from *(ii)*

$$100x - x = 13$$

$$99x = 13$$

Thus $x = \dfrac{13}{99}$

$\therefore\quad 0.\overline{13} = \dfrac{13}{99}$

6(iii) $0.\overline{67}$

Let $x = 0.\overline{67}$

or $x = 0.67676767\ldots$

Here, a block of *67* is being repeated, we multiply both sides by *100*.

$$x = 0.67676767\ldots\ldots(i)$$

$$100x = 67.67676767\ldots\ldots(ii)$$

Subtracting *(i)* from *(ii)*

$$100x - x = 67$$

$$99x = 67$$

Thus $x = \dfrac{67}{99}$

$\therefore\quad 0.\overline{67} = \dfrac{67}{99}$

Properties of Real Numbers

(a) ***Properties of real numbers w.r.t. addition.***

(i) Closure Property

$a + b \in R,\ \forall\, a, b \in R$

(ii) Commutative Property

$a + b = b + a,\ \forall\, a, b \in R$

(iii) Associative Property

$(a + b) + c = a + (b + c),\ \forall\, a, b, c \in R$

(iv) **Additive Identity**

$a + 0 = a = 0 + a \quad \forall a \in R$

(0 is called additive identity)

(v) **Additive Inverse**

$a + (-a) = 0 = (-a) + a$

Properties of real numbers under multiplication

(i) **Closure Property**

$ab \in R \quad \forall a, b \in R$

(ii) **Commutative Property**

$ab = ba \quad \forall a, b \in R$

(iii) **Associative Property**

$(ab)c = a(bc) \quad \forall a, b, c \in R$

(iv) **Multiplicative Identity**

$a \times 1 = a = 1 \times a \quad \forall a \in R$

(v) **Multiplicative Inverse**

$a \times \frac{1}{a} = 1 = \frac{1}{a} \times a \quad \forall a \in R$ and $a \neq 0$

(vi) **Multiplication is distributive over addition and subtraction.**

$\forall a, b, c \in R$

$a(b+c) = ab + ac$ (Left distributive law)

$(a+b)c = ac + bc$ (Right distributive law)

Properties of Equality of Real Numbers

(i) **Reflexive property**

$a = a \quad \forall a \in R$

(ii) **Symmetric Property**

If $a = b$ then $b = a \quad \forall a, b \in R$

(iii) **Transitive Property**

If $a = b$ and $b = c$, then $a = c, \quad \forall a, b, c \in R$

(iv) **Cancellation Property** for Addition

If $a = c$, then $a + c = b + c, \quad \forall a, b, c \in R$

(v) **Multiplicative Property**

If $a = b$, then $ac = bc, \quad \forall a, b, c \in R$

(vi) **Cancellation Property** for Addition

If $a + b = b + c$, then $a = b, \quad \forall a, b, c \in R$

(vii) **Cancellation Property** for Multiplication

If $ac = bc, c \neq 0$ then $a = b$, $\forall a, b, c \in R$

(c) *Properties of Inequalities of Real Numbers*

Properties of inequalities of real numbers are as follows:

(i) *Trichotomy Property*

$\forall \ a, b \in R$

$a < b$ or $a = b$ or $a > b$

(ii) *Transitive Property*

$\forall \ a, b, c \in R$

(a) $a < b$ and $b < c \Rightarrow a < c$

(b) $a > b$ and $b > c \Rightarrow a > c$

(iii) *Additive Property*

$\forall \ a, b, c \in R$

(a) $a < b \Rightarrow a + c < b + c$ and

$a < b \Rightarrow c + a < c + b$

(b) $a > b \Rightarrow a + c > b + c$

$a > b \Rightarrow c + a > c + b$

(iv) *Multiplicative Property*

(a) $\forall \ a, b, c \in R$ and $c > 0$

(i) $a > b \Rightarrow ac > bc$ and

$a > b \Rightarrow ca > cb$

(ii) $a < b \Rightarrow ac < bc$ and

$a < b \Rightarrow ca < cb$

(b) $\forall \ a, b, c \in R$ and $c < 0$

(i) $a > b \Rightarrow ac < bc$ and

$a > b \Rightarrow ca < cb$

(ii) $a < b \Rightarrow ac > bc$ and

$a < b \Rightarrow ca > cb$

(v) *Multiplicative Inverse Property*

$\forall \ a, b \in R$ and $a \neq 0, b \neq 0$

(a) $a < b \Leftrightarrow \dfrac{1}{a} > \dfrac{1}{b}$

(b) $a > b \Leftrightarrow \dfrac{1}{a} < \dfrac{1}{b}$

Exercise 2.2

Q.1. Identify the property used in the following.

(i) $a + b = b + a$ …………

(ii) $(ab)c = a(bc)$ …………

(iii) $7 \times 1 = 7$…………

(iv) $x > y$ or $x = y$ or $x < y$…………

(v) $ab = ba$…………

(vi) $a + c = b + c \Rightarrow a = b$ …………

(vii) $5 + (-5) = 0$ …………

(viii) $7 \times \frac{1}{7} = 1$ …………

(ix) $a > b \Rightarrow ac > bc$ $(c > 0)$ …………

Solution:-

(i) $a + b = b + a$ ………… (Commutative w.r.t addition)

(ii) $(ab)c = a(bc)$ ………… (Associative w.r.t multiplication)

(iii) $7 \times 1 = 7$………… (Multiplicative w.r.t Identity)

(iv) $x > y$ or $x = y$ or $x < y$………… (Trichotomy)

(v) $ab = ba$………… (Commutative w.r.t Multiplication)

(vi) $a + c = b + c \Rightarrow a = b$ …………
(Cancellation property of addition)

(vii) $5 + (-5) = 0$ ………… (Additive Inverse)

(viii) $7 \times \frac{1}{7} = 1$ ………… (Multiplicative inverse)

(ix) $a > b \Rightarrow ac > bc$ $(c > 0)$ …………
(Multiplicative property)

Q.2. Fill in the following blanks by stating the properties of real numbers used.

$3x + 3(y - x)$

$= 3x + 3y - 3x,$ ……………… (i)

$= 3x - 3x + 3y,$ ……………… (ii)

$= 0 + 3y,$ ……………… (iii)

$= 3y,$ ……………… (iv)

2(i) $3x + 3(y - x)$

$= 3x + 3y - 3x$

Distributive property of multiplication over subtraction.

(ii) $3x + 3y - 3x$

$= 3x - 3x + 3y$ (Commutative)

(iii) $3x - 3x + 3y$

$= 0 + 3y$ (Addition Inverse)

(iv) $0 + 3y$

$= 3y$ (Addition Identity)

Q.3. *Give the name of property used in the following.*

(i) $\sqrt{24} + 0 = \sqrt{24}$ ....

(ii) $-\frac{2}{3}\left(5 + \frac{7}{2}\right) = \left(-\frac{2}{3}\right)(5) + \left(-\frac{2}{3}\right)\left(\frac{7}{2}\right)$ ....

(iii) $\pi + (-\pi) = 0$ ....

(iv) $\sqrt{3} \cdot \sqrt{3}$ ....

(v) $\left(-\frac{5}{8}\right)\left(-\frac{8}{5}\right) = 1$ ....

Solution:-

(i) $\sqrt{24} + 0 = \sqrt{24}$ .... (Additive Identity)

(ii) $-\frac{2}{3}\left(5 + \frac{7}{2}\right) = \left(-\frac{2}{3}\right)(5) + \left(-\frac{2}{3}\right)\left(\frac{7}{2}\right)$ ....

Distributive property of multiplication over addition.

(iii) $\pi + (-\pi) = 0$ .... (Additive Inverse)

(iv) $\sqrt{3} \cdot \sqrt{3}$ is a real number.

(Closure property w.r.t multiplication)

(v) $\left(-\frac{5}{8}\right)\left(-\frac{8}{5}\right) = 1$ ....

(Multiplication Inverse)

Concept of Radicals and Radicands

If n is positive integer greater than 1 and a is a real number, then any real number x such that $x^n = a$ is called the nth root of a, and in symbols is written as

$x = \sqrt[n]{a}$, or $x = (a)^{1/n}$.

In the radical $\sqrt[n]{a}$, the symbol $\sqrt{\ }$ is called the radical sign, n is called the index of the radical and the real number a under the radical sign is called the radicand or base.

Note

$\boxed{\sqrt[2]{a} \text{ is usually written as } \sqrt{a}}$

Properties of Radicals

Let $a, b \in R$ and m, n be positive integers. Then,

(i) $\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$ *(ii)* $\sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}$

(iii) $\sqrt[n]{\sqrt[m]{a}} = \sqrt[nm]{a}$ *(iv)* $\sqrt[n]{a^m} = (\sqrt[n]{a})^m$

(v) $\sqrt[n]{a^n} = a$

Exercise 2.3

1. Write each radical expression in exponential notation and each exponential expression in radical notation. Do not simplify.

(i) $\sqrt[3]{-64}$ *(ii)* $2^{3/5}$

(iii) $-7^{1/3}$ *(iv)* $y^{-2/3}$

(i) $\sqrt[3]{-64} = (-64)^{1/3}$ (Exponential form)

(ii) $2^{3/5} = \sqrt[5]{2^3}$ (Radical form)

(iii) $-7^{1/3} = -\sqrt[3]{7}$ (Radical form)

(iv) $y^{-2/3} = \sqrt[3]{y^{-2}}$ (Radical form)

Q.2. Tell whether the following statements are true or false?

(i) $5^{1/5} = \sqrt{5}$ *(ii)* $2^{2/3} = \sqrt[3]{4}$

(iii) $\sqrt{49} = \sqrt{7}$ *(iv)* $\sqrt[3]{x^{27}} = x^3$

Solutions:-

2(i) $5^{1/5} = \sqrt{5}$ **(False)**

$5^{1/5} = \sqrt[5]{5}$ **Explanation**

2(ii) $2^{2/3} = \sqrt[3]{4}$ **(True)**

$2^{2/3} = \sqrt[3]{2^2} = \sqrt[3]{4}$ **Explanation**

2(iii) $\sqrt{49} = \sqrt{7}$ **(False)**

$\sqrt{49} = 7$ **Explanation**

2(iv) $\sqrt[3]{x^{27}} = x^3$ **(False)**

$\sqrt[3]{x^{27}} = (x^{27})^{1/3}$ **Explanation**

$$= x^{27\times 1/3} = x^{9}$$

Q.3. Simplify the following radical expressions.

(i) $\sqrt[3]{-125}$ (ii) $\sqrt[4]{32}$

(iii) $\sqrt[5]{\dfrac{3}{32}}$ (iv) $\sqrt[3]{-\dfrac{8}{27}}$

Solutions:-

3(i) $\sqrt[3]{-125}$

$$= \sqrt[3]{(-5)^3} = [(-5)^3]^{1/3}$$
$$= (-5)^{3\times 1/3}$$
$$= (-5)$$

3(ii) $\sqrt[4]{32}$

$$= \sqrt[4]{2^5} = \sqrt[4]{2^4\times 2}$$
$$= (2^4\times 2)^{1/4}$$
$$= 2^{4\times 1/4}\times 2^{1/4}$$
$$= 2\sqrt[4]{2}$$

3(iii) $\sqrt[5]{\dfrac{3}{32}}$

$$= \sqrt[5]{\frac{3}{32}} = \left(\frac{3}{2^5}\right)^{1/5}$$
$$= \frac{\sqrt[5]{3}}{2}$$

3(iv) $\sqrt[3]{-\dfrac{8}{27}}$

$$= \sqrt[3]{\left(-\frac{2}{3}\right)^3} = \left[\left(-\frac{2}{3}\right)^3\right]^{1/3}$$

$$-\left(-\frac{2}{3}\right)^{3\times 1,\,3} = -\frac{2}{3}$$

Base and Exponent

In the exponential notation a^n (read as a to the nth power) we call 'a' as the base and 'n' exponent or the power to which the base is raised.

Laws of Exponents

If $a, b \in R$ and m, n are positive integers then

$(a^m)^n = a^{mn}$, $(ab)^n = a^n b^n$

$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$, $b \neq 0$

$a^m a^n = a^{m+n}$

$\frac{a^m}{a^n} = a^{m-n}$, $m > n$, $a \neq 0$

$a^0 = 1$, $a^{-n} = \frac{1}{a^n}$, $a \neq 0$

Exercise 2.4

Q.1. Use laws of exponents to simplify:

(i) $\dfrac{(243)^{-2/3}(32)^{-1/5}}{\sqrt{(196)^{-1}}}$ (ii) $(2x^5y^{-4})(-8x^{-3}y^2)$

(iii) $\left(\dfrac{x^{-2}y^{-1}z^{-4}}{x^4y^{-3}z^0}\right)^{-3}$ (iv) $\dfrac{(81)^n.3^5-(3)^{4n-1}(243)}{(9^{2n})(3^3)}$

Solution:-

1(i) $\dfrac{(243)^{-2/3}(32)^{-1/5}}{\sqrt{(196)^{-1}}}$

$= \dfrac{(3^5)^{-2/3}.(2^5)^{-1/5}}{[(14^2)^{-1}]^{1/2}}$

$= \dfrac{3^{5(-2/3)}.2^{5(-1/5)}}{14^{2(-1)(1/2)}}$

$= \dfrac{3^{-10/3}\,2^{-1}}{14^{-1}} = \dfrac{3^{-10/3}\,2^{-1}}{2^{-1}\times 7^{-1}}$

$= \dfrac{7}{3^{10/3}} = \dfrac{7}{\sqrt[3]{3^{10}}}$

$= \dfrac{7}{\sqrt[3]{3^3.3^3.3^3.3}} = 3^{9+1}$

$= \dfrac{7}{3^3\sqrt[3]{3}} = \dfrac{7}{27\sqrt[3]{3}}$

1(ii) $(2x^5y^{-4})(-8x^{-3}y^2)$

$= (2)(-8)\,x^5y^{-4}x^{-3}y^2$

$= -16\,x^{5-3}y^{-4+2}$

$= -16\,x^2y^{-2}$

$= -\dfrac{16x^2}{y^2}$

1(iii) $\left(\dfrac{x^{-2}y^{-1}z^{-4}}{x^4y^{-3}z^0}\right)^{-3}$

$$=\left(\frac{y^{-1+3}}{x^{4+2}z^{4+0}}\right)^{-3}$$

$$=\left(\frac{y^{2}}{x^{6}z^{4}}\right)^{-3}$$

$$=\frac{y^{2(-3)}}{x^{6(-3)}z^{4(-3)}}=\frac{y^{-6}}{x^{-18}z^{-12}}=\frac{x^{18}z^{12}}{y^{6}}$$

(iv) $$=\frac{(81)^{n}.3^{5}-(3)^{4n-1}(243)}{(9^{2n})(3^{3})}$$

$$=\frac{(3^{4})^{n}.3^{5}-3^{4n-1}(3)^{5}}{(3^{2})^{2n}(3^{3})}$$

$$=\frac{3^{4n+5}-3^{4n-1+5}}{3^{4n}\times 3^{3}}$$

$$=\frac{3^{4n+5}-3^{4n+4}}{3^{4n+3}}=\frac{3^{4n+4}.3^{1}-3^{4n+4}}{3^{4n+3}}$$

$$=\frac{3^{4n+4}(3-1)}{3^{4n+3}}=3^{4n+4-4n-3}\times(2)$$

$$\therefore 3\times 2=6$$

Q.2. Show that:

$$\left(\frac{x^{a}}{x^{b}}\right)^{a+b}\times\left(\frac{x^{b}}{x^{c}}\right)^{b+c}\times\left(\frac{x^{c}}{x^{a}}\right)^{c+a}=1$$

$$L.H.S=\left(\frac{x^{a}}{x^{b}}\right)^{a+b}\times\left(\frac{x^{b}}{x^{c}}\right)^{b+c}\times\left(\frac{x^{c}}{x^{a}}\right)^{c+a}$$

$$=(x^{a-b})^{a+b}\times(x^{b-c})^{b+c}\times(x^{c-a})^{c+a}$$

$$=x^{(a-b)(a+b)}\times x^{(b-c)(b+c)}\times x^{(c-a)(c+a)}$$

$$=x^{a^{2}-b^{2}}\times x^{b^{2}-c^{2}}\times x^{c^{2}-a^{2}}$$

$$=x^{a^{2}-b^{2}+b^{2}-c^{2}+c^{2}-a^{2}}$$

$$=x^{0}=1$$

Q.3. Simplify:

(i) $\dfrac{2^{1/3}.(27)^{1/3}.(60)^{1/2}}{(180)^{1/2}.(4)^{-1/3}.(9)^{1/4}}$ (ii) $\sqrt{\dfrac{(216)^{2/3}.(25)^{1/2}}{(.04)^{-1/2}}}$

(iii) $5^{2^{3}}\div(5^{2})^{3}$ (iv) $(x^{3})^{2}\div x^{3^{2}},x\neq 0$

3(i) $\dfrac{2^{1/3}.(27)^{1/3}.(60)^{1/2}}{(180)^{1/2}.(4)^{-1/3}.(9)^{1/4}}$

$= \dfrac{2^{1/3}.(3^3)^{1/3}.(2^2.3.5)^{1/2}}{(2^2.3^2.5)^{1/2}.(2^2)^{-1/3}.(3^2)^{1/4}}$

$= \dfrac{2^{1/3}.3^{3\times1/3}.2^{2\times1/2}.3^{1/2}.5^{1/2}}{2^{2\times1/2}.3^{2\times1/2}.5^{1/2}.2^{2(-1/3)}.3^{2\times1/4}}$

$= \dfrac{2^{1/3}.3.2.3^{1/2}.5^{1/2}}{2.3.5^{1/2}.2^{-2/3}.3^{1/2}} = 2^{1/3+2/3}$

$= 2^{\frac{1+2}{3}} = 2^{\frac{3}{3}} = 2^1 = 2$

3(ii) $\sqrt{\dfrac{(216)^{2/3}.(25)^{1/2}}{(.04)^{-1/2}}}$

$= \sqrt{\dfrac{(6^3)^{2/3}.(5^2)^{1/2}}{(\frac{4}{100})^{-1/2}}}$

$= \sqrt{\dfrac{6^{3\times2/3}.5^{2\times1/2}}{(\frac{1}{25})^{-1/2}}}$

$= \sqrt{\dfrac{6^2\times5}{(\frac{1}{5^2})^{-1/2}}}$

$= \sqrt{\dfrac{6^2\times5}{(5^{-2})^{-1/2}}} = \sqrt{\dfrac{6^2\times5}{5^{(-2)(-1/2)}}}$

$= \sqrt{\dfrac{6^2\times5}{5}} = (6^2)^{1/2} = 6$

3(iii) $5^{2^3} \div (5^2)^3$

$= 5^8 \div 5^{2(3)}$

$= 5^8 \div 5^6$

$= \dfrac{5^8}{5^6} = 5^{8-6} = 5^2 = 25$

3(iv) $(x^3)^2 \div x^{3^2}, x \neq 0$

$= x^{3(2)} \div x^9$

$\therefore x^6 \div x^9 = \frac{x^6}{x^9} = \frac{1}{x^{9-6}} = \frac{1}{x^3}$

Complex Numbers

A number $\sqrt{-1}$ has been found that square of it is "-1". It is called the imaginary unit. It is denoted by i (iota) that $i^2 = -1$. It is not a real number. The swiss mathematician Leonard Euler (1707 – 1783) was the first to use the symbol i for the number $\sqrt{-1}$.

Numbers like $\sqrt{-1}$, $\sqrt{-3}$, $\sqrt{-4}$ etc are called imaginary numbers.

Definition of a Complex Numbers

A number of the form $z = a + bi$ where a and b are real numbers and $i = \sqrt{-1}$, is called a complex number.

Set of Complex Numbers

Set of complex numbers is denoted by C.

$C = \{z \mid z = a + bi, \text{ where } a, b \in R \text{ and } i = \sqrt{-1}\}$

The number 'a' is real part and b is imaginary part of the complex number $a + bi$.

These are denoted as $a = Re(z)$ and $b = Im(z)$

Conjugate of a Complex Number

Conjugate of $a + bi$ is $a - bi$, then $z = a + bi$ and

$\bar{z} = a - bi$

Now $\bar{\bar{z}} = a + bi$ i.e; conjugate of a original complex number.

Equality of Complex Numbers

$\forall a, b, c, d \in R$

If $a + bi = c + di$ then $a = c$ and $b = d$

Properties of Complex Numbers.

(i) $z_1 = z_1$ ....(Reflexive Law)

(ii) $z_1 = z_2 \Rightarrow z_2 = z_1$ ....(Symmetric Law)

(iii) $z_1 = z_2, z_2 = z_3 \Rightarrow z_1 = z_3$ ....(Transitive Law)

Exercise 2.5

Q.1. Evaluate:

(i) i^7 (ii) i^{50} (iii) i^{12}

(iv) $(-i)^8$ (v) $(-i)^5$ (vi) i^{27}

1(i) i^7

$= i^6 \,.\, i$

$= (i^2)^3 \,.\, i$

$= (-1)^3 \,.\, i = -i$

1(ii) i^{50}

$= (i^2)^{25}$

$(-1)^{25} = (-1)^{24}\,(-1) = -1$

1(iii) i^{12}

$= (i^2)^6$

$= (-1)^6 = 1$

1(iv) $(-i)^8$

$= (-i)^8$

$= [(-1)^2]^4 = (-1)^4 = 1$

1(v) $(-i)^5$

$= (-i)^4 \times (i)$

$= i^4 \times (-i) = (i^2)^2 \times (-i)$

$= (-1)^2 \times (-i) = -i$

1(vi) i^{27}

$= (i^2)^{13} \times i$

$= (-1)^{13} \times i = -i$

Q.2. Write the conjugate of the following numbers.

(i) $2+3i$ (ii) $3-5i$ (iii) $-i$

(iv) $-3+4i$ (v) $-4-i$ (vi) $i-3$

Solution:-

2(i) $z = 2+3i$

$\bar{z} = 2-3i$

2(ii) $z = 3-5i$

$\bar{z} = 3+5i$

2(iii) $z = -i$

$\bar{z} = i$

2(vi) $z = -3+4i$

$\bar{z} = -3-4i$

2(v) $z = -4-i$

$\bar{z} = -4+i$

2(vi) $z = i-3$

$\bar{z} = -i-3$

Q.3. Write the real and imaginary part of the following numbers.

(i) $1+i$ (ii) $-1+2i$ (iii) $-3i+2$

(iv) $-2-2i$ (v) $-3i$ (vi) $2+0i$

Solutions:-

3(i) $z = 1+i$

$Re\ (z) = 1\ ;\ Im\ (z) = 1$

3(ii) $z = -1 + 2i$

$Re\ (z) = -1\ ;\ Im\ (z) = 2$

3(iii) $z = -3i + 2$

$Re\ (z) = 2\ ;\ Im\ (z) = -3$

3(iv) $z = -2 - 2i$

$Re\ (z) = -2\ ;\ Im\ (z) = -2$

3(v) $z = -3i$

$Re\ (z) = 0\ ;\ Im\ (z) = -3$

3(vi) $z = 2 + 0i$

$Re\ (z) = 2\ ,\ Im\ (z) = 0$

Q.4. *Find the value of x and y if*

$x + iy + 1 = 4 - 3i$

Here $iy = -3i$

$\therefore \quad y = -3$

and $x + 1 = 4$

$x = 4 - 1$

$\therefore \quad x = 3$

Exercise 2.6

Q.1. Identify the following statements as true or false.

(i) $\sqrt{-3}\sqrt{-3} = 3$ *(ii)* $i^{73} = -i$

(iii) $i^{10} = -1$

(iv) Complex conjugate of $(-6i + i^2)$ is $(-1 + 6i)$

(v) Difference of a complex number z = a + bi and its conjugate is a real number.

(vi) If $(a-1) - (b+3)i = 5 + 8i$, then $a = 6$ and $b = -11$

(vii) Product of a complex number and its conjugate is always a non-negative real number.

Solutions:-

1(i) $\sqrt{-3}\sqrt{-3} = 3$ **False**

$\sqrt{3i} \times \sqrt{-3i}$ **Explanation**

$\sqrt{-3}.\sqrt{-3}.i^2.$

$= 3.(-1) = -3$

1(ii) $i^7 = -i$ **False**

i^{73} **Explanation**

$= (i)^{72} \times i$

$= (i^2)^{36} \times i$

$= (-1)^{36} \times i = i$

1(iii) $i^{10} = -1$ **True**

i^{10} **Explanation**

$= (i^2)^5$

$= (-1)^5 = -1$

1(iv) Complex Conjugate of $(-6i + i^2)$ is $(-1 + 6i)$

True

$-6i + i^2$ **Explanation**

$= -6i - 1 = -1 - 6i$

1(v) Difference of a complex number $z = a + bi$ and its conjugate is a real number. **False**

$z = a + bi$ **Explanation**

and

$$\bar{z} = a - bi$$

$$z - \bar{z} = (a + bi) - (a - bi)$$
$$= a + bi - a + bi$$
$$= 2bi \text{ (an imaginary number)}$$

1(vi) If $(a - 1) - (b + 3i) = 5 + 8i$

Solution:-

$$(a - 1) - (b + 3i) = 5 + 8i$$
$$a - 1 - b - 3i = 5 + 8i$$
$$a - b = 5 + 8i + 1 + 3i$$
$$a - b = 6 + 11i$$
$$\therefore \quad a = 6$$
and $\quad b = -11 \quad$ **(True)**

1(vii) Product of a complex number and its conjugate is always a real non-negative real number.

Example:

$(2 - 3i)(2 + 3i)$
$= 2(2 + 3i) - 3i(2 + 3i)$
$= 4 + 6i - 6i - 9i^2$
$= 4 - 9i^2$
$= 4 - 9(-1) = 4 + 9 = 13$

It is a non-negative real number. **(True)**

Q.2. ***Express each complex number in the standard form a + bi, where a and b are real numbers.***

(i) $(2 + 3i) + (7 - 2i)$
(ii) $2(5 + 4i) - 3(7 + 4i)$
(iii) $-(-3 + 5i) - (4 + 9i)$
(iv) $2i^2 + 6i^3 + 3i^{16} - 6i^{19} + 4i^{25}$

Solution:-

2(i) $(2 + 3i) + (7 - 2i)$
$= 2 + 3i + 7 - 2i$

$= 2 + 7 + 3i - 2i = 9 + i$

2(iii) $= -(-3 + 5i) - (4 + 9i)$

$= 3 - 5i - 4 - 9i$

$= 3 - 4 - 5i - 9i = -1 - 14i$

2(iv) $2i + 6i^3 + 3i^{16} - 6i^{19} + 4i^{25}$

$= 2i^2 + 6i^3 + 3i^{16} - 6i^{19} + 4i^{25}$

$= 2(-1) + 6i^2 i + 3(i^2)^8 - 6(i^2)^9 i + 4(i^2)^{12} i$

$= -2 + 6(-1)i + 3(-1)^8 - 6(-1)^9 i + 4(-1)^{12} i$

$= -2 - 6i + 3(1) - 6(-1)i + 4(1)i$

$= -2 - 6i + 3 + 6i + 4i$

$= -2 + 3 - 6i + 6i + 4i$

$= 1 + 4i$

Q.3. ***Simplify and write your answer in the form $a + bi$.***

(i) $(-7 + 3i)(-3 + 2i)$

(ii) $(2 - \sqrt{-4})(3 - \sqrt{-4})$

(iii) $(\sqrt{5} - 3i)^2$

(iv) $(2 + 3i)(\overline{3 - 2i})$

Solution:-

3(i) $(-7 + 3i)(-3 + 2i)$

$= (-7)(-3 + 2i) + 3i(-3 + 2i)$

$= 21 - 14i - 9i + 6i^2$

$= 21 - 14i - 9i + 6(-1)$

$= 21 - 23i - 6 = 15 - 23i$

3(ii) $(2 - \sqrt{-4})(3 - \sqrt{-4})$

$= (2 - 2i)(3 - 2i)$

$= 2(3 - 2i) - 2i(3 - 2i)$

$= 6 - 4i - 6i + 4i^2$

$= 6 - 10i + 4(-1)$

$= 6 - 10i - 4 = 2 - 10i$

3(iii) $(\sqrt{5} - 3i)^2$

$= (\sqrt{5})^2 + (3i)^2 - 2\sqrt{5} \times 3i$

$= 5 + 9i^2 - 6\sqrt{5}\, i$

$= 5 + 9(-1) - 6\sqrt{5}\, i$

$= 5 - 9 - 6\sqrt{5}\, i$

$= -4 - 6\sqrt{5}\, i$

3(iv) $(2 - 3i)(\overline{3 - 2i})$

$= (2 - 3i)(3 + 2i)$ (Taking conjugate)

$= 2(3 + 2i) - 3i(3 + 2i)$

$= 6 + 4i - 9i - 6i^2$

$= 6 + 4i - 9i - 6(-1)$

$= 6 + 4i - 9i + 6$

$= 12 - 5i$

Q.4. ***Simplify and write your answer in the form a + bi.***

(i) $\dfrac{-2}{1 + i}$ (ii) $\dfrac{2 + 3i}{4 - i}$

(iii) $\dfrac{9 - 7i}{3 + i}$ (iv) $\dfrac{2 - 6i}{3 + i} - \dfrac{4 + i}{3 + i}$

(v) $\left(\dfrac{1 + i}{1 - i}\right)^2$ (vi) $\dfrac{1}{(2 + 3i)(1 - i)}$

Solution:-

4(i) $\dfrac{-2}{1 + i}$

$= \dfrac{-2}{1 + i} \times \dfrac{1 - i}{1 - i}$

$= \dfrac{-2(1 - i)}{1 - i^2} = \dfrac{-2 + 2i}{1 - (-1)}$

$= \dfrac{-2 + 2i}{1 + 1} = \dfrac{-2 + 2i}{2}$

$= -1 + 1i = -1 + i$

4(ii) $\dfrac{2 + 3i}{4 - i}$

$$= \frac{2+3i}{4-i} \times \frac{4+i}{4+i}$$

$$= \frac{(2+3i)(4+i)}{4^2 - i^2}$$

$$= \frac{2(4+i)+3i(4+i)}{16-(-1)}$$

$$= \frac{8+2i+12i+3i^2}{16+1}$$

$$= \frac{8+14i+3(-1)}{17}$$

$$= \frac{8+14i-3}{17}$$

$$= \frac{5+14i}{17} = \frac{5}{17} + \frac{14}{17}i$$

4(iii) $\frac{9-7i}{3+i}$

$$= \frac{9-7i}{3+i} \times \frac{3-i}{3-i}$$

$$= \frac{(9-7i) \times (3-i)}{3^2 - i^2}$$

$$= \frac{9(3-i)-7i(3-i)}{9-(-1)}$$

$$= \frac{27-9i-21i+7i^2}{9+1}$$

$$= \frac{27-30i+7(-1)}{10} = \frac{20-30i}{10} = 2-3i$$

4(iv) $\frac{2-6i}{3+i} - \frac{4+i}{3+i}$

$$= \left(\frac{2-6i}{3+i} \times \frac{3-i}{3-i}\right) - \left(\frac{4+i}{3+i} \times \frac{3-i}{3-i}\right)$$

$$= \left(\frac{2(3-i)-6i(3-i)}{3^2-i^2}\right) - \left(\frac{4(3-i)+i(3-i)}{3^2-i^2}\right)$$

$$= \frac{6-2i-18i+6i^2}{9-(-1)} - \frac{12-4i+3i-i^2}{9-(-1)}$$

$$= \frac{6-20i+6(-1)}{10} - \frac{12-i-(-1)}{10}$$

$$= \frac{6-20i-6}{10} - \frac{12-i+1}{10}$$

$$= \frac{-20i}{10} - \frac{13-i}{10} = \frac{-20i-13+i}{10}$$

$$= \frac{-13-19i}{10} = \frac{-13}{10} - \frac{19}{10}i$$

4(v) $\left(\frac{1+i}{1-i}\right)^2$

$$= \left[\frac{1+i}{1-i} \times \frac{1+i}{1+i}\right]^2$$

$$= \left[\frac{(1+i)^2}{1-i^2}\right]^2 = \left[\frac{1+2i+i^2}{1-(-1)}\right]^2$$

$$= \left[\frac{1+2i-1}{2}\right]^2$$

$$= \left[\frac{2i}{2}\right]^2 = i^2 = -1$$

4(vi) $\frac{1}{(2+3i)(1-i)}$

$$= \frac{1}{2+3i} \times \frac{1}{1-i}$$

$$= \frac{1(2-3i)}{(2+3i)(2-3i)} \times \frac{(1+i)}{(1-i)(1+i)}$$

$$= \frac{(2-3i)(1+i)}{(4-9i^2)(1-i^2)} = \frac{2(1+i)-3i(1+i)}{[4-9(-1)][1-(-1)]}$$

$$= \frac{2+2i-3i-3i^2}{(4+9)(1+1)} = \frac{2-i-3(-1)}{(13)(2)}$$

$$= \frac{2+3-i}{26} = \frac{5-i}{26} = \frac{5}{26} - \frac{1}{26}i$$

Q.5. ***Calculate (a) $\bar{z}$ (b) $z + \bar{z}$ (c) $z - \bar{z}$ (d) $z\bar{z}$, for each of the following.***

(i) $z = -i$ (ii) $z = 2 + i$

(iii) $z = \dfrac{1+i}{1-i}$ (iv) $z = \dfrac{4-3i}{2+4i}$

Solution:-

5(i) $z = -i$

(a) $z = 0 - i$

$\bar{z} = 0 + i$

(b) $z + \bar{z}$

$= -i + i$

$= 0$

(c) $z - \bar{z}$

$= -i - (i)$

$= -i - i$

$= -2i$

(d) $z\bar{z}$

$= (-i)(i)$

$= -i^2$

$= -(-1)$

$= 1$

5(ii) $z = 2 + i$

(a) $\bar{z} = 2 - i$

(b) $z + \bar{z}$

$= (2 + i) + (2 - i)$

$= 2 + i + 2 - i$

$= 4$

(c) $z - \bar{z}$

$= (2 + i) - (2 - i)$

$= 2 + i - 2 + i$

$= 2i$

(d) $z\bar{z}$

$= (2 + i)(2 - i)$

$= 4 - (i^2)$

$= 4 - (-1)$

$= 4 + 1 = 5$

5(iii) $z = \dfrac{1+i}{1-i}$

(a) $= \dfrac{1+i}{1-i} \times \dfrac{1+i}{1+i}$

$$= \frac{(1+i)^2}{1-i^2} = \frac{1-i^2+2i}{1+1}$$

$$= \frac{1-1+2i}{2} = i = 0 + i$$

$\therefore \quad \overline{z} = 0 - i$

(b) $z + \overline{z}$

$= i - i = 0$

(c) $z - \overline{z}$

$= i - (-i)$

$= i + i = 2i$

(d) $z\,\overline{z}$

$= (i)(-i)$

$= (-i)^2 = -(-1) = 1$

5(iv) $z = \dfrac{4-3i}{2+4i}$

$$= \frac{4-3i}{2+4i} \times \frac{(2-4i)}{2-4i}$$

$$= \frac{(4-3i)(2-4i)}{(2)^2-(4i)^2}$$

$$= \frac{4(2-4i)-3i(2-4i)}{4-16i^2}$$

$$= \frac{8-16i-6i+12i^2}{4-16(-1)}$$

$$= \frac{8-22i+12(-1)}{4+16} = \frac{-4-22i}{20}$$

$$= \frac{-4}{20} - \frac{22}{20}i$$

$$z = -\frac{1}{5} - \frac{11}{10}i$$

(a) $\overline{z} = -\dfrac{1}{5} - \dfrac{11}{10}i$

(b) $z+\bar{z} = \left(-\frac{1}{5}-\frac{11}{10}i\right) + \left(-\frac{1}{5}-\frac{11}{10}i\right)$

$= -\frac{1}{5}-\frac{11}{10}i-\frac{1}{5}+\frac{11}{10}i$

$= -\frac{1}{5}-\frac{1}{5} = \frac{-2}{5}$

(c) $z-\bar{z} = \left(-\frac{1}{5}-\frac{11}{10}i\right)-\left(-\frac{1}{5}-\frac{11}{10}i\right)$

$= -\frac{1}{5}-\frac{11}{10}i+\frac{1}{5}-\frac{11}{10}i$

$= -\frac{11}{10}i-\frac{11}{10}i$

$= -\frac{-22}{10}i = -\frac{11}{5}i$

(d) $z\bar{z} = (-\frac{1}{5}-\frac{11}{10}i)(-\frac{1}{5}+\frac{11}{10}i)$

$= \left(-\frac{1}{5}\right)^2 - \left(\frac{11}{10}i\right)^2$

$= \frac{1}{25}-\frac{121}{100}i^2$

$= \frac{1}{25}-\frac{121}{100}(-1)$

$= \frac{1}{25}+\frac{121}{100} = \frac{4+121}{100}$

$= \frac{125}{100} = \frac{5}{4}$

Q.6. ***If $z = 2+3i$ and $w = 5-4i$, show that***

(i) $\overline{z+w} = \bar{z}+\bar{w}$ **(ii)** $\overline{z-w} = \bar{z}-\bar{w}$

(iii) $\overline{zw} = \bar{z}\,\bar{w}$ **(iv)** $\overline{\left(\frac{z}{w}\right)} = \frac{\bar{z}}{\bar{w}}$ $w \neq 0$

(v) $\frac{1}{2}(z+\bar{z})$ is a real part of z

(vi) $\frac{1}{2i}(z - \bar{z})$ is the imaginary part of z

Solutions:-

6(i) $z = 2 + 3i$

and

$w = 5 - 4i$

$\overline{z + w} = \bar{z} + \bar{w}$ (To show)

$\bar{z} = 2 - 3i$

$\bar{w} = 5 + 4i$

Now,

$z + w = 2 + 3i + 5 - 4i$

$= 7 - i$

$\overline{z + w} = 7 + i$ *(i)*

$\bar{z} + \bar{w} = (2 - 3i) + (5 + 4i)$

$= 2 - 3i + 5 + 4i$

$= 7 + i$ *(ii)*

$\overline{z + w} = \bar{z} + \bar{w}$ (From i and ii)

6(ii) $\overline{z - w} = \bar{z} - \bar{w}$

We find $z - w$

$z - w = (2 + 3i) - (5 - 4i)$

$= 2 + 3i - 5 + 4i$

$= -3 + 7i$

$\overline{z - w} = -3 - 7i$ *(i)*

$\bar{z} = 2 - 3i$

$\bar{w} = 5 + 4i$

$\bar{z} - \bar{w} = (2 - 3i) - (5 + 4i)$

$= 2 - 3i - 5 - 4i$

$= -3 - 7i$ (ii)

$\overline{z - w} = \overline{z} - \overline{w}$ (From i and ii)

6(iii) $\overline{zw} = \overline{z}\ \overline{w}$

We find zw

$zw = (2 + 3i)(5 - 4i)$

$= 2(5 - 4i) + 3i(5 - 4i)$

$= 10 - 8i + 15i - 12i^2$

$= 10 + 7i - 12(-1)$

$= 10 + 7i + 12$

$= 22 + 7i$

$\overline{zw} = 22 - 7i$ (i)

$\overline{z} = 2 - 3i$

$\overline{w} = 5 + 4i$

$\overline{z}\ \overline{w} = (2 - 3i)(5 + 4i)$

$= 2(5 + 4i) - 3i(5 + 4i)$

$= 10 + 8i - 15i - 12i^2$

$= 10 - 7i - 12(-1)$

$= 10 - 7i + 12$

$= 22 - 7i$ (ii)

$\overline{z}\ \overline{w} = \overline{z}\ \overline{w}$ (From i and ii)

6(iv) $\overline{\left(\dfrac{z}{w}\right)} = \dfrac{\overline{z}}{\overline{w}}$ $(w \neq 0)$

We find $\dfrac{z}{w}$

$\dfrac{z}{w} = \dfrac{2 + 3i}{5 - 4i}$

$= \dfrac{2 + 3i}{5 - 4i} \times \dfrac{5 + 4i}{5 + 4i}$

$$= \frac{(2+3i)(5+4i)}{(5)^2-(4i)^2}$$

$$= \frac{2(5+4i)+3i(5+4i)}{25-16i^2}$$

$$= \frac{10+8i+15i+12i^2}{25-16(-1)}$$

$$= \frac{10+23i+12(-1)}{41} = \frac{10-23i-12}{41}$$

$$= \frac{-2+23i}{41}$$

$$\frac{z}{w} = \frac{-2}{41} + \frac{23}{41}i$$

$$\overline{\left(\frac{z}{w}\right)} = -\frac{2}{41} - \frac{23}{41}i \qquad (i)$$

Now,

$$\frac{\bar{z}}{\bar{w}} = \frac{2-3i}{5+4i}$$

$$= \frac{2-3i}{5+4i} \times \frac{5-4i}{5-4i}$$

$$\frac{\bar{z}}{\bar{w}} = \frac{(2-3i)(5-4i)}{(5)^2+(4i)^2}$$

$$= \frac{2(5-4i)-3i(5-4i)}{25-16i^2}$$

$$= \frac{10-8i-15i+12i^2}{25-16(-1)}$$

$$\frac{10-23i-12}{41}$$

$$= \frac{-2-23i}{41}$$

$$= \frac{-2}{41} - \frac{23}{41}i \qquad (ii)$$

$\overline{\left(\frac{z}{w}\right)} = \frac{\bar{z}}{\bar{w}}$ (From i and ii)

6(v) $\frac{1}{2}(z + \bar{z})$

$= \frac{1}{2}[2 + 3i + 2 - 3i]$

$= \frac{1}{2}[4]$

$= 2$ (2 is a real part of z)

6(vi) $\frac{1}{2}(z - \bar{z})$

$= \frac{1}{2}[(2 + 3i) - (2 - 3i)]$

$= \frac{1}{2}[2 + 3i - 2 + 3i]$

$= \frac{1}{2}[6i]$

$= 3i$ (3 is imaginary part of z)

Q.7. *Solve the following equations for real x and y.*

(i) $(2 - 3i)(x + yi) = 4 + i$

(ii) $(3 - 2i)(x + yi) = 2(x - 2yi) + 2i - 1$

(iii) $(3 + 4i)^2 - 2(x - yi) = x + yi$

Solution:-

7(i) $(2 - 3i)(x + yi) = 4 + i$

$2(x + yi) - 3i(x + yi) = 4 + i$

$2x + 2yi - 3xi - 3yi^2 = 4 + i$

$2x + 2yi - 3xi - 3y(-1) = 4 + i$

$2x + 2yi - 3xi + 3y = 4 + i$

$2x + 3y + 2yi - 3xi = 4 + i$

$(2x + 3y) + (2y - 3x)i = 4 + i$

$2x + 3y = 4$ (i)

and $2y - 3x = 1$ (ii)

Multiply eq. *(i)* by *3* and eq *(ii)* by *2*.

$$6x + 9y = 12 \qquad (iii)$$
$$4y - 6x = 2 \qquad (iv)$$

From *(iv)* + *(iii)*

$$13y = 14$$
$$y = \frac{14}{13}$$

Putting $y = \frac{14}{13}$ in *(i)*

$$2x + 3\left(\frac{14}{13}\right) = 4$$
$$2x + \frac{42}{13} = 4$$
$$2x = 4 - \frac{42}{13}$$
$$= \frac{52 - 42}{13}$$
$$2x = \frac{10}{13}$$
$$x = \frac{10}{2 \times 13}$$
$$x = \frac{5}{13}$$

Thus $x = \frac{5}{13}, y = \frac{14}{13}$

7(ii) $$(3 - 2i)(x + yi) = 2(x - 2yi) + 2i - 1$$
$$3x + 3yi - 2xi - 2yi^2 = 2x - 4yi + 2i - 1$$
$$3x + 3yi - 2xi - 2y(-1) = 2x - 4yi + 2i - 1$$
$$3x + 3yi - 2xi + 2y = 2x - 4yi + 2i - 1$$
$$3x - 2x + 3yi + 4yi - 2xi + 2y = -1 + 2i$$
$$\Rightarrow x + 2y + 7yi - 2xi = -1 + 2i$$
$$(x + 2y) + (7y - 2x)i = -1 + 2i$$

Thus $$x + 2y = -1 \qquad (i)$$

and $7y \quad 2x \quad 2$ *(ii)*

(iii) $2x + 4y \quad -2$

Multiplying eq. (i) by 2.

$11y = 0$ Adding *(iii)*, *(ii)*

$y \quad 0$

$x + 2(0) \quad -1$

Putting $y \quad 0$ in eq. *(i)*

$x + 0 = -1$

$x = -1$

Thus $x = -1, y = 0$

7(iii) $(3 + 4i)^2 - 2(x - yi) = x + yi$

$9 + 24i + 16i^2 - 2x + 2yi = x + yi$

$9 + 24i + 16(-1) \quad 2x + 2yi - x - yi = 0$

$9 + 24i - 16 \quad 3x + yi = 0$

$-3x + yi \quad 16 - 9 \quad 24i$

$-3x + yi \quad 7 - 24i$

Thus $-3x = 7$ *(i)*

and $y = -24$ *(ii)*

$x = -\frac{7}{3}$ From *(i)*

Thus $x = -\frac{7}{3}, y = -24$

Unit 3 LOGARITHMS

Scientific Notation:

Scientist have developed a concise precise and convenient method to write very small or very large numbers, that is called scientific notation of expressing an ordinary number.

A number in the form $a \times 10^n$ where $1 \le a \le 10$ and n is an integer in called the scientific notation.

Exercise 3.1

Q.1. Express each of the following numbers in scientific notation.

(i)	*5700*	**(ii)**	*49,800,000*
(iii)	*96,000, 000*	**(iv)**	*416.9*
(v)	*83, 000*	**(vi)**	*0.00643*
(vii)	*0.0074*	**(viii)**	*60, 000, 000*
(ix)	*0.00000000395*	**(x)**	$\frac{275,000}{0.0025}$

Solution:-

1(i) *5700*

$= 5.700 \times 1000$

$= 5.7 \times 10^3$

1(ii) *49, 800, 000*

$= 4.9800000 \times 10000000$

$= 4.98 \times 10^7$

1(iii) 96, 000, 000

$9.6000000 \times 10000000$

$= 9.6 \times 10^7$

1(iv) 416.9

$= 4.169 \times 100$

$= 4.169 \times 10^2$

1(v) 83000

$= 8.3000 \times 10000$

$= 8.3 \times 10^4$

1(vi) 0.00643

$= \frac{6.43}{1000} = \frac{6.43}{10^3} = 6.43\ 10^{-3}$

1(vii) .0074

$= \frac{7.4}{1000} = \frac{7.4}{10^3} = 7.4\ 10^{-3}$

1(viii) 60, 000, 000

$= 6.0000000 \times 10000000$

$= 6 \times 10^7$

1(ix) 0.00000000395

$= \frac{3.95}{1000000000} = \frac{3.95}{10^9}$

$= 3.95\ 10^{-9}$

1(x) $\frac{275000}{.0025}$

$= \frac{2.7500 \times 100000}{\frac{2.5}{1000}}$

$= \frac{2.75 \times 10^5}{2.5 \times 10^{-3}}$

Q.2. *Express the following numbers in ordinary notation.*

(i) 6×10^{-4} (ii) 5.06×10^{10}

(iii) 9.018×10^{-6} *(iv)* 7.865×10^{8}

Solution:-

2(i) 6×10^{-4}

$$= 6 \times \frac{1}{10^4} = .0006$$

2(ii) 5.06×10^{10}

$$= 50600000000$$

2(iii) 9.018×10^{-6}

$$= 9.018 \times \frac{1}{10^6} = .000009018$$

2(iv) 7.865×10^{8}

$$= 786500000$$

Logarithm of a Real Number

If $a^x = y$, then x is called the logarithm of 'y' to the base 'a' and is written as $log_a y = x$, where $a > 0$, $a \neq 1$ and $y > 0$.

If $a^x = y$

then $log_a y = x$ and

if $log_a y = x$

then $a^x = y$

Thus $a^x = y$ and $log_a y = x$ are equivalent equations.

$a^x = y$ is exponential form and

$log_a y = x$ is logarithmic form.

Remember:

$a^0 = 1 \Rightarrow log_a 1 = 0$ and

$a^1 = a \Rightarrow log_a a = 1$

Exercise 3.2

Q.1. Find the common logarithms of the following numbers.

(i)	*232.92*	*(ii)*	*29.326*
(iii)	*0.00032*	*(iv)*	*0.3206*

Solution:-

1(i) $\log 232.92$

(i) Integral part has three *(3)* digits.

$\therefore \quad ch = 3 - 1 = 2$

(ii) To find mantissa, we round off *232.92* to *232.9*

Log Table | **Mean Differences**

	0 1	2	3	4	5	6	7	8	9	1 2 3 4 5 6 7 8 9
23		3655								17

(iii) We see the log table and find the row corresponding to *23*.

(iv) We proceed horizontally and reach column corresponding to *2*. The number at the intersection in *3655*.

(v) Proceeding ahead we find a number in the mean difference column under *9*. It is *17*. Mantissa is $3655 + 17 = 3672$.

$\therefore \log 232.92 = 2.3672$

1(ii) $\log 29.326$

(i) Integral part has *(2)* digits.

$\therefore \quad ch = 2 - 1 = 1$

(ii) To find mantissa, we round off *29.326* to *29.32*

Log Table | **Mean Differences**

	0	1	2	3	4	5	6	7	8	9	1 2 3 4 5 6 7 8 9
29				4669							4

(iii) We read the log table and find the row corresponding to *29*.

(iv) We proceed horizontally and reach column corresponding to *3*. The number at the intersection is *4669*.

(v) Proceed ahead and we find a number in the mean differences column under *3*. It is *4*. Mantissa is $4669 + 4 = 4673$

$\therefore \log 29.326 = 1.4673$

1(iii) $\log 0.00032$

(i) There are three zeros on the right of the decimal point:-

$\therefore \quad ch = \bar{4}$

Log Table

	0	1	2	3	4	5	6	7	8	9	**Mean Differences** 123456789
32	↓ 5051										

(ii) Corresponding to *32* in the log table, we proceed horizontally and under *0* find a number *5051*.

Mantissa $= 5051$

Thus $\log 0.00032 = \bar{4}.5051$

1(iv) $\log 0.3206$

(i) The integral part of 0.3206 is 'o' and there is no zero on the right of decimal point.

$\therefore ch = \bar{1}$

Log Table

	0	1	2	3	4	5	6	7	8	9	**Mean Differences** 123456789
32	↓ 5051										↓ 8

(ii) Corresponding to *32* in the log table, we proceed horizontally and under *0* find a number *5051*.

(iii) We procced ahead and find in the mean difference column under 6 number 8.

$\therefore$ $mantissa = 5051 + 8 = 5059$

$$\log 0.3206 = \bar{1}.5059$$

Q.2. If $\log 31.09 = 1.4926$, find the values of following.

(i) $\log 3.109$ *(ii)* $\log 310.9$

(iii) $\log 0.003109$ *(iv)* $\log 0.3109$

Solution:-

(2) $\log 31.09 = 1.4926$ (Given)

2(i) $\log 3.109 = 0.4926$ (Mantissa does not change)

2(ii) $\log 310.9 = 2.4926$

2(iii) $\log 0.003109 = \bar{3}.4926$

2(iv) $\log 0.3109 = \bar{1}.4926$

Q.3. Find the numbers whose common logarithms are:

(i) 3.5621 *(ii)* $\bar{1}.7427$

3(i) Let the number $= x$

$\log x = 3.5622$

here ch 3

Anti-log Table

	0 1 2 3 4 5 6 7 8 9	**Mean Differences** 123456789
.56	3648	2

(i) Corresponding to *.56* in the **antilog table**, we proceed horizontally and under *2*, find a number *3648*.

(ii) We proceed ahead and find in the mean difference column under *2* number *2*.

Now $3648 + 2 = 3650$

$ch = 3$, therefore, there are four digits in the number.

Required Number $x = 3650$

3(ii) Let the number $= x$

$log\, x = \bar{2}.7427$

here $ch = \bar{2}$.

Therefore, there will be ONE zeros on the right side of decimal point.

	Anti-log Table 0 1 2 3 4 5 6 7 8 9	**Mean Differences** 1 2 3 4 5 6 7 8 9
.74	5521	9

(i) Corresponding to $.\overline{74}$ in the antilog table, we proceed horizontally and under 2, find a number 5521.

(ii) We proceed ahead and find in the mean difference column under 7 number 9.

Now $5521 + 9 = 5530$

$ch = \bar{2}$, therefore, there are four digits in the number.

Required Number $x = .05530$

Q.4. What replacement for the unknown in each of the following will make the statement true?

(i) $log_3 81 = L$ *(ii)* $log_a 6 = 0.5$

(iii) $log_5 n = 2$ *(iv)* $10^p = 40$

4(i) $log_3 81 = L$

$3^L = 81$ (Writing in the exponential form)

$3^L = 3^4$

$\therefore L = 4$

4(ii) $log_a 6 = 0.5$

$6 = a^{.5}$ (Writing in the exponential form)

$6 = a^{5/10}$

$6 = a^{1/2}$

$6^2 = (a^{1/2})^2$ (Squaring)

$36 = a^{1/2 \times 2}$

$36 = a$

Thus $a = 36$

4(iii) $\log_5{}^n = 2$

$n = 5^2$ (Writing in the exponential form)

$n = 25$

4(iv) $10^p = 40$

$3^L = \log 40$ (Taking log)

$p \log 10 = 1.6021$ (using log table)

$p = 1.6021$ $(\log 10 = 1)$

Q.5. Evaluate:

(i) $\log_2 \frac{1}{128}$ (ii) $\log 512$ to the base $2\sqrt{2}$

5(i) $\log_2 \frac{1}{128}$

Let $x = \log_2{}^{1/28}$

$2^x = \frac{1}{128}$ (Writing in exponential form)

$= \frac{1}{2^7}$

$2^x = 2^{-7}$

Thus $x = -7$

5(ii) $\log 512$ to the base $2\sqrt{2}$

Let $x = \log_{2\sqrt{2}}{}^{512}$

$(2\sqrt{2})^x = 512$ (Writing in exponential form)

$(\sqrt{2 \times 2 \times 2})^x = 512$

$(\sqrt{8})^x = 512$

$(8^{1/2})^x = 8^3$

$8^{x/2} = 8^3$

Thus $\frac{x}{2} = 3$

Hence $x = 3 \times 2 = 6$

Q.6. Find the value of x from the following statements.

(i) $\log_2{}^x = 5$ (ii) $\log_{81}{}^9 = x$

(iii) $\log_{64}{}^8 = \frac{x}{2}$ (iv) $\log_x{}^{64} = 2$

(v) $\log_3{}^x = 4$

6(i) $\log_2 x = 5$

$x = 2^5$ (Writing in exponential form)

$x = 32$

6(ii) $\log_{81}{}^9 = 5$

$(81)^x = 9$ (Writing in exponential form)

$(9^2)^x = 9$

$9^{2x} = 9$

$\therefore 2x = 1$

or $x = \frac{1}{2}$

6(iii) $\log_{64}{}^8 = \frac{x}{2}$

$(64)^{\frac{x}{2}} = 8$ (Writing in exponential form)

$8^{2(\frac{x}{2})} = 8$

$8^x = 8$

Thus $x = 1$

6(iv) $\log_x{}^{64} = 2$

$x^2 = 64$ (Writing in exponential form)

$x^2 = 8^2$

Thus $x = 8$

6(v) $\log_3{}^x = 4$

$x = 3^4$ (Writing in exponential form)

Thus $x = 81$

(Laws of Logarithm)

Law (i)

$$\log_a{(mn)} = \log_a m + \log_a n$$

Proof

Let $\log_a m = x, \log_a n = y$

$\therefore \quad a^x = m \quad (i)$

and $\quad a^y = n \quad (ii)$

$a^x \times a^y = m \times n$ from (i) × (ii)

$a^{x+y} = mn$

$\log_a mn = x + y$ (Writing log form)

$= \log_a m + \log_a n$ (Writing values of x, y)

Law (ii)

$$\log_a m/n = \log_a m - \log_a n$$

Proof:

Let $\log_a m = x$

and $\log_a n = y$ (Writing in exponential form)

$a^x = m \quad (i)$

$a^y = n \quad (ii)$ Dividing (i) by (ii)

$$\frac{a^x}{a^y} = \frac{m}{n}$$

$a^{x-y} = \frac{m}{n}$ (Writing in log form)

$\log_a m/n = x - y$ (Putting values of x, y)

$\log_a m/n = \log_a m - \log_a n$

Law (iii)

$$\log_a m^n = n \log_a m$$

Proof:

Let $\log_a m^n = x$

$or\ a^x = m^n$

Let $\log_a^m = y$

or $a^y = m$ (i)

$a^x = m^n$

$= (a^y)^n$ From (i)

$a^x = a^{yn}$

$a^x = a^{yn}$

So $x = yn = ny$

$\therefore \log_a^{m^n} = n \log_a^m$

Law (iv) ***(Change of base)***

$$\log_a^n = \log_b^n \times \log_a^b = \frac{\log_b^n}{\log_b^a}$$

Proof:

Let $\log_b^n = x \Rightarrow n = b^x$

$\log_a^n = \log_a{}^{b^x} = x \log_a^b$ (Taking log to base a)

$\therefore \log_a^n = \log_b^n \log_a^b$

$\log_b^a \times \log_a^b = \log_a^a = 1$

or $\log_a^b = \dfrac{1}{\log_b^a}$ Putting value of $\log_a^b$ in (i)

$$\log_a^n = \frac{\log_b^n}{\log_b^a}$$

Exercise 3.3

Q.1. Write the following into sum or difference

(i) $\log (A \times B)$ (ii) $\log \frac{15.2}{30.5}$

(iii) $\log \frac{21 \times 5}{8}$ (iv) $\log \sqrt[3]{\frac{7}{15}}$

(v) $\log \frac{(22)^{1/3}}{5^3}$ (vi) $\log \frac{25 \times 47}{29}$

Solution:-

1(i) $\log (A \times B)$

$= \log A + \log B$

1(ii) $\log \frac{15.2}{30.5}$

$= \log 15.2 - \log 30.5$

1(iii) $\log \frac{25 \times 5}{8}$

$= \log 25 + \log 5 - \log 8$

1(iv) $\log \sqrt[3]{\frac{7}{15}}$

$= \log \left(\frac{7}{15}\right)^{1/3}$

$= \frac{1}{3} [\log 7 - \log 15]$

1(v) $\log \frac{(22)^{1/3}}{5^3}$

$= \log (22)^{1/3} - \log 5^3$

$= \frac{1}{3} \log 22 - 3\log 5$

1(vi) $\log \frac{25 \times 49}{29}$

$= \log 25 + \log 47 - \log 29$

Q.2. Express as a single logarithm.

$\log x - 2\log x + 3\log(x+1) - \log(x^2-1)$

Solution:-

$$\log x - 2\log x + 3\log(x+1) - \log(x^2-1)$$
$$= \log x - \log x^2 + \log(x+1)^3 - \log(x^2-1)$$
$$= \log \frac{x(x+1)^3}{x^2(x^2-1)}$$
$$= \log \frac{(x+1)^3}{x(x^2-1)}$$
$$= \log \frac{(x+1)^3}{x(x+1)(x-1)}$$
$$= \log \frac{(x+1)^2}{x(x-1)}$$

Q.3. Write the following in the form of a single logarithm.

(i) $\log 21 + \log 5$ (ii) $\log 25 - 2\log 3$

(iii) $2\log x - 3\log y$ (iv) $\log 5 + \log 6 - \log 2$

Solution:-

3(i) $\log 21 + \log 5$

$= \log 21 \times 5$

3(ii) $\log 25 - 2\log 3$

$= \log 25 - \log 3^2$

$= \log \frac{25}{3^2}$

3(iii) $2\log x - 3\log y$

$= \log x^2 - \log y^3$

$= \log \frac{x^2}{y^3}$

3(iv) $\log 5 + \log 6 - \log 2$

$= \log \frac{5 \times 6}{2}$

Q.4. Calculate the following:

(i) $\log_3 2 \times \log_2 81$ (ii) $\log_5 3 \times \log_3 25$

Solution:-

4(i) $\log_3 2 \times \log_2 81$

$= \dfrac{\log 81}{\log 3} = \dfrac{\log 3^4}{\log 3}$

$= \dfrac{4 \log 3}{\log 3} = 4$

4(ii) $\log_5 3 \times \log_3 25$

$= \dfrac{\log 25}{\log 5} = \dfrac{\log 5^2}{\log 5}$

$= \dfrac{2 \log 5}{\log 5} = 2$

Q.5. If log 2 = 0.3010, log 3 = 0.4771, log 5 = 0.6990 then find the values of the following.

(i) $\log 32$ (ii) $\log 24$

(iii) $\log\sqrt{3\frac{1}{3}}$ (iv) $\log\frac{8}{3}$

(v) $\log 30$

Solution:-

5(i) $\log 32$

$= \log 2^5$

$= 5 \log 2 = 5(0.3010) = 1.5050$

5(ii) $\log 24$

$= \log 8 \times 3$

$= \log 2^3 \times 3 = \log 2^3 + \log 3$

$= 3 \log 2 + \log 3$

$= 3\,(0.3010) + 0.4771$

$= 0.9030 + 0.4771 = 1.3801$

5(iii) $\log\sqrt{3\frac{1}{3}}$

$$= \log\left(\frac{10}{3}\right)^{1/2}$$

$$= \frac{1}{2}\log\frac{10}{3}$$

$$= \frac{1}{2}\log\frac{2\times5}{3}$$

$$= \frac{1}{2}[\log 2 + \log 5 - \log 3]$$

$$= \frac{1}{2}[0.3010 + 0.6990 - 0.4771]$$

$$= \frac{1}{2}(.5229) = 0.2615$$

5(iv) $\log\frac{8}{3}$

$$= \log\frac{2^3}{3}$$

$$= \log 2^3 - \log 3$$

$$= 3\log 2 - \log 3 = 3(0.3010) - 0.4771$$

$$= 0.9030 - 0.4771 = 0.4259$$

5(v) $\log 30$

$$= \log 2\times3\times5$$

$$= \log 2 + \log 3 + \log 5$$

$$= 0.3010 + 0.4771 + 0.6990$$

$$= 1.4771$$

Exercise 3.4

Q.1. Use log tables to find the value of:

(i) 0.8176×13.64 (ii) $(789.5)^{1/8}$

(iii) $\dfrac{0.678 \times 9.01}{0.0234}$ (iv) $\sqrt[5]{2.709} \times \sqrt[7]{1.239}$

(v) $\dfrac{(1.23)(0.6975)}{(0.0075)(1278)}$ (vi) $\sqrt[3]{\dfrac{0.7214 \times 20.37}{60.8}}$

(vii) $\dfrac{83 \times \sqrt[3]{92}}{127 \times \sqrt[5]{246}}$ (viii) $\dfrac{(438)^3\sqrt{0.056}}{(388)^4}$

1(i) 0.8176×13.64

Let $x = 0.8176 \times 13.64$

$\log x = \log(.8176 \times 13.64)$ (Taking log)

$= \log .8176 + \log 13.64$

$= \bar{1}.9125 + 1.1348$

$\log x = 1.0473$

Antilog $(\log x) =$ *Antilog* 1.0473 (Taking anti-log)

$x = 11.15$

1(ii) $(789.5)^{1/8}$

Let $x = (789.5)^{1/8}$

$\log x = \log (789.5)^{1/8}$

$= \frac{1}{8} \log 789.5$

$= \frac{1}{8}[2.8974] = 0.3622$

Antilog $(\log x) =$ *Antilog* $(0.3622) = 2.302$

1(iii) $\dfrac{0.678 \times 9.01}{0.0234}$

Let $x = \dfrac{0.678 \times 9.01}{0.0234}$

$\log x = \log \frac{0.678 \times 9.01}{0.0234}$ Taking log

$= \log 0.678 + \log 9.01 - \log 0.0234$

$= \bar{1}.8312 + 0.9547 - (\bar{2}.3692)$

$= \bar{1}.8312 + 0.9547 - \bar{2} - .3692$

$= -1 + .8312 + 0.9547 + 2 - .3692 = 2.4167$

$Antilog (\log x) = Antilog\ 2.4167$ Taking anti-log

$x = 261$

1(iv) $\sqrt[5]{2.709} \times \sqrt[7]{1.239}$

Let $x = \sqrt[5]{2.709} \times \sqrt[7]{1.239}$

$x = (2.709)^{1/5} \times (1.239)^{1/7}$ (Taking log)

$\log x = \log (2.709)^{1/5} \times (1.239)^{1/7}$

$= \frac{1}{5} \log 2.709 + \frac{1}{7} \log 1.239$

$= \frac{1}{5}(0.4328) + \frac{1}{7}(0.0931)$

$= 0.0866 + 0.0133$

$\log x = 0.0999$ (Taking anti-log)

$Antilog (\log x) = Antilog\ 0.0999$

$x = 1.258$

1(v) $\frac{(1.23)(0.6975)}{(0.0075)(1278)}$

Let $x = \frac{(1.23)(0.6975)}{(0.0075)(1278)}$

$\log x = \log \frac{(1.23)(0.6975)}{(0.0075)(1278)}$ (Taking log)

$\log x = \log 1.23 + \log 0.6975 - \log 0.0075 - \log 1278$

$= 0.0899 + \bar{1}.8435 - (\bar{3}.8751) - (3.1065)$

$= 0.0899 - 1 + .8435 + 3 - .8751 - 3.1065$

$= -1.0482 = -1.0482 - 1 + 1$

$\log x = \bar{2}.9518$

$Antilog(\log x) = Antilog\ \bar{2}.9518$ (Taking anti-log)

$x = .0895$

$1(vi)$ $\sqrt[3]{\dfrac{0.7214 \times 20.37}{60.8}}$

Let $x = \sqrt[3]{\dfrac{0.7214 \times 20.37}{60.8}}$

$x = \log\left(\dfrac{0.7214 \times 20.37}{60.8}\right)^{1/3}$ (Taking log)

$\log x = \dfrac{1}{3}[\log .7214 + \log 20.37 - \log 60.8]$

$= \dfrac{1}{3}[\bar{1}.8582 + 1.3090 - 1.7839]$

$= \dfrac{1}{3}[-1 + .8582 + 1.3090 - 1.7839]$

$= \dfrac{1}{3}[-0.6167] = -0.2056$

$= -0.2056 - 1 + 1 = \bar{1}.7944$

$Antilog(\log x) = Antilog\ \bar{1}.7944$ (Taking anti-log)

$x = .6229$

$1(vii)$ $\dfrac{83 \times \sqrt[3]{92}}{127 \times \sqrt[5]{246}}$

Let $x = \dfrac{83 \times \sqrt[3]{92}}{127 \times \sqrt[5]{246}}$

$x = \dfrac{83 \times (92)^{1/3}}{127 \times (246)^{1/5}}$ (Taking log)

$\log x = \log 83 + \dfrac{1}{3}\log 92 - \log 127 - \dfrac{1}{5}\log$

$$= 1.9191 + \frac{1}{3}(1.9638) - (2.1038) - \frac{1}{5}(2.3909)$$

$$= 1.9191 + 0.6546 - 2.1038 - 0.4782$$

$$= 2.5737 - 2.582 = -.0083 = -.0083 - 1 + 1$$

$$\log x = \bar{1}.9917$$

$$Antilog\ (\log x) = Antilog\ \bar{1}.9917 \quad \text{(Taking anti-log)}$$

$$x = .9811$$

1(viii) $\dfrac{(438)^3\sqrt{0.056}}{(388)^4}$

Let $x = \dfrac{(438)^3\sqrt{0.056}}{(388)^4}$

$$x = \frac{(438)^3 \times (.056)^{1/2}}{(388)^4}$$

$$\log x = 3\log 438 + \frac{1}{2}\log(.056) - 4\log 388 \quad \text{(Taking log)}$$

$$= 3(2.6415) + \frac{1}{2}(\bar{2}.7482) - 4(2.5888)$$

$$= 7.9245 + \bar{1}.3741 - 10.3552$$

$$= 7.9245 - 1 + .3741 - 10.3552$$

$$\log x = -3.0566 = -3.0566 - 1 + 1$$

$$= \bar{4}.9434$$

$$Antilog\ (\log x) = Antilog\ \bar{4}.9434 \quad \text{(Taking anti-log)}$$

$$x = 0.0008778$$

Q.2. A gas is expanding according to the law $pv^n = C$, find C when $p = 80$, $v = 3.1$ $n = \frac{5}{4}$

Solution:-

$$C = pv^n$$

Putting values of *p, v, n*

$$C = 80(3.1)^{5/4}$$

$$\log C = \log 80 + \frac{5}{4}\log 3.1 \qquad \text{(Taking log)}$$

$$= 1.9031 + \frac{5}{4}(0.4970)$$

$$= 1.9031 + \frac{2.4570}{4}$$

$$= 1.9031 + 0.6143$$

$$\log C = 2.5174$$

$$\text{Antilog}(\log C) = \text{Antilog } 2.5174 \qquad \text{(Taking anti-log)}$$

$$C = 329.2$$

Q.3. The formula $p = 90\,(5)^{-q/10}$ applies to the demand of a product, where q is the number of units and p is the price of one unit. How many units will be demanded if the price is Rs 18.00?

Solution:-

$$p = 90(5)^{-q/10}$$

$$18 = \log 90(5)^{-q/10} \qquad \textit{Putting } p = 18.00$$

$$\log 18 = 90(5)^{-q/10} \quad \text{(Taking log)}$$

$$= \log 90 + \log 5^{-q/10}$$

$$\log 18 = \log 90 - \frac{q}{10}\log 5$$

$$1.2552 = 1.9542 - \frac{q}{10}(0.6990)$$

$$= 1.9542 - .0699\,q$$

$$.0699q = 1.9542 - 1.2552$$

$$.0699q = .6990$$

$$q = \frac{.06990}{.0699} = \frac{6990}{0699}$$

$$= 10 \qquad \text{Units}$$

Q.4. If $A = \pi r^2$, find A, when $\pi = \frac{22}{7}$ and $r = 15$.

Solution:-

$A = \pi r^2$

Putting $\pi = \frac{22}{7}$, $r = 15$

$A = \frac{22}{7} \times (15)^2$ (Taking log)

$= \log 22 - \log 7 + 2 \log 15$

$= 1.3424 - 0.8451 + 2(1.1761)$

$= 1.3424 - 0.8451 + 2.3522$

$\log A = 2.8494$

$\text{Antilog} (\log A) = \text{Antilog } 2.8495$ (Taking anti-log)

$A = 707.1$

Q.5. If $V = \frac{1}{3}\pi r^2 h$, find V, when $\pi = \frac{22}{7}$, $r = 2.5$ and $h = 4.2$

Solution:-

$V = \frac{1}{3}\pi r^2 h$

Putting values of h, r, π

$V = \frac{1}{3} \times \frac{22}{7} \times (2.5)^2 \times 4.2$

$\log V = \log \frac{1}{3} \times \frac{22}{7} \times (2.5)^2 \times 4.2$ (Taking log)

$= \log 22 + 2 \log 2.5 + \log 4.2 - \log 3 - \log 7$

$= 1.3424 + 2(0.3979) + 0.6232 - 0.4771 - 0.8451$

$= 1.3424 + .7958 + .6232 - 0.4771 - 0.8451$

$\log V = 1.4392$

$\text{Antilog} (\log V) = \text{Antilog } 1.4392$ (Taking anti-log)

$V = 27.49$

Unit 4 ALGEBRAIC EXPRESSIONS AND ALGEBRAIC FORMULAS

Types of Algebraic Expressions

(i) Polynomial expressions.

(ii) Rational Expression.

(iii) Irrational expression.

(i) Polynomial expressions.

A polynomial consists of one or more than one terms. The exponent of variable or variables is zero or positive integer. For example $12, -x, 3x - 4y, x^2 - 2x + 1, \sqrt{3}x$ are polynomial expressions, where as $\frac{2}{x^3}, x + \frac{1}{x}, \sqrt{3x} - \frac{1}{\sqrt{3x}}$ and $35x - \frac{6}{x}$ are not polynomials.

(ii) Rational Expression.

$\frac{p(x)}{q(x)}$ is a rational expression where as p(x), q(x) are polynomials and $q(x) \neq 0$.

(iii) Irrational expression.

If an algebraic expression cannot be written in the form $\frac{p(x)}{q(x)}$ where p(x), q(x) are polynomials and $q(x) \neq 0$ in called an irrational expression. For example $\sqrt{x}$, $x^{2/3}$, $\sqrt[3]{x^2y^2}$ and $\sqrt{x^2+1}$, $\frac{1}{\sqrt{x}+2}$ are irrational expressions.

Exercise 4.1

Q.1 Identify whether the following algebraic expressions are polynomials (Yes or No)

(i) $3x^2 + \frac{1}{x} - 5$ (ii) $3x^3 - 4x^2 - x\sqrt{x} + 3$

(iii) $x^2 \quad 3x + \sqrt{2}$ (iv) $\frac{3x}{2x-1} + 8$

Solutions:

1(i) $3x^2 + \frac{1}{x} - 5$ ***No*** Reason: $\frac{1}{x}$

1(ii) $3x^3 - 4x^2 - x\sqrt{x} + 3$ ***No*** Reason: $\sqrt{x}$

1(iii) $x^2 \quad 3x + \sqrt{2}$ ***Yes***

1(iv) $\frac{2x}{2x-1} + 8$ ***No*** Reason:

$\frac{1}{2x-1}$

Q.2 State whether each of the following expression is a rational expression or not.

(i) $\frac{3\sqrt{x}}{3\sqrt{x}+5}$ (ii) $\frac{x^3 - 2x^2 + \sqrt{3}}{2 + 3x - x^2}$

(iii) $\frac{x^2 + 6x + 9}{x^2 - 9}$ (iv) $\frac{2\sqrt{x} + 3}{2\sqrt{x} - 3}$

Solutions:

2(i) $\frac{3\sqrt{x}}{3\sqrt{x}+5}$

Since $3\sqrt{x}$ and $3\sqrt{x} + 5$ are not polynomials therefore, the given expression is not an algebraic rational

2(ii) $\dfrac{x^3 - 2x^2 + \sqrt{3}}{2 + 3x - x^2}$

It is rational expressions as $x^3 - 2x^2 + \sqrt{3}$ and $2 + 3x - x^2$ are polynomials.

2(iii) $\dfrac{x^2 + 6x + 9}{x^2 - 9}$

It is a rational expression. Both denominator and numerator are polynomials.

2(iv) $\dfrac{2\sqrt{x} + 3}{2\sqrt{x} - 3}$

It is not a rational expression. $2\sqrt{x} + 3$ and $2\sqrt{x} - 3$ are not polynomials.

Q.3 ***Reduce the following rational expressions to the lowest forms.***

(i) $\dfrac{120x^2y^3z^5}{30x^3yz^2}$ *(ii)* $\dfrac{8a(x+1)}{2(x^2-1)}$

(iii) $\dfrac{(x+y)^2 - 4xy}{(x-y)^2}$ *(iv)*

$\dfrac{(x^3 - y^3)(x^2 - 2xy + y^2)}{(x-y)(x^2 + xy + y^2)}$

(v) $\dfrac{(x+2)(x^2-1)}{(x+1)(x^2-4)}$ *(vi)* $\dfrac{x^2 - 4x + 4}{2x^2 - 8}$

(vii) $\dfrac{64x^5 - 64x}{(8x^2+8)(2x+2)}$ *(viii)* $\dfrac{9x^2 - (x^2-4)^2}{4 + 3x - x^2}$

Solutions:

3(i) $\dfrac{120x^2y^3z^5}{30x^3yz^2}$

$$= \frac{30 \times 4y^{3-1}z^{1-2}}{30x^{3-2}} \quad 4y\ z$$

3(ii) $\frac{8a(x+1)}{2(x^2-1)}$

$$= \frac{2 \times 4a(x+1)}{2(x+1)(x-1)} = \frac{4a}{x-1}$$

3(iii) $\frac{(x+y)^2 - 4xy}{(x-y)^2}$

$$= \frac{x^2 + y^2 + 2xy - 4xy}{(x-y)^2}$$

$$= \frac{x^2 - 2xy + y^2}{x^2 - 2xy + y^2} = 1$$

3(iv) $\frac{(x^3 - y^3)(x^2 - 2xy + y^2)}{(x-y)(x^2 + xy + y^2)}$

$$= \frac{(x^3 - y^3)(x^2 - 2xy + y^2)}{(x^3 - y^3)}$$

$$= x^2 - 2x + y^2 = (x-y)^2$$

3(v) $\frac{(x+2)(x^2-1)}{(x+1)(x^2-4)}$

$$= \frac{(x+2)(x^2-1^2)}{(x+1)(x^2-2^2)}$$

$$= \frac{(x+2)(x+1)(x-1)}{(x+1)(x+2)(x-2)} = \frac{x-1}{x-2}$$

3(vi) $\frac{x^2 - 4x + 4}{2x^2 - 8}$

$$x^2 - 2x - 2x + 4$$

$$\frac{x(x-2)-2(x-2)}{2(x^2-2^2)}$$

$$=\frac{(x-2)(x-2)}{2(x-2)(x+2)}$$

$$=\frac{x-2}{2(x+2)}$$

3(vii) $$\frac{64x^5-64x}{(8x^2+8)(2x+2)}$$

$$=\frac{64x(x^4-1)}{8(x^2+1)(2)(x+1)}$$

$$=\frac{64x[(x^2)^2-(1)^2]}{16(x^2+1)(x+1)}$$

$$=\frac{64x(x^2+1)(x^2-1)}{16(x^2+1)(x+1)}$$

$$=\frac{4x(x^2+1)(x+1)(x-1)}{(x^2+1)(x+1)}$$

$$=4x(x-1)$$

3(viii) $$\frac{9x^2-(x^2-4)^2}{4+3x-x^2}$$

$$=\frac{(3x)^2-(x^2-4)^2}{4+3x-x^2}$$

$$=\frac{[3x+(x^2-4)][3x-(x^2-4)]}{(4+3x-x^2)}$$

$$=\frac{(3x+x^2-4)(3x-x^2+4)}{(4+3x-x^2)}$$

$$=\frac{(x^2+3x-4)(4+3x-x^2)}{(4+3x-x^2)}$$

$= x^2 + 3x - 4$

Q4 ***Evaluate (a)*** $\dfrac{x^3y - 2z}{xz}$ ***for***

(i) $x = 3, y = -1, z = -2$ ***(ii)*** $x = -1, y = -9, z = 4$

(b) $\dfrac{x^2y^3 - 5z^4}{xyz}$ ***for*** $x = 4, y = -2, z = -1$

Solutions:

4(a)(i) $\dfrac{x^3y - 2z}{xz}$

Putting $x = 3, y = -1, z = -2$

$$= \frac{(3)^3(-1) - 2(-2)}{(3)(-2)}$$

$$= \frac{(27)(-1) + 4}{-6}$$

$$= \frac{-27 + 4}{-6} = \frac{-23}{-6}$$

$$= \frac{23}{6} = 3\frac{5}{6}$$

4(a)(ii) $\dfrac{x^3y - 2z}{xz}$

Putting $x = -1, y = -9, z = 4$

$$= \frac{(-1)^3(-9) - 2(4)}{(-1)(4)}$$

$$= \frac{(-1)(-9) - 8}{-4}$$

$$= \frac{9 - 8}{-4} = -\frac{1}{4}$$

4(b) $\dfrac{x^2y^3 - 5z^4}{xyz}$

Putting $x = 4, y = -2, z = -1$

$= \dfrac{(4)^2(-2)^3 - 5(-1)^4}{(4)(-2)(-1)}$

$= \dfrac{(16)(-8) - 5(1)}{8}$

$= \dfrac{-128 - 5}{8}$

$= \dfrac{-133}{8} = -16\dfrac{5}{8}$

Q.5 ***Perform the indicated operation and simplify.***

(i) $\dfrac{15}{2x-3y} - \dfrac{4}{3y-2x}$ ***(ii)*** $\dfrac{1+2x}{1-2x} - \dfrac{1-2x}{1+2x}$

(iii) $\dfrac{x^2-25}{x^2-36} - \dfrac{x+5}{x+6}$ ***(iv)***

$\dfrac{x}{x-y} - \dfrac{y}{x+y} - \dfrac{2xy}{x^2-y^2}$

(v) $\dfrac{x-2}{x^2+6x+9} - \dfrac{x+2}{2x^2-18}$

(vi) $\dfrac{1}{x-1} - \dfrac{1}{x+1} - \dfrac{2}{x^2+1} - \dfrac{4}{x^4-1}$

Solutions:

5(i) $\dfrac{15}{2x-3y} - \dfrac{4}{3y-2x}$

$= \dfrac{15}{2x-3y} + \dfrac{4}{2x-3y}$ (N.T.S)

$= \dfrac{15+4}{2x-3y}$

$$= \frac{19}{2x-3y}$$

5(ii) $\frac{1+2x}{1-2x} - \frac{1-2x}{1+2x}$

$$= \frac{(1+2x)(1+2x)-(1-2x)(1-2x)}{(1-2x)(1+2x)}$$

$$= \frac{(1+4x+4x^2)-(1-4x+4x^2)}{(1)^2-(2x)^2}$$

$$= \frac{1+4x+4x^2-1+4x-4x^2}{1-4x^2}$$

$$= \frac{8x}{1-4x^2}$$

5(iii) $\frac{x^2-25}{x^2-36} - \frac{x+5}{x+6}$

$$= \frac{x^2-5^2}{x^2-6^2} - \frac{x+5}{x+6}$$

$$= \frac{(x+5)(x-5)}{(x+6)(x-6)} - \frac{x+5}{x+6}$$

$$= \frac{(x+5)(x-5)-(x+5)(x-6)}{(x+6)(x-6)}$$

$$= \frac{x^2-25-(x^2-x-30)}{x^2-6^2}$$

$$= \frac{x^2-25-x^2+x+30}{x^2-36}$$

$$= \frac{x+5}{x^2-36}$$

5(iv) $\frac{x}{x-y}-\frac{y}{x+y}-\frac{2xy}{x^2-y^2}$

$$=\frac{x}{x-y}-\frac{y}{x+y}-\frac{2xy}{(x-y)(x+y)}$$

$$=\frac{x(x+y)-y(x-y)-2xy}{(x-y)(x+y)}$$

$$=\frac{x^2+xy-xy+y^2-2xy}{(x-y)(x+y)}$$

$$=\frac{x^2+y^2-2xy}{(x-y)(x+y)}$$

$$=\frac{(x-y)^2}{(x-y)(x+y)}$$

$$=\frac{x-y}{x+y}$$

5(v) $\frac{x-2}{x^2+6x+9}-\frac{x+2}{2x^2-18}$

$$=\frac{x-2}{x^2+3x+3x+9}-\frac{x+2}{2(x^2-9)}$$

$$=\frac{x-2}{x(x+3)+3(x+3)}-\frac{x+2}{2(x^2-3^2)}$$

$$=\frac{x-2}{(x+3)(x+3)}-\frac{x+2}{2(x+3)(x-3)}$$

$$=\frac{2(x-2)(x-3)-(x+2)(x+3)}{2(x+3)(x+3)(x-3)}$$

$$=\frac{2x^2-10x+12-x^2-5x-6}{2(2x+3)^2(x-3)}$$

$$=\frac{x^2-15x+6}{2(x+3)(x-3)}$$

5(vi) $\frac{1}{x-1}-\frac{1}{x+1}-\frac{2}{x^2+1}-\frac{4}{x^4-1}$

$$=\left(\frac{1}{x-1}-\frac{1}{x+1}\right)-\left(\frac{2}{x^2+1}+\frac{4}{(x^2)^2-1^2}\right)$$

$$=\left(\frac{x+1-x+1}{(x-1)(x+1)}\right)-\left(\frac{2}{x^2+1}+\frac{4}{(x^2+1)(x^2-1)}\right)$$

$$=\frac{2x}{x^2-1}-\left(\frac{2(x^2-1)+4}{(x^2+1)(x^2-1)}\right)$$

$$=\frac{2x}{x^2-1}-\frac{2x^2-2+4}{(x^2+1)(x^2-1)}$$

$$=\frac{2x}{x^2-1}-\frac{2x^2+2}{(x^2+1)(x^2-1)}$$

$$=\frac{2x}{x^2-1}-\frac{2(x^2+1)}{(x^2+1)(x^2-1)}$$

$$=\frac{2x}{x^2-1}-\frac{2}{x^2-1}=0$$

Q.6 ***Perform the indicated operation and simplify***

(i) $(x^2-49).\ \frac{5x+2}{x+7}$ ***(ii)*** $\frac{4x-12}{x^2-9}\div\frac{18-2x^2}{x^2+6x+9}$

(iii) $\frac{x^6-y^6}{x^2-y^2}\div(x^4+x^2y^2+y^4)$ ***(iv)*** $\frac{x^2-1}{x^2+2x+1}\cdot\frac{x+5}{1-x}$

(v) $\dfrac{x^2+xy}{y(x+y)} \cdot \dfrac{x^2+xy}{y(x+y)} \div \dfrac{x^2-x}{xy-2y}$

Solutions:

6(i) $(x^2-49).\ \dfrac{5x+2}{x+7}$

$= (x^2-7^2) \times \dfrac{5x+2}{x+7}$

$= (x-7)(x+7) \times \dfrac{5x+2}{x+7}$

$= (x-7)(5x+2)$

$= 5x^2 - 33x - 14$

6(ii) $\dfrac{4x-12}{x^2-9} \div \dfrac{18-2x^2}{x^2+6x+9}$

$= \dfrac{4(x-3)}{x^2-3^2} \div \dfrac{2(9-x^2)}{x^2+3x+3x+9}$

$= \dfrac{4(x-3)}{(x-3)(x+3)} \div \dfrac{2(3^2-x^2)}{x(x+3)+3(x+3)}$

$= \dfrac{4}{x+3} \div \dfrac{2(3+x)(3-x)}{(x+3)(x+3)}$

$= \dfrac{4}{x+3} \div \dfrac{2(3-x)}{x+3}$

$= \dfrac{4}{x+3} \times \dfrac{x+3}{2(3-x)}$

$= \dfrac{2}{3-x}$

$x^6 - y^6$

$$=\frac{(x^2)^3-(y^2)^3}{x^2-y^2}\div(x^4+x^2y^2+y^4)$$

$$=\frac{(x^2-y^2)(x^4+x^2y^2+y^4)}{(x^2-y^2)}\times\frac{1}{x^4+x^2y^2+y^4}$$

$$=1$$

6(iv) $\dfrac{x^2-1}{x^2+2x+1}\cdot\dfrac{x+5}{1-x}$

$$=\frac{x^2-1^2}{x^2+x+x+1}\times\frac{x+5}{-(x-1)}$$ N.T.S

$$=\frac{(x-1)(x+1)}{x(x+1)+1(x+1)}\times\frac{-(x+5)}{(x-1)}$$

$$=\frac{(x-1)(x+1)}{(x+1)(x+1)}\times\frac{-(x+5)}{(x-1)}$$

$$=\frac{-(x+5)}{x+1}$$

6(v) $\dfrac{x^2+xy}{y(x+y)}\cdot\dfrac{x^2+xy}{y(x+y)}\div\dfrac{x^2-x}{xy-2y}$

$$=\frac{x(x+y)}{y(x+y)}\times\frac{x(x+y)}{y(x+y)}\times\frac{xy-2y}{x^2-x}$$

$$=\frac{x(x+y)}{y(x+y)}\times\frac{x(x+y)}{y(x+y)}\times\frac{y(x-2)}{x(x-1)}$$

$$=\frac{x(x-2)}{y(x-1)}$$

Algebraic Formulae

(i) $(a+b)^2 = a^2 + 2ab + b^2$

(ii) $(a-b)^2 = a^2 - 2ab + b^2$

(iii) $(a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$

(iv) $(a+b)^2 - (a-b)^2 = 4ab$

(v) $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$= a^2 + b^2 + c^2 + 2(ab + bc + ca)$

(vi) $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$= a^3 + b^3 + 3ab(a+b)$

(vii) $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$= a^3 - b^3 - 3ab(a-b)$

(viii) $a^3 - b^3 = (a+b)(a^2 - ab + b^2)$

$= (a-b)(a^2 + ab + b^2)$

Exercise 4.2

Q.1(i) If $a + b = 10$ and $a - b = 6$, then find the value of $a^2 + b^2$

(ii) If $a + b = 5$ and $a - b = \sqrt{17}$ the find the value of ab.

1(i) If $a + b = 10$ and $a - b = 6$, then find the value of $a^2 + b^2$

Solutions: $a + b = 10$ Given

$a - b = 6$

$a^2 + b^2 = ?$

We know that

$$2(a^2 + b^2) = (a + b)^2 + (a - b)^2$$

Putting value of a + b and a − b

$$= (10)^2 + (6)^2$$

$$= 100 + 36$$

$$2(a^2 + b^2) = 136$$

$$a^2 + b^2 = \frac{136}{2} \quad \text{(Dividing by 2)}$$

$$\therefore \quad a^2 + b^2 = 68$$

1(ii) If $a + b = 5$ and $a - b = \sqrt{17}$ the find the value of ab.

Solution: $a + b = 5$ Given

$a - b = \sqrt{17}$

$ab = ?$

We know that

$$(a + b)^2 - (a - b)^2 = 4ab$$

Putting values of a + b and a − b

$$(5)^2 - (\sqrt{17})^2 = 4ab$$

$$25 - 17 = 4ab$$

$$8 = 4ab$$

or $\quad 4ab = 8$

or $\quad ab = \frac{8}{4}$

$\therefore \quad ab = 2$

Q.2 If $a^2 + b^2 + c^2 = 45$ and $a + b + c = -1$, find the value of $ab + bc + ca$

Solution: $a^2 + b^2 + c^2 = 45$ Given

$a + b + c = 1$

$ab + bc + ca = ?$

$(a + b + c)^2 = (a^2 + b^2 + c^2) + 2(ab + bc + ca)$

Putting $a + b + c = -1$, $a^2 + b^2 + c^2 = 45$

$(-1)^2 = 45 + 2(ab + bc + ca)$

$1 - 45 = 2(ab + bc + ca)$

$-44 = 2(ab + bc + ca)$

$-22 = ab + bc + ca$ (Dividing by 2)

or $\quad ab + bc + ca = -22$

Q.3 If $m + n + p = 10$ and $mn + np + mp = 27$, find the value of $m^2 + n^2 + p^2$

Solution: $m + n + p = 10$ Given

$mn + np + mp = 27$

$m^2 + n^2 + p^2 = ?$

Formula

$(m + n + p)^2 = m^2 + n^2 + p^2 + 2(mn + np + mp)$

Putting $mn + np + mp = 27$, $m + n + p = 10$

$(10)^2 = m^2 + n^2 + p^2 + 2(27)$

$100 = m^2 + n^2 + p^2 + 54$

or $\quad m^2 + n^2 + p^2 = 100 - 54$

$= 46$

Q.4 ***If $x^2 + y^2 + z^2 = 78$ and $xy + yz + zx = 59$, Find the value of $x + y + z$***

Solution:

$$x^2 + y^2 + z^2 = 78 \quad \text{Given}$$
$$xy + yz + zx = 59$$
$$x + y + z = ?$$

Formula

$$(x + y + z)^2 = x^2 + y^2 + z^2 + 2(xy + yz + zx)$$

Putting $xy + yz + zx = 59$, $x^2 + y^2 + z^2 = 78$

$$(x + y + z)^2 = 78 + 2(59)$$
$$= 78 + 118$$
$$(x + y + z)^2 = 196$$
$$x + y + z = \pm\sqrt{196} \quad \text{(Taking square root)}$$
$$x + y + z = \pm 14$$

Q.5 ***If $x + y + z = 12$ and $x^2 + y^2 + z^2 = 64$, find the value of $xy + yz + zx$***

Solution:

$$x + y + z = 12 \quad \text{Given}$$
$$x^2 + y^2 + z^2 = 64$$
$$xy + yz + zx = ?$$

Formula

$$(x + y + z)^2 = x^2 + y^2 + z^2 + 2(xy + yz + zx)$$

Putting value of $x + y + z = 12$, $x^2 + y^2 + z^2 = 64$

$$(12)^2 = 64 + 2(xy + yz + zx)$$
$$144 = 64 + 2(xy + yz + zx)$$
$$144 - 64 = 2(xy + yz + zx)$$
$$80 = 2(xy + yz + zx)$$

Thus

$$2(xy + yz + zx) = 80$$
$$xy + yz + zx = 40 \quad \text{(Dividing by 2)}$$

Q.6 If $x + y = 7$ and $xy = 12$, then find the value of $x^3 + y^3$.

Solution:

$$x + y = 7 \quad \text{Given}$$
$$xy = 12$$
$$x^3 + y^3 = ?$$

Fourmula

$$(x + y)^3 \quad x^3 + y^3 + 3xy(x + y)$$

Putting value of $x + y = 7$, $xy = 12$

$$(7)^3 = x^3 + y^3 + 3(12)(7)$$
$$343 = x^3 + y^3 + 252$$
$$343 - 252 = x^3 + y^3$$
$$91 = x^3 + y^3$$

or
$$x^3 + y^3 = 91$$

Q.7 If $3x + 4y = 11$ and $xy = 12$, then find the value of $27x^3 + 64y^3$.

Solution:

$$3x + 4y = 11 \quad \text{Given}$$
$$xy = 12$$
$$27x^3 + 64y^3 = ?$$

Formula

$$(3x + 4y)^3 = (3x)^3 + (4y)^3 + 3(3x)(4y)(3x + 4y)$$

Putting value of $3x + 4y = 11$, $xy = 12$

$$(11)^3 = 27x^3 + 64y^3 + 3 \times 3 \times 4(xy)(11)$$
$$1331 = 27x^3 + 64y^3 + 3 \times 3 \times 4 \times 12 \times 11$$
$$1331 = 27x^3 + 64y^3 + 4752$$
$$1331 - 4752 = 27x^3 + 64y^2$$
$$-3421 = 27x^3 + 64y^3$$

or
$$27x^3 + 64y^3 = -3421$$

Q.8 If $x - y = 4$ and $xy = 21$, then find the value of $x^3 - y^3$

Solution:

$$x - y = 4 \text{ Given}$$
$$xy = 21$$
$$x^3 - y^3 = ?$$

Formula

$$(x-y)^3 = x^3 - y^3 - 3xy(x-y)$$

Putting value of $x - y = 4$, $xy = 21$

$$(4)^3 = x^3 - y^3 - 3(21)(4)$$
$$64 = x^3 - y^3 - 252$$
$$64 + 252 = x^3 - y^3$$
$$316 = x^3 - y^3$$

or $$x^3 - y^3 = 316$$

Q.9 If $5x - 6y = 13$ and $xy = 6$, then find the value of $125x^3 - 216y^3$

Solution: $5x - 6y = 13$ Given

$$xy = 6$$
$$125x^3 - 216y^3 = ?$$

Formula

$$(5x-6y)^3 = (5x)^3 - (6y)^3 - 3(5x)(6y)(5x-6y)$$

Putting value of $5x - 6y = 13$, $xy = 6$

$$(13)^3 = 125x^3 - 216y^3 - 3 \times 5 \times 6 + xy \times (13)$$
$$2197 = 125x^3 - 216y^3 - 1170 \times 6$$
$$2197 = 125x^3 - 216y^3 - 7020$$
$$2197 + 7020 = 125x^3 - 216y^3$$
$$9217 = 125x^3 - 216y^3$$

or $$125x^3 - 216y^3 = 9217$$

***Q.10* If $x + \frac{1}{x} = 3$, then find the value of $x^3 + \frac{1}{x^3}$**

Solution: $x + \frac{1}{x} = 3$ Given

$$x^3 + \frac{1}{x^3} = ?$$

Formula

$$\left(x+\frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3(x)\left(\frac{1}{x}\right)\left(x+\frac{1}{x}\right)$$

Putting value of $x + \frac{1}{x} = 3$

$$(3)^3 = x^3 + \frac{1}{x^3} + 3(3)$$

$$27 = x^3 + \frac{1}{x^3} + 9$$

$$27 - 9 = x^3 + \frac{1}{x^3}$$

$$18 = x^3 + \frac{1}{x^3}$$

or $$x^3 + \frac{1}{x^3} = 18$$

Q.11 **If $x - \frac{1}{x} = 7$, then find the value of $x^3 - \frac{1}{x^3}$**

Solution: $x + \frac{1}{x} = 7$ Given

$$x^3 - \frac{1}{x^3} = ?$$

Formula

$$\left(x-\frac{1}{x}\right)^3 = x^3 - \frac{1}{x^3} - 3(x)\left(\frac{1}{x}\right)\left(x-\frac{1}{x}\right)$$

Putting value of $x - \frac{1}{x} = 3$

$$(7)^3 = x^3 - \frac{1}{x^3} - 3(7)$$

$$343 = x^3 - \frac{1}{x^3} - 21$$

$$343 + 21 = x^3 - \frac{1}{x^3}$$

$$364 = x^3 - \frac{1}{x^3}$$

or $$x^3 - \frac{1}{x^3} = 364$$

Q.12 If $\left(3x+\frac{1}{3x}\right) = 5$***, then find the value of*** $27x^3 + \frac{1}{27x^3}$

Solution: $$3x + \frac{1}{3x} = 5 \quad \text{Given}$$

$$\left(3x+\frac{1}{3x}\right)^3 = (5)^3 \quad \text{(Cubing)}$$

$$(3x)^3 + \left(\frac{1}{3x}\right)^3 + 3(3x)\left(\frac{1}{3x}\right)\left(3x+\frac{1}{3x}\right) = 125$$

$$27x^3 + \frac{1}{27x^3} + 3 \times (5) = 125$$

$$27x^3 + \frac{1}{27x^3} = 125 - 15$$

$$27x^3 + \frac{1}{27x^3} = 110$$

Q.13 If $\left(5x-\frac{1}{5x}\right) = 6$***, then find the value of***

$\left(125x^3 - \frac{1}{125x^3}\right)$

Solution: $$5x - \frac{1}{5x} = 6 \quad \text{(Given)}$$

$$\left(5x-\frac{1}{5x}\right)^3 = (6)^3 \quad \text{(Cubing)}$$

$$(5x)^3 - \frac{1}{(5x)^3} - 3(5x)\left(\frac{1}{5x}\right)\left(5x - \frac{1}{5x}\right) = 216$$

$$125x^3 - \frac{1}{125x^3} - 3(6) = 216$$

$$125x^3 - \frac{1}{125x^3} = 216 + 18$$

$$125x^3 - \frac{1}{125x^3} = 234$$

Q.14 Factorize:

(i) $x^3 - y^3 - x + y$

(ii) $8x^3 - \dfrac{1}{27y^3}$

14(i) $x^3 - y^3 - x + y$

Solution:

$$= x^3 - y^3 - (x - y)$$

$$= (x - y)(x^2 + xy + y^2) - (x - y)$$

$$= (x - y)[x^2 + xy + y^2 - 1]$$

14(ii) $8x^3 - \dfrac{1}{27y^3}$

Solution:

$$= (2x)^3 - \left(\frac{1}{3y}\right)^3$$

$$= \left(2x - \frac{1}{3y}\right)\left[(2x)^2 + (2x)\left(\frac{1}{3y}\right) + \left(\frac{1}{3y}\right)^2\right]$$

$$= \left(2x - \frac{1}{3y}\right)\left[4x^2 + \frac{2x}{3y} + \frac{1}{9y^2}\right]$$

(ii) $(x^3 - y^3)(x^6 + x^3y^3 + y^6)$

(iii) $(x - y)(x + y)(x^2 + y^2)(x^2 + xy + y^2)(x^2 - xy + y^2)(x^4 - x^2y^2 + y^4)$

(iv) $(2x^2 - 1)(2x^2 + 1)(4x^4 + 2x^2 + 1)(4x^4 - 2x^2 + 1)$

Solutions: -

15(i) $(x^2 - y^2)(x^4 - x^2y^2 + y^4)$

Solution: $= (x^2 + y^2)(x^4 - x^2y^2 + y^4)$

Formula $[(x + y)(x^2 - xy + y^2) = x^3 + y^3]$

$= (x^2)^3 + (y^2)^3$

$= x^6 + y^6$

15(ii) $(x^3 - y^3)(x^6 + x^3y^3 + y^6)$

Solution: $[(x - y)(x^2 + xy + y^2) = x^3 - y^3]$ Formula

$= (x^3)^3 - (y^3)^3$

$= x^{3\times3} - y^{3\times3}$

$= x^9 - y^9$

15(iii) $(x - y)(x + y)(x^2 + y^2)(x^2 + xy + y^2)(x^2 - xy + y^2)(x^4 - x^2y^2 + y^4)$

Solution: $= [(x - y)(x^2 - xy + y^2)]\,[(x + y)(x^2 - xy + y^2)]\,[(x^2 + y^2)(x^4 - x^2y^2 + y^4]$

$= [x^3 - y^3][x^3 + y^3]\,[(x^2)^3 + (y^2)^3]$

$= [(x^3 - y^3)(x^3 + y^3)]\,[x^6 + y^6]$

$= [(x^3)^2 - (y^3)^2]\,[x^6 + y^6]$

$= (x^6 - y^6)(x^6 + y^6)$

$= (x^6)^2 - (y^6)^2$

$= x^{12} - y^{12}$

15(iv) $(2x^2 - 1)(2x^2 + 1)(4x^4 + 2x^2 + 1)(4x^4 - 2x^2 + 1)$

Solution: $= [(2x^2-1)(4x^4 +2x^2+1)]\,[(2x^2 + 1)(4x^4 - 2x^2+1)]$

$= [(2x^2)^3 - (1)^3]\,[(2x^2)^3 + (1)^3]$

SURDS

An irrational radical with rational radicand is called a surd.

Hence $\sqrt{2}, \sqrt{3}, \sqrt{\frac{3}{7}}$ are surds.

$\sqrt[n]{a}$ is surd if

(i) a is rational

(ii) the result $\sqrt[n]{a}$ is irrational.

e.g., $\sqrt{\pi}$ and $\sqrt{3+\sqrt{17}}$ **are** not surds as π and $3 + \sqrt{17}$ are not rational.

In $\sqrt[n]{a}$ n is degree of the surd.

$a - \sqrt{b}$ is called conjugate of $a + \sqrt{b}$.

If we change the denominator from its surd form to rational form, then this process is rationalization.

Exercise 4.3

Q.1 Express each of the following surd in the simplest form.

(i) $\sqrt{180}$ (ii) $3\sqrt{162}$

(iii) $\frac{3}{4}\sqrt[3]{128}$ (iv) $\sqrt[5]{96x^6y^7z^8}$

1(i) $\sqrt{180}$

Solution:

$$= \sqrt{2\times 2\times 3\times 3\times 5}$$
$$= \sqrt{2^2\times 3^2\times 5}$$
$$= 2\times 3\ \sqrt{5}$$
$$= 6\sqrt{5}$$

1(ii) $3\sqrt{162}$

Solution:

$$= 3\sqrt{9\times 9\times 2}$$
$$= 3\sqrt{9^2\times 2}$$
$$= 3\times 9\ \sqrt{2}$$
$$= 27\sqrt{2}$$

1(iii) $\frac{3}{4}\sqrt[3]{128}$

Solution:

$$= \frac{3}{4}\sqrt[3]{2^3\times 2^3\times 2}$$
$$= \frac{3}{4}\times 2\times 2\times\sqrt[3]{2}$$
$$= 3\sqrt[3]{2}$$

1(iv) $\sqrt[5]{96x^6y^7z^8}$

$= 2xyz\sqrt[5]{3xy^2z^3}$

Q.2 Simplify

(i) $\dfrac{\sqrt{18}}{\sqrt{3}\sqrt{2}}$ *(ii)* $\dfrac{\sqrt{21}\sqrt{9}}{\sqrt{63}}$

(iii) $\sqrt[5]{243x^5y^{10}z^{15}}$ *(iv)* $\dfrac{4}{5}\sqrt[3]{125}$

(v) $\sqrt{21}\times\sqrt{7}\times\sqrt{3}$

Solutions:

2(i) $\dfrac{\sqrt{18}}{\sqrt{3}\sqrt{2}}$

Solution: $= \sqrt{\dfrac{18}{3\times2}}$

$= \sqrt{\dfrac{3\times2\times3}{3\times2}} = \sqrt{3}$

2(ii) $\dfrac{\sqrt{21}\sqrt{9}}{\sqrt{63}}$

Solution: $= \sqrt{\dfrac{21\times9}{63}}$

$= \sqrt{\dfrac{21\times3\times3}{21\times3}} = \sqrt{3}$

2(iii) $\sqrt[5]{243x^5y^{10}z^{15}}$

Solution: $= (3^5x^5y^{10}z^{15})^{1/5}$

$= 3^{5\times1/5}x^{5\times1/5}y^{10\times1/5}z^{15\times1/5}$

$= 3xy^2z^3$

2(iv) $\dfrac{4}{5}\sqrt[3]{125}$

Solution: $= \frac{4}{5}\sqrt[3]{5^3}$

$= \frac{4}{5}\times 5^{3\times\frac{1}{3}}$

$= \frac{4}{5}\times 5 = 4$

2(v) $\sqrt{21}\times\sqrt{7}\times\sqrt{3}$

Solution: $= \sqrt{21\times 7\times 3}$

$= \sqrt{21\times 21}$

$[(21)^2]^{1/2} = (21)^{2\times 1/2} = 21$

Q.3 Simplify by combining similar terms.

(i) $\sqrt{45}-3\sqrt{20}+4\sqrt{5}$

(ii) $4\sqrt{12}+5\sqrt{27}-3\sqrt{75}+\sqrt{300}$

(iii) $\sqrt{3}(2\sqrt{3}+3\sqrt{3})$

(iv) $2(6\sqrt{5}-3\sqrt{5})$

3(i) $\sqrt{45}-3\sqrt{20}+4\sqrt{5}$

Solution: $= \sqrt{9\times 5}-3\sqrt{4\times 5}+4\sqrt{5}$

$= \sqrt{3^2\times 5}-3\sqrt{2^2\times 5}+4\sqrt{5}$

$= 3\sqrt{5}-3\times 2\sqrt{5}+4\sqrt{5}$

$\sqrt{5}[3-6+4]$

$= \sqrt{5}(1) = \sqrt{5}$

3(ii) $4\sqrt{12}+5\sqrt{27}-3\sqrt{75}+\sqrt{300}$

Solution: $4\sqrt{4\times 3}+5\sqrt{9\times 3}-3\sqrt{25\times 3}+\sqrt{100\times 3}$

$4\sqrt{2^2\times 3}+5\sqrt{3^2\times 3}-3\sqrt{5^2\times 3}+\sqrt{10^2\times 3}$

$8\sqrt{3}+15\sqrt{3}\quad 15\sqrt{3}+10\sqrt{3}$

$\sqrt{3}[8+15\quad 15+10]$

$\sqrt{3}(18)\quad 18\sqrt{3}$

3(iii) $\sqrt{3}(2\sqrt{3}+3\sqrt{3})$

Solution: $\left(\sqrt{3}\right)\left(\sqrt{3}\right)(2+3)$

$3\cdot 5\quad 15\quad \left(\sqrt{3}\right)^2(5)$

3(iv) $2(6\sqrt{5}-3\sqrt{5})$

Solution: $2(\sqrt{5})(6\quad 3)$

$2(\sqrt{5})(3)\quad 6\sqrt{5}$

Q.4 ***Simplify:***

(i) $(3+\sqrt{3})(3-\sqrt{3})$ ***(ii)*** $(\sqrt{5}+\sqrt{3})^2$

(iii) $(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})$ ***(iv)*** $\left(\sqrt{2}+\frac{1}{\sqrt{3}}\right)\left(\sqrt{2}-\frac{1}{\sqrt{3}}\right)$

(v) $(\sqrt{x}+\sqrt{y})(\sqrt{x}-\sqrt{y})(x+y)(x^2+y^2)$

4(i) $(3+\sqrt{3})(3-\sqrt{3})$

Solution: $(3)^2\quad(\sqrt{3})^2$

$9\quad 3\quad 6$

4(ii) $(\sqrt{5}+\sqrt{3})^2$

Solution: $(\sqrt{5})^2+(\sqrt{3})^2+2\sqrt{5}\times\sqrt{3}$

$5+3+2\sqrt{5\times 3}$

$8+2\sqrt{15}$

4(iii) $(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})$

Solution: $(\sqrt{5})^2-(\sqrt{3})^2$

4(iv) $\left(\sqrt{2}+\frac{1}{\sqrt{3}}\right)\left(\sqrt{2}-\frac{1}{\sqrt{3}}\right)$

Solution: $(\sqrt{2})^2-\left(\frac{1}{\sqrt{3}}\right)^2$

$2-\frac{1}{3}$

$\frac{6-1}{3}=\frac{5}{3}$

4(v) $(\sqrt{x}+\sqrt{y})(\sqrt{x}-\sqrt{y})(x+y)(x^2+y^2)$

Solution: $\left(\sqrt{x}+\sqrt{y}\right)\left(\sqrt{x}-\sqrt{y}\right)(x+y)(x^2+y^2)$

$\left[\left(\sqrt{x}\right)^2-\left(\sqrt{y}\right)^2\right](x+y)(x^2+y^2)$

$(x-y)(x+y)(x^2+y^2)$

$[(x-y)(x+y)](x^2+y^2)$

$(x^2-y^2)(x^2+y^2)$

$(x^2)^2-(y^2)^2=x^4-y^4$

Rationalization of Surds Definitions

(i) A surd which contains a single term is caved a monomial surd.

(ii) A surd which contains sum of two monomial surds or sum of a monomial surd and a rational number is called a binomial surd.

(iii) The conjugate of $x+\sqrt{y}$ is $x-\sqrt{y}$

Exercise 4.4

Q.1 ***Rationalize the denominator of the following:***

(i) $\frac{3}{4\sqrt{3}}$ *(ii)* $\frac{14}{\sqrt{98}}$

(iii) $\frac{6}{\sqrt{8}\sqrt{27}}$ *(iv)* $\frac{1}{3+2\sqrt{5}}$

(v) $\frac{15}{\sqrt{31}-4}$ *(vi)* $\frac{2}{\sqrt{5}-\sqrt{3}}$

(vii) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$ *(viii)* $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$

1(i) $\frac{3}{4\sqrt{3}}$

Solution: $\frac{3}{4\sqrt{3}} \times \left(\frac{\sqrt{3}}{\sqrt{3}}\right)$

$$= \frac{3\sqrt{3}}{4\times 3} = \frac{1}{4}\sqrt{3}$$

1(ii) $\frac{14}{\sqrt{98}}$

Solution: $\frac{14}{\sqrt{98}} \times \left(\frac{\sqrt{98}}{\sqrt{98}}\right)$

$$= \frac{14\sqrt{98}}{98}$$

$$= \frac{\sqrt{98}}{7} = \frac{\sqrt{7\times 7\times 2}}{7} = \frac{7\sqrt{2}}{7}$$

1(iii) $\frac{6}{\sqrt{8}\sqrt{27}}$

Solution: $= \frac{6}{\sqrt{8}\sqrt{27}} \times \left(\frac{\sqrt{8}\sqrt{27}}{\sqrt{8}\sqrt{27}}\right)$

$= \frac{6 \times \sqrt{8} \times \sqrt{27}}{8 \times 27}$

$= \frac{6 \times \sqrt{2^2 \times 2} \times \sqrt{3^2 \times 3}}{8 \times 27}$

$= \frac{6 \times 2\sqrt{2} \times 3\sqrt{3}}{8 \times 27}$

$= \frac{6 \times 2 \times 3\sqrt{2} \times \sqrt{3}}{8 \times 27}$

$= \frac{1}{6} \times \sqrt{2 \times 3} = \frac{1}{6} \times \sqrt{6} = \frac{\sqrt{6}}{6}$

1(iv) $\frac{1}{3 + 2\sqrt{5}}$

Solution: $= \frac{1}{3 + 2\sqrt{5}} \times \left(\frac{3 - 2\sqrt{5}}{3 - 2\sqrt{5}}\right)$

$= \frac{3 - 2\sqrt{5}}{(3)^2 - (2\sqrt{5})^2}$

$= \frac{3 - 2\sqrt{5}}{9 - 20}$

$= -\frac{1}{11}(3 - 2\sqrt{5})$

1(v) $\frac{15}{\sqrt{31} - 4}$

$\frac{15}{\sqrt{31} - 4} \times \left(\frac{\sqrt{31} + 4}{\ldots}\right)$

$$= \frac{15\left(\sqrt{31}+4\right)}{\left(\sqrt{31}\right)^2-(4)^2}$$

$$= \frac{15\left(\sqrt{31}+4\right)}{31-16}$$

$$= \frac{15\left(\sqrt{31}+4\right)}{15} = \sqrt{31}+4$$

1(vi) $\dfrac{2}{\sqrt{5}-\sqrt{3}}$

Solution: $$= \frac{2}{\sqrt{5}-\sqrt{3}} \times \left(\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}\right)$$

$$= \frac{2\left(\sqrt{5}+\sqrt{3}\right)}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}$$

$$= \frac{2\left(\sqrt{5}+\sqrt{3}\right)}{5-3}$$

$$= \frac{2\left(\sqrt{5}+\sqrt{3}\right)}{2} = \sqrt{5}+\sqrt{3}$$

1(vii) $\dfrac{\sqrt{3}-1}{\sqrt{3}+1}$

Solution: $$= \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\left(\sqrt{3}-1\right)}{\left(\sqrt{3}-1\right)}$$

$$= \frac{\left(\sqrt{3}-1\right)^2}{}$$

$$\frac{(\sqrt{3})^2+(1)^2-2\sqrt{3}(1)}{3-1}$$

$$\frac{3+1-2\sqrt{3}}{2}$$

$$\frac{4-2\sqrt{3}}{2}$$

$$\frac{2(2-\sqrt{3})}{2}=2-\sqrt{3}$$

1(viii) $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$

Solution: $\frac{(\sqrt{5}+\sqrt{3})}{(\sqrt{5}-\sqrt{3})}\times\frac{(\sqrt{5}+\sqrt{3})}{(\sqrt{5}+\sqrt{3})}$

$$\frac{(\sqrt{5}+\sqrt{3})^2}{(\sqrt{5})^2-(\sqrt{3})^2}$$

$$\frac{(\sqrt{5})^2+(\sqrt{3})^2+2\sqrt{5}\sqrt{3}}{5-3}$$

$$\frac{5+3+2\sqrt{15}}{2}=\frac{8+2\sqrt{15}}{2}$$

$$\frac{2(4+\sqrt{15})}{2}=4+\sqrt{15}$$

Q.2 Find the conjugate of $x+\sqrt{y}$

(i) $3+\sqrt{7}$ (ii) $4-\sqrt{5}$

(iii) $2+\sqrt{3}$ (iv) $2+\sqrt{5}$

(vii) $7 - \sqrt{6}$ **(viii)** $9 + \sqrt{2}$

2(i) $3 + \sqrt{7}$

Solution: Conjugate $= 3 - \sqrt{7}$

2(ii) $4 - \sqrt{5}$

Solution: Conjugate $= 4 + \sqrt{5}$

2(iii) $2 + \sqrt{3}$

Solution: Conjugate $= 2 - \sqrt{3}$

2(iv) $2 + \sqrt{5}$

Solution: Conjugate $= 2 - \sqrt{5}$

2(v) $5 + \sqrt{7}$

Solution: Conjugate $= 5 - \sqrt{7}$

2(vi) $4 - \sqrt{15}$

Solution: Conjugate $= 4 + \sqrt{15}$

2(vii) $7 - \sqrt{6}$

Solution: Conjugate $= 7 + \sqrt{6}$

2(viii) $9 + \sqrt{2}$

Solution: Conjugate $= 9 - \sqrt{2}$

Q.3 **(i)** **If $x = 2 - \sqrt{3}$, find $\frac{1}{x}$**

(ii) **If $x = 4 - \sqrt{17}$, find $\frac{1}{x}$**

(iii) **If $x = \sqrt{3} + 2$, find $x + \frac{1}{x}$**

3(i) $x = 2 - \sqrt{3}$

Solution: $\frac{1}{x} = \frac{1}{2-\sqrt{3}}$

$$= \frac{1}{\left(2-\sqrt{3}\right)} \times \frac{\left(2+\sqrt{3}\right)}{\left(2+\sqrt{3}\right)}$$

$$= \frac{2+\sqrt{3}}{(2)^2 - \left(\sqrt{3}\right)^2}$$

$$= \frac{2+\sqrt{3}}{4-3} = 2+\sqrt{3}$$

3(ii) $x = 4 - \sqrt{17}$

Solution: $\frac{1}{x} = \frac{1}{4-\sqrt{17}}$

$$= \frac{1}{4-\sqrt{17}} \times \frac{\left(4+\sqrt{17}\right)}{\left(4+\sqrt{17}\right)}$$

$$= \frac{4+\sqrt{17}}{(4)^2 - \left(\sqrt{17}\right)^2}$$

$$= \frac{4+\sqrt{17}}{16-17}$$

$$= \frac{4+\sqrt{17}}{-1} = -4-\sqrt{17}$$

3(iii) $x = \sqrt{3} + 2$

Solution: $\frac{1}{x} = \frac{1}{\sqrt{3}+2}$

$$= \frac{1}{\ } \times \frac{\left(\sqrt{3}+2\right)}{\ }$$

$$= \frac{\sqrt{3}-2}{\left(\sqrt{3}\right)^2-(2)^2}$$

$$= \frac{\sqrt{3}-2}{3-4}$$

$$= \frac{\sqrt{3}-2}{-1} = -\sqrt{3}+2$$

Now $\quad x = \sqrt{3}+2$

And $\quad \frac{1}{x} = -\sqrt{3}+2$

$$\therefore \quad x+\frac{1}{x} = (\sqrt{3}+2)+(-\sqrt{3}+2)$$

$$= \sqrt{3}+2-\sqrt{3}+2$$

$$= 4$$

Q.4 Simplify:

(i) $\dfrac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}}+\dfrac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}$

(ii) $\dfrac{1}{2+\sqrt{3}}+\dfrac{2}{\sqrt{5}-\sqrt{3}}+\dfrac{1}{2+\sqrt{5}}$

(iii) $\dfrac{2}{\sqrt{5}+\sqrt{3}}+\dfrac{1}{\sqrt{3}+\sqrt{2}}+\dfrac{3}{\sqrt{5}+\sqrt{2}}$

4(i) $\dfrac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}}+\dfrac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}$

Solution:

$$\frac{\left(1+\sqrt{2}\right)}{\left(\sqrt{5}+\sqrt{3}\right)}\times\frac{\left(\sqrt{5}-\sqrt{3}\right)}{\left(\sqrt{5}-\sqrt{3}\right)}+\frac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}\times\frac{\left(\sqrt{5}+\sqrt{3}\right)}{\left(\sqrt{5}+\sqrt{3}\right)}$$

$$\frac{(1+\sqrt{2})(\sqrt{5}-\sqrt{3})}{(\sqrt{5})^2-(\sqrt{3})^2}+\frac{(1-\sqrt{2})(\sqrt{5}+\sqrt{3})}{(\sqrt{5})^2-(\sqrt{3})^2}$$

$$\frac{(1+\sqrt{2})(\sqrt{5}-\sqrt{3})}{5-3}+\frac{(1-\sqrt{2})(\sqrt{5}+\sqrt{3})}{5-3}$$

$$\frac{\sqrt{5}-\sqrt{3}+\sqrt{2\times5}-\sqrt{2\times3}}{2}+\frac{\sqrt{5}+\sqrt{3}-\sqrt{2\times5}-\sqrt{2\times3}}{2}$$

$$\frac{\sqrt{5}-\sqrt{3}+\sqrt{10}-\sqrt{6}+\sqrt{5}+\sqrt{3}-\sqrt{10}-\sqrt{6}}{2}$$

$$\frac{2\sqrt{5}-2\sqrt{6}}{2}$$

$$\frac{2(\sqrt{5}-\sqrt{6})}{2}=\sqrt{5}-\sqrt{6}$$

4(ii) $\frac{2}{\sqrt{5}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{2}}-\frac{3}{\sqrt{5}+\sqrt{2}}$

Solution:

$$\frac{1(2-\sqrt{3})}{(2+\sqrt{3})(2-\sqrt{3})}+\frac{2(\sqrt{5}+\sqrt{3})}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})}+\frac{1(2-\sqrt{5})}{(2+\sqrt{5})(2-\sqrt{5})}$$

$$\frac{2-\sqrt{3}}{(2)^2-(\sqrt{3})^2}+\frac{2(\sqrt{5}+\sqrt{3})}{(\sqrt{5})^2-(\sqrt{3})^2}+\frac{2-\sqrt{5}}{(2)^2-(\sqrt{5})^2}$$

$$\frac{2-\sqrt{3}}{4-3}+\frac{2(\sqrt{5}+\sqrt{3})}{(\sqrt{5})^2-(\sqrt{3})^2}+\frac{2-\sqrt{5}}{(2)^2-(\sqrt{5})^2}$$

$$\cdots\sqrt{3}+\frac{2(\sqrt{5}+\sqrt{3})}{\cdots}+\frac{2-\sqrt{5}}{\cdots}$$

$$\frac{2-\sqrt{3}}{1}+\frac{2(\sqrt{5}+\sqrt{3})}{2}+\frac{2-\sqrt{5}}{-1}$$

$$2-\sqrt{3}+\sqrt{5}+\sqrt{3}-2+\sqrt{5}$$

$$2\sqrt{5}$$

4(iii) $\frac{2}{\sqrt{5}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{2}}-\frac{3}{\sqrt{5}+\sqrt{2}}$

Solution: $A=\frac{2}{\sqrt{5}+\sqrt{3}}$, $B=\frac{1}{\sqrt{3}+\sqrt{2}}$, $C=\frac{3}{\sqrt{5}+\sqrt{2}}$

$$A=\frac{2(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})}$$

$$=\frac{2(\sqrt{5}-\sqrt{3})}{(\sqrt{5})^2-(\sqrt{3})^2}$$

$$=\frac{2(\sqrt{5}-\sqrt{3})}{5-3}$$

$$=\frac{2(\sqrt{5}-\sqrt{3})}{2}=\sqrt{5}-\sqrt{3}$$

$$B=\frac{1}{\sqrt{3}+\sqrt{2}}$$

$$=\frac{1(\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})}$$

$$=\frac{1(\sqrt{3}-\sqrt{2})}{(\sqrt{3})^2-(\sqrt{2})^2}$$

$$= \frac{\sqrt{3}-\sqrt{2}}{3-2} = \sqrt{3}-\sqrt{2}$$

$$C \quad \frac{3\left(\sqrt{5}-\sqrt{2}\right)}{\left(\sqrt{5}+\sqrt{2}\right)\left(\sqrt{5}-\sqrt{2}\right)}$$

$$\frac{3\left(\sqrt{5}-\sqrt{2}\right)}{\left(\sqrt{5}\right)^2-\left(\sqrt{2}\right)^2}$$

$$= \frac{3\left(\sqrt{5}-\sqrt{2}\right)}{5-2}$$

$$\frac{3\left(\sqrt{5}-\sqrt{2}\right)}{3} = \sqrt{5}-\sqrt{2}$$

$A + B - C$

$\left(\sqrt{5}-\sqrt{3}\right)+\left(\sqrt{3}-\sqrt{2}\right)-\left(\sqrt{5}-\sqrt{2}\right)$

$= \sqrt{5}-\sqrt{3}+\sqrt{3}-\sqrt{2}-\sqrt{5}+\sqrt{2}$

$= 0$

Q.5 **(i)** If $x = 2+\sqrt{3}$, then find the value of $x - \frac{1}{x}$ an $\left(x-\frac{1}{x}\right)^2$

(ii) If $x = \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}$, then find the value of $x+\frac{1}{x}$, $x^2+\frac{1}{x^2}$ and $x^3+\frac{1}{x^3}$

5(i) **If $x = 2 + \sqrt{3}$, then find the value of $x - \frac{1}{x}$ and $\left(x - \frac{1}{x}\right)^2$**

Solution:

$$x = 2 + \sqrt{3}$$

$$\frac{1}{x} = \frac{1}{2+\sqrt{3}}$$

$$= \frac{1(2-\sqrt{3})}{(2+\sqrt{3})(2-\sqrt{3})}$$

$$= \frac{2-\sqrt{3}}{(2)^2 - (\sqrt{3})^2}$$

$$= \frac{2-\sqrt{3}}{4-3} = 2 - \sqrt{3}$$

Now $$x - \frac{1}{x} = (2+\sqrt{3}) - (2-\sqrt{3})$$

$$= 2 + \sqrt{3} - 2 + \sqrt{3}$$

Thus $$x - \frac{1}{x} = 2\sqrt{3}$$

$$\left(x - \frac{1}{x}\right)^2 = (2\sqrt{3})^2 \qquad \text{(Squaring)}$$

$$= 4 \times 3 = 12$$

5(ii) **If $x = \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}$, then find the value of $x + \frac{1}{x}$, $x^2 + \frac{1}{x^2}$ and $x^3 + \frac{1}{x^3}$**

Solution:

$$x = \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}$$

$$\frac{1}{x} = \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$$

Now $$x+\frac{1}{x} = \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}+\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$$

$$= \frac{\left(\sqrt{5}-\sqrt{2}\right)^2+\left(\sqrt{5}+\sqrt{2}\right)^2}{\left(\sqrt{5}+\sqrt{2}\right)\left(\sqrt{5}-\sqrt{2}\right)}$$

$$= \frac{5+2-2\sqrt{5\times 2}+5+2+2\sqrt{5\times 2}}{\left(\sqrt{5}\right)^2-\left(\sqrt{2}\right)^2}$$

$$= \frac{7-2\sqrt{10}+7+2\sqrt{10}}{5-2}$$

$$= \frac{14}{5-2}=\frac{14}{3}$$

$$x^2+\frac{1}{x^2} = ?$$

$$\left(x+\frac{1}{x}\right)^2 = \left(\frac{14}{3}\right)^2$$

$$x^2+\frac{1}{x^2}+2 = \frac{196}{9}$$

$$x^2+\frac{1}{x^2} = \frac{196}{9}-2$$

$$= \frac{196-18}{9}=\frac{178}{9}$$

$$x^3+\frac{1}{x^3} = ?$$

Now $$\left(x+\frac{1}{x}\right)^3 = \left(\frac{14}{3}\right)^3$$

$$x^3 + \frac{1}{x^3} + 3x \times \frac{1}{x}\left(x + \frac{1}{x}\right) = \frac{2744}{27}$$

$$x^3 + \frac{1}{x^3} + 3\left(\frac{14}{3}\right) = \frac{2744}{27}$$

$$x^3 + \frac{1}{x^3} + 14 = \frac{2744}{27}$$

$$x^3 + \frac{1}{x^3} = \frac{2744}{27} - 14$$

$$= \frac{2744 - 378}{27} = \frac{2366}{27}$$

Q.6 Determine the rational number a and b if

$$\frac{\sqrt{3}-1}{\sqrt{3}+1} + \frac{\sqrt{3}+1}{\sqrt{3}-1} = a + b\sqrt{3}$$

$$\frac{\sqrt{3}-1}{\sqrt{3}+1} + \frac{\sqrt{3}+1}{\sqrt{3}-1} = a + b\sqrt{3}$$

$$\frac{\left(\sqrt{3}-1\right)\left(\sqrt{3}-1\right)+\left(\sqrt{3}+1\right)\left(\sqrt{3}+1\right)}{\left(\sqrt{3}+1\right)\left(\sqrt{3}-1\right)} = a + b\sqrt{3}$$

$$\frac{\left(\sqrt{3}-1\right)^2+\left(\sqrt{3}+1\right)^2}{\left(\sqrt{3}\right)^2-(1)^2} = a + b\sqrt{3}$$

$$\frac{\left(3+1-2\sqrt{3}\right)+3+1+2\sqrt{3}}{3-1} = a + b\sqrt{3}$$

$$\frac{8}{2} = a + b\sqrt{3}$$

$$\Rightarrow \quad 4 + (0)\sqrt{3} = a + b\sqrt{3}$$

Unit 5 FACTORIZATION

The process of writing a polynomial as a product of two or more than two polynomials is called factorization. Every new polynomial is called a factor of the given polynomial.

For example: In $ab + bc = b(a + c)$, $b(a + c)$ is the factorization of $ab + bc$ and b, $(a + c)$ are its factors.

Exercise 5.1

Factorize:

Q.1. (i) $2abc - 4abx + 2abd$

(ii) $9xy - 12x^2y + 18y^2$

(iii) $-3x^2y - 3x + 9xy^2$

(iv) $5ab^2c^3 - 10a^2b^3c - 20a^3bc^2$

(v) $3x^3y(x - 3y) - 7x^2y^2(x - 3y)$

(vi) $2xy^3(x^2 + 5) + 8xy^2(x^2 + 5)$

Solutions:-

1(i) $2abc - 4abx + 2abd$

$= 2ab(c - 2x + d)$

1(ii) $9xy - 12x^2y + 18y^2$

$= 3y(3x - 4x^2 + 6y)$

1(iii) $-3x^2y - 3x + 9xy^2$

$= -3x(xy + 1 - 3y^2)$

1(iv) $5ab^2c^3 - 10a^2b^3c - 20a^3bc^2$

$= 5abc(bc^2 - 2ab^2 - 4a^2c)$

1(v) $3x^3y(x - 3y) - 7x^2y^2(x - 3y)$

$= x^2y(x - 3y)(3x - 7y)$

1(vi) $2xy^3(x^2 + 5) + 8xy^2(x^2 + 5)$

$= 2xy^2(x^2 + 5)(y + 4)$

Q.2. Factorize:

(i) $5ax - 3ay - 5bx + 3by$

(ii) $3xy + 2y - 12x - 8$

(iii) $x^3 + 3xy^2 - 2x^2y - 6y^3$

(iv) $(x^2 - y^2)z + (y^2 - z^2)x$

Solution:-

2(i) $5ax - 3ay - 5bx + 3by$

$(5ax - 3ay) - (5bx - 3by)$

$= a(5x - 3y) - b(5x - 3y)$

$= (5x - 3y)(a - b)$

2(ii) $3xy + 2y - 12x - 8$

$(3xy + 2y) - (12x + 8)$

$= y(3x + 2) - 4(3x + 2)$

$(3x + 2)(y - 4)$

2(iii) $x^3 + 3xy^2 - 2x^2y - 6y^3$

$= (x^3 + 3xy^2) - (2x^2y + 6y^3)$

$= x(x^2 + 3y^2) - 2y(x^2 + 3y^2)$

$(x^2 + 3y^2)(x - 2y)$

2(iv) $(x^2 - y^2)z + (y^2 - z^2)x$

$x^2z - y^2z + y^2x - z^2x$

$= x^2z + xy^2 - z^2x - y^2z$

$= x(xz + y^2) - z(xz + y^2)$

$(xz + y^2)(x - z)$

Q.3. Factorize:

(i) $144a^2 + 24a + 1$

(ii) $\frac{a^2}{b^2} - 2 + \frac{b^2}{a^2}$

(iii) $(x + y)^2 - 14z(x + y) + 49z^2$

(iv) $12x^2 - 36x + 27$

Solutions:-

3(i) $144a^2 + 24a + 1$

$= 144a^2 + 12a + 12a + 1$

$(144a^2 + 12a) + (12a + 1)$

$12a(12a + 1) + 1(12a + 1)$

$(12a + 1)(12a + 1) = (12a + 1)^2$

3(ii) $\frac{a^2}{b^2} - 2 + \frac{b^2}{a^2}$

$\left(\frac{a}{b}\right)^2 - 2\left(\frac{a}{b}\right)\left(\frac{b}{a}\right) + \left(\frac{b}{a}\right)^2$

$\left(\frac{a}{b} - \frac{b}{a}\right)^2$

$\frac{a^2}{b^2} - 2 + \frac{b^2}{a^2}$

$\frac{a^2}{b^2} - 2\frac{a}{b} \times \frac{b}{a} + \frac{b^2}{a^2}$

$\frac{a^2}{b^2} - \frac{a}{b} \times \frac{b}{a} - \frac{a}{b} \times \frac{b}{a} + \frac{b^2}{a^2}$

$\left(\frac{a^2}{b^2} - \frac{a}{b} \times \frac{b}{a}\right) - \left(\frac{a}{b} \times \frac{b}{a} - \frac{b^2}{a^2}\right)$

$\frac{a}{b}\left(\frac{a}{b} - \frac{b}{a}\right) - \frac{b}{a}\left(\frac{a}{b} - \frac{b}{a}\right)$

$\left(\frac{a}{b} - \frac{b}{a}\right)\left(\frac{a}{b} - \frac{b}{a}\right)$

$\left(\frac{a}{b} - \frac{b}{a}\right)^2$

3(iii) $(x + y)^2 - 14z(x + y) + 49z^2$

$(x + y)^2 - 7z(x + y) - 7z(x + y) + 49z^2$

$[(x + y)^2 - 7z(x + y)] - [7z(x + y) - 49z^2]$

$(x + y)(x + y - 7z) - (7z)(x + y - 7z)$

$(x + y - 7z)(x + y - 7z)$

$(x + y - 7z)^2$

3(iv) $12x^2 - 36x + 27$

$3(4x^2 - 12x + 9)$

$3[2x(2x - 3) - 3(2x - 3)]$

$= 3(2x - 3)(2x - 3)$

$3(2x-3)^2$

Q.4. Factorize:

(i) $3x^2 - 75y^2$ (ii) $x(x-1) - y(y-1)$

(iii) $128am^2 - 242an^2$ (iv) $3x - 243x^3$

Solution:-

4(i) $3x^2 - 75y^2$

$= 3(x^2 - 25y^2)$

$= 3[(x)^2 - (5y)^2]$

$= 3(x+5y)(x-5y)$

4(ii) $x(x-1) - y(y-1)$

$= x^2 - x - y^2 + y$

$= x^2 - y^2 - x + y$

$= (x^2 - y^2) - (x - y)$

$= (x-y)(x+y) - 1(x-y)$

$= (x-y)[(x+y) - 1]$

$= (x-y)(x+y-1)$

4(iii) $128am^2 - 242an^2$

$= 2a(64m^2 - 121n^2)$

$= 2a[(8m)^2 - (11n)^2]$

$= 2a(8m+11n)(8m-11n)$

4(iv) $3x - 243x^3$

$= 3x(1 - 81x^2)$

$= 3x[(1)^2 - (9x)^2]$

$= 3x(1+9x)(1-9x)$

Q.5. Factorize:

(i) $x^2 - y^2 - 6y - 9$ (ii) $x^2 - a^2 + 2a - 1$

(iii) $4x^2 - y^2 - 2y - 1$ (iv) $x^2 - y^2 - 4x - 2y +$

(v) $25x^2 - 10x + 1 - 36z^2$

(vi) $x^2 - y^2 - 4xz + 4z^2$

Solutions:-

5(i) $x^2 - y^2 - 6y - 9$

$= x^2 - (y^2 + 6y + 9)$

$= (x)^2 - [y^2 + 3y + 3y + 9]$

$= (x)^2 - [y(y + 3) + 3(y + 3)]$

$= (x)^2 - (y + 3)(y + 3)$

$= (x)^2 - (y + 3)^2$

$= [x + (y + 3)].[x - (y + 3)]$

5(ii) $x^2 - a^2 + 2a - 1$

$= x^2 - (a^2 - 2a + 1)$

$= (x)^2 - [a^2 - a - a + 1]$

$= (x)^2 - [a(a - 1) - 1(a - 1)]$

$= (x)^2 - (a - 1)(a - 1)$

$= (x)^2 - (a - 1)^2$

$= (x + a - 1)(x - a + 1)$

5(iii) $4x^2 - y^2 - 2y - 1$

$= (2x)^2 - (y^2 + 2y + 1)$

$= (2x)^2 - [y^2 + y + y + 1]$

$= (2x)^2 - [y(y + 1) + 1(y + 1)]$

$= (2x)^2 - (y + 1)(y + 1)$

$= (2x)^2 - (y + 1)^2$

$= (2x + y + 1)(2x - y - 1)$

5(iv) $x^2 - y^2 - 4x - 2y + 3$

Re-arranging

$x^2 - 4x + 4 - y^2 - 2y - 1$ N.T.S

$= (x^2 - 4x + 4) - (y^2 + 2y + 1)$

$= [x^2 - 2x - 2x + 4] - [y^2 - y - y + 1]$

$= [x(x - 2) - 2(x - 2)] - [y(y - 1) - 1(y - 1)]$

$= [(x - 2)(x - 2)] - [(y - 1)(y - 1)]$

$= (x-2)^2 - (y-1)^2$

$= (x-2+y+1)(x-2-y-1)$

$= (x+y-1)(x-y-3)$

5(v) $25x^2 - 10x + 1 - 36z^2$

$= (25x^2 - 10x + 1) - (36z^2)$

$= [25x^2 - 5x - 5x + 1] - (6z)^2$

$= [5x(5x-1) - 1(5x-1)] - (6z)^2$

$= (5x-1)(5x-1) - (6z)^2$

$= (5x-1)^2 - (6z)^2$

$= (5x-1+6z)(5x-1-6z)$

5(vi) $x^2 - y^2 - 4xz + 4z^2$

$= (x^2 - 4xz + 4z^2) - (y^2)$

$= [x^2 - 2xz - 2xz + 4z^2] - (y)^2$

$= [x(x-2z) - 2z(x-2z)] - y^2$

$= (x-2z)(x-2z) - y^2$

$= (x-2z)^2 - (y)^2$

$= (x-2z+y)(x-2z-y)$

$= (x+y-2y)(x-y-2z)$

Exercise 5.2

Q.1. Factorize:

(i) $x^4 + \frac{1}{x^4} - 3$ (ii) $3x^4 + 12y^4$

(iii) $a^4 + 3a^2b^2 + 4b^4$ (iv) $4x^4 + 81$

(v) $x^4 + x^2 + 25$ (vi) $x^4 + 4x^2 + 16$

Solutions:-

1(i) $x^4 + \frac{1}{x^4} - 3$

$= x^4 + \frac{1}{x^4} - 2 - 1$

$= \left[(x^2)^2 - 2 + \left(\frac{1}{x^2}\right)^2\right] - 1$

$= \left(x^2 - \frac{1}{x^2}\right)^2 - (1)^2$

$= \left(x^2 - \frac{1}{x^2} + 1\right)\left(x^2 - \frac{1}{x^2} - 1\right)$

1(ii) $3x^4 + 12y^4$

$= 3(x^4 + 4y^4)$

Completing the square

$= 3[(x^2)^2 + (2y^2)^2 + 4x^2y^2 - 4x^2y^2]$

$= 3[\{(x^2)^2 + 2(x^2)(2y^2) + (2y^2)^2\} - (2xy)^2]$

$= 3[(x^2 + 2y^2)^2 - (2xy)^2]$

$= 3(x^2 + 2y^2 + 2xy)(x^2 + 2y^2 - 2xy)$

$= 3(x^2 + 2xy + 2y^2)(x^2 - 2xy + 2y^2)$

1(iii) $a^4 + 3a^2b^2 + 4b^4$

Completing the square

$= (a^2)^2 + 2(a^2)(2b^2) + (2b^2)^2 - a^2b^2$

$= (a^2 + 2b^2)^2 - (ab)^2$

$= (a^2 + 2b^2 + ab)(a^2 + 2b^2 - ab)$

1(iv) $4x^4 + 81$

Completing the square

$= (2x^2)^2 + (9)^2$

$= (2x^2)^2 + (9)^2 + 2(2x^2)(9) - 36x^2$

$= (2x^2 + 9 - 6x)(2x^2 + 9 - 6x)$

$= (2x^2 - 6x + 9)(2x^2 - 6x + 9)$

1(v) $x^4 + x^2 + 25$

$= (x^2)^2 + (5)^2 + x^2$

Completing the square

$= (x^2)^2 + (5)^2 + 2(x^2)(5) - 9x^2$

$= (x^2 + 5)^2 - (3x)^2$

$= (x^2 + 5 + 3x)(x^2 + 5 - 3x)$

$= (x^2 + 3x + 5)(x^2 - 3x + 5)$

1(vi) $x^4 + 4x^2 + 16$

$= (x^2)^2 + (4)^2 + 2(x^2)(4) - 4x^2$

Completing the square

$= (x^2)^2 + (4)^2 + 2(x^2)(4) - 4x^2$

$= (x^2 + 4)^2 - (2x)^2$

$= (x^2 + 4 + 2x)(x^2 + 4 - 2x)$

$= (x^2 + 2x + 4)(x^2 - 2x + 4)$

Q.2. *Factorize:*

(i) $x^2 + 14x + 48$ (ii) $x^2 - 21x + 108$

(iii) $x^2 - 11x - 42$ (iv) $x^2 + x - 132$

Solutions:-

2(i) $x^2 + 14x + 48$

$= x^2 + 8x + 6x + 48$

$= x(x + 8) + 6(x + 8)$

$= (x + 8)(x + 6)$

2(ii) $x^2 - 21x + 108$

$= x^2 - 12x - 9x + 108$

$= x(x - 12) - 9(x - 12)$

$= (x - 12)(x - 9)$

2(iii) $x^2 - 11x - 42$

$= x^2 - 11x - 42$

$= x^2 - 14x + 3x - 42$

$= x(x - 14) + 3(x - 14)$

$= (x - 14)(x + 3)$

2(iv) $x^2 + x - 132$

$= x^2 + 12x - 11x - 132$

$= x(x + 12) - 11(x + 12)$

$= (x + 12)(x - 11)$

Q.3. Factorize:

(i)	$4x^2 + 12x + 5$	(ii)	$30x^2 + 7x - 15$
(iii)	$24x^2 - 65x + 21$	(iv)	$5x^2 - 16x - 21$
(v)	$4x^2 - 17xy + 4y^2$	(vi)	$3x^2 - 38xy - 13y^2$
(vii)	$5x^2 + 33xy - 14y^2$		

(viii) $\left(5x - \frac{1}{x}\right)^2 + 4\left(5x - \frac{1}{x}\right) + 4,\ x \neq 0$

Solutions:-

3(i) $4x^2 + 12x + 5$

$= 4x^2 + 10x + 2x + 5$

$= 2x(2x + 5) + 1(2x + 5)$

$= (2x + 5)(2x + 1)$

3(ii) $30x^2 + 7x - 15$

$= 30x^2 + 25x - 18x - 15$

$= 5x(6x + 5) - 3(6x + 5)$

$= (6x + 5)(5x - 3)$

3(iii) $24x^2 - 65x + 21$

$= 24x^2 - 56x - 9x + 21$

$= 8x(3x - 7) - 3(3x - 7)$

$= (3x - 7)(8x - 3)$

3(iv) $5x^2 - 16x - 21$

$= 5x^2 - 21x + 5x - 21$

$= x(5x - 21) + 1(5x - 21)$

$= (5x - 21)(x + 1)$

3(v) $4x^2 - 17xy + 4y^2$

$= 4x^2 - 16xy - xy + 4y^2$

$= 4x(x - 4y) - y(x - 4y)$

$= (x - 4y)(4x - y)$

3(vi) $3x^2 - 38xy - 13y^2$

$= 3x^2 + xy - 39xy - 13y^2$

$= x(3x + y) - 13y(3x + y)$

$= (3x + y)(x - 13y)$

3(vii) $5x^2 + 33xy - 14y^2$

$5x^2 + 35xy - 2xy - 14y^2$

$= 5x(x + 7y) - 2y(x + 7y)$

$= (x + 7y)(5x - 2y)$

3(viii) $\left(5x - \frac{1}{x}\right)^2 + 4\left(5x - \frac{1}{x}\right) + 4, \ x \neq 0$

$= \left(5x - \frac{1}{x}\right)^2 + 2\left(5x - \frac{1}{x}\right) + 2\left(5x - \frac{1}{x}\right) + 4$

$= \left[\left(5x - \frac{1}{x}\right)^2 + 2\left(5x - \frac{1}{x}\right)\right] + 2\left(5x - \frac{1}{x}\right) + 4$

$= \left(5x - \frac{1}{x}\right)\left(5x - \frac{1}{x} + 2\right) + 2\left[5x - \frac{1}{x} + 2\right]$

$= \left(5x - \frac{1}{x} + 2\right)\left(5x - \frac{1}{x} + 2\right)$

Q.4. Factorize:

(i) $(x^2 + 5x + 4)(x^2 + 5x + 6) - 3$

(ii) $(x^2 - 4x)(x^2 - 4x - 1) - 20$

(iii) $(x + 2)(x + 3)(x + 4)(x + 5) - 15$

(iv) $(x + 4)(x - 5)(x + 6)(x - 7) - 504$

(v) $(x + 1)(x + 2)(x + 3)(x + 6) - 3x^2$

Solution:-

4(i) $(x^2 + 5x + 4)(x^2 + 5x + 6) - 3$

Let $x^2 + 5x = y$

$(x^2 + 5x + 4)(x^2 + 5x + 6) - 3$

$= (y + 4)(y + 6) - 3$

$= y^2 + 10y + 24 - 3$

$= y^2 - 10y + 21$

$= y^2 + 7y + 3y + 21$

$= y(y + 7) + 3(y + 7)$

$= (y + 7)(y + 3)$

Putting $y = x^2 + 5x$

$= (x^2 + 5x + 7)(x^2 + 5x + 3)$

4(ii) $(x^2 - 4x)(x^2 - 4x - 1) - 20$

Let $x^2 - 4x = y$

$\therefore (x^2 - 4x)(x^2 - 4x - 1) - 20$ (given expression)

$= (y)(y - 1) - 20$

$= y^2 - y - 20$

$= y^2 - 5y + 4y - 20$

$= y(y - 5) + 4(y - 5)$

Putting $y = x^2 - 4x$

$= (x^2 - 4x - 5)(x^2 - 4x + 4)$

$= [x^2 - 5x + x - 5][x^2 - 2x - 2x + 4]$

$= [x(x - 5) + 1(x - 5)][x(x - 2) - 2(x - 2)]$

$= (x - 5)(x + 1)(x - 2)(x - 2)$

$= (x - 5)(x + 1)(x - 2)^2$

4(iii) $(x + 2)(x + 3)(x + 4)(x + 5) - 15$

Here $2 + 5 = 3 + 4$

$(x + 2)(x + 5)(x + 3)(x + 4) - 15$

$= (x^2 + 7x + 10)(x^2 + 7x + 12) - 15$

Let $x^2 + 7x = y$

$\therefore (x^2 + 7x + 10)(x^2 + 7x + 12) - 15$

$= (y + 10)(y + 12) - 15$

$= y^2 + 22y + 120 - 15$

$= y^2 + 22y + 105$

$= y^2 + 15y + 7y + 105$

$= y(y + 15) + 7(y + 15)$

$= (y + 15)(y + 7)$

Putting $y = x^2 + 7x$

$= (x^2 + 7x + 15)(x^2 + 7x + 7)$

4(iv) $(x + 4)(x - 5)(x + 6)(x - 7) - 504$

Here $+4 - 5 = 6 - 7$

$(x^2 - x - 20)(x^2 - x - 42) - 504$

Let $x^2 - x = y$

$(x^2 - x - 20)\ (x^2 - x - 42) - 504$ (given expression)

$= (y - 20)(y - 42) - 504$

$= y^2 - 62y + 840 - 504$

$= y^2 - 62y + 336$

$= y^2 - 56y - 6y + 316$

$= y(y - 56) - 6(y - 56)$

$= (y - 56)(y - 6)$

Putting $y = x^2 - x$

$= (x^2 - x - 56)(x^2 - x - 6)$

$= (x^2 - 8x + 7x - 56)(x^2 - 3x + 2x - 6)$

$= [x(x - 8) + 7(x - 8)][x(x - 3) + 2(x - 3)]$

$= (x - 8)(x + 7)(x - 3)(x + 2)$

4(v) $(x + 1)(x + 2)(x + 3)(x + 6) - 3x^2$

$1 \times 6 = 2 \times 3$

$(x + 1)(x + 6)(x + 2)(x + 3) - 3x^2$

$(x^2 + 7x + 6)(x^2 + 5x + 6) - 3x^2$

$(x^2 + 6 + 7x)(x^2 + 6 + 5x) - 3x^2$

Let $x^2 + 6 = y$

$= (y + 7x)(y + 5x) - 3x^2$ (Expression)

$= y^2 + 12xy + 35x^2 - 3x^2$

$= y^2 + 12xy + 32x^2$

$= y^2 + 8xy + 4xy + 32x^2$

$= y(y + 8x) + 4x(y + 8x)$

$= (y + 8x)(y + 4x)$

Putting $y = x^2 + 6$

$= (x^2 + 6 + 8x)(x^2 + 6 + 4x)$

$= (x^2 + 8x + 6)(x^2 + 4x + 6)$

Q.5. Factorize:

(i) $x^3 + 48x - 12x^2 - 64$

(ii) $8x^3 + 60x^2 + 150x + 125$

(iii) $x^3 - 18x^2 + 108x - 216$

(iv) $8x^3 - 125y^3 - 60x^2y + 150xy^2$

Solutions:-

5(i) $x^3 + 48x - 12x^2 - 64$

$= x^3 - 64 - 12x^2 + 48x$

$= (x)^3 - (4)^3 - 3(x^2)(4) + 3(x)(4^2)$

$= (x - 4)^3$ $[(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3]$ (Formula)

5(ii) $8x^3 + 60x^2 + 150x + 125$

$= 8x^3 + 125 + 60x^2 + 150x$

$= (2x)^3 + (5)^3 + 3(2x)^2(5) + 3(2x)(5)^2$

$= (2x + 5)^3$

5(iii) $x^3 - 18x^2 + 108x - 216$

$= x^3 - 216 - 18x^2 + 108x$

$= (x)^3 - (6)^3 - 3(x)^2(6) + 3(x)(6)^2$

$= (x - 6)^3$

5(iv) $8x^3 - 125y^3 - 60x^2y + 150xy^2$

$= (2x)^3 - (5y)^3 - 3(2x)^2(5y) + 3(2x)(5y)^2$

$= (2x - 5y)^3$

.6. Factorize:

(i) $27 + 8x^3$ (ii) $125x^3 - 216y^3$

(iii) $64x^3 + 27y^3$ (iv) $8x^3 + 125y^3$

Solutions:-

6(i) $27 + 8x^3$

$= (3)^3 + (2x)^3$

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ (Formula)

$= (3 + 2x)\,[(3)^2 - (3)(2x) + (2x)^2]$

$= (3 + 2x)(9 - 6x + 4x^2)$

6(ii) $125x^3 - 216y^3$

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$ (Formula)

$= (5x)^3 - (6y)^3$

$= (5x - 6y)[(5x)^2 + (5x)(6y) + (6y)^2]$

$= (5x - 6y)(25x^2 + 30xy + 36y^2)$

6(iii) $64x^3 + 27y^3$

$= (4x)^3 + (3y)^3$

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ (Formula)

$= (4x + 3y)[(4x)^2 - (4x)(3y) + (3y)^2]$

$= (4x + 3y)(16x^2 - 12xy + 9y^2)$

6(iv) $8x^3 + 125y^3$

$= (2x)^3 + (5y)^3$

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ (Formula)

$= (2x + 5y)[(2x)^2 - (2x)(5y) + (5y)^2]$

$= (2x + 5y)(4x^2 - 10xy + 25y^2)$

Remainder Theorem

If a polynomial $p(x)$ is divided by a linear divisor $(x - a)$, then the remainder is $p(a)$.

Exercise 5.3

Q.1. Use the remainder theorem to find the remainder when

(i)	$3x^3 - 10x^2 + 13x - 6$	is divided by $(x-2)$
(ii)	$4x^3 - 4x - 3$	is divided by $(2x-1)$
(iii)	$6x^4 + 2x^3 - x + 2$	is divided by $(x+2)$
(iv)	$(2x-1)^3 + 6(3+4x)^2 - 10$	is divided by $(2x+1)$
(v)	$x^3 - 3x^2 + 4x - 14$	is divided by $(x+2)$

Solutions:-

1(i) $p(x) = 3x^3 - 10x^2 + 13x - 6$

From $x = 2;\ x - 2 = 0$

$p(2) = 3(2)^3 - 10(2)^2 + 13(2) - 6$

$= 24 - 40 + 26 - 6$

$= 4\ (R)$

1(ii) $p(x) = 4x^3 - 4x + 3$

From $2x - 1 = 0;\ x = \frac{1}{2}$

$p(\frac{1}{2}) = 4(\frac{1}{2})^3 - 4(\frac{1}{2}) + 3$

$= 4 \times \frac{1}{8} - 2 + 3$

$= \frac{1}{2} - 2 + 3 = \frac{1}{2} + 1 = \frac{3}{2}$

1(iii) $p(x) = 64x^4 + 2x^3 - x + 2$

From $x + 2 = 0;\ x = -2$

$p(-2) = 6(-2)^4 + 2(-2)^3 - (-2) + 2$

$= 6(16) + 2(-8) + 2 + 2$

$= 96 - 16 + 2 + 2$

$= 84\ (R)$

1(v) $p(x) = (2x-1)^3 + 6(3+4x)^2 - 10$

From $2x + 1 = 0;\ x = -\frac{1}{2}$

$$p(-\frac{1}{2}) = \left[2(-\frac{1}{2}) - 1\right]^3 + 6\left[3 + 4(-\frac{1}{2})\right]^2 - 10$$
$$= [-1-1]^3 + 6[3-2]^2 - 10$$
$$= (-2)^3 + 6(1)^2 - 10$$
$$= -8 + 6 - 10$$
$$= -12 \ (R)$$

1(v) $p(x) = x^3 - 3x^2 + 4x - 14$

From $x + 2 = 0;\ x = -2$

$$p(-2) = (-2)^3 - 3(-2)^2 + 4(-2) - 14$$
$$= -8 - 12 - 8 - 14$$
$$= -42 \ (R)$$

Q.2. (i) If $(x + 2)$ is a factor of $3x^2 - 4kx - 4k^2$, then find the value (s) of k.

(ii) If $(x - 1)$ is a factor of $x^3 - kx^2 + 11x - 6$, then find the value of k.

Solutions:-

2(i) $p(x) = 3x^2 - 4kx - 4k^2$

From $x + 2 = 0,\ x = -2$

$$p(-2) = 3(-2)^2 - 4k(-2) - 4k^2$$
$$= 3(4) + 8k - 4k^2$$
$$= 12 + 8k - 4k^2 \ (R)$$

If $(x + 2)$ is a factor then $R = 0$

Thus $12 + 8k - 4k^2 = 0$ *divided by '4'*

$$3 + 2k - k^2 = 0$$
$$3 + 3k - k - k^2 = 0$$
$$3(1 + k) - k(1 + k) = 0$$
$$(1 + k)(3 - k) = 0$$

From $1 + k = 0$

$k = -1$

and $3 - k = 0$

give $k = 3$

$\therefore\ k = 3, -1$

2(ii) $p(x) = x^3 - kx^2 + 11x - 6$

From $x - 1 = 0;\ x = 1$

$p(1) = (1)^3 - k(1)^2 + 11(1) - 6$

$= 1 - k + 11 - 6$

$= 6 - k \quad (R)$

If $x - 1$ is factor, then $R = 0$

From

$6 - k = 0$

$k = 6$

Q.3. *Without actual long division determine whether*

(i) $(x - 2)$ and $(x - 3)$ are factors of $p(x) = x^3 - 12x^2 + 44 - 48$

(ii) $(x-2), (x+3)$ and $(x-4)$ are factors of $q(x) = x^3 + 2x^2 - 5x - 6$

3(i) $p(x) = x^3 - 12x^2 + 44x - 48$

From $x - 2 = 0;\ x = 2$

$p(2) = (2)^3 - 12(2)^2 + 44(2) - 48$

$= 8 - 48 + 88 - 48$

$= 96 - 96 = 0 \quad (R)$

$R = 0$; therefore, $x - 2$ is a factor of the polynomial

$p(x) = x^3 - 12x^2 + 44x - 48$

From $(x - 3)$; $x = 3,\ x - 3 = 0$

$p(3) = (3)^3 - 12(3)^2 + 44(3) - 48$

$= 27 - 108 + 132 - 48$

$R = 159 - 156 = 3 \neq 0$

$R \neq 0$; therefore, $x - 3$ is NOT factor

3(ii) $p(x) = x^3 + 2x^2 - 5x - 6$

From $x - 2 = 0;\ x = 2$

$p(2) = (2)^3 + 2(2)^2 - 5(2) - 6$

$= 8 + 8 - 10 - 6$

$= 16 - 16 = 0 \quad (R)$

$R = 0$, therefore, $x - 2$ is factor

$p(x) = x^3 + 2x^2 - 5x - 6$

from $x + 3 = 0,\ x = -3$

$p(-3) = (-3)^3 + 2(-3)^2 - 5(-3) - 6$

$= -27 + 18 + 15 - 6$

$= 33 - 33 = 0 \quad (R)$

$\because R = 0$; $x + 3$ is factor

$p(x) = x^3 + 2x^2 - 5x - 6$

From $(x - 4) = 0;\ x = 4$

$p(4) = (4)^3 + 2(4)^2 - 5(4) - 6$

$= 64 + 32 - 20 - 6$

$= 96 - 26 \neq 0 \quad (R)$

$R \neq 0$; therefore, $x - 4$ is not factor

Q.4. For what value of m is the polynomial $p(x) = 4x^3 - 7x^2 + 6x - 3m$ exactly divisible by $x + 2$?

Solution:-

$p(x) = 4x^3 - 7x^2 + 6x - 3m$

From $x + 2 = 0;\ x = -2$

$p(-2) = 4(-2)^3 - 7(-2)^2 + 6(-2) - 3m$

$= -32 - 28 - 12 - 3m$

$= -72 - 3m \quad (R)$

If $x + 2$ is factor, then $R = 0$

$\therefore -72 - 3m = 0$

or $24 + m = 0$

thus $m = -24$

Q.5. Determine the value of k if $p(x) = kx^3 + 4x^2 + 3x - 4$ and $q(x) = x^3 - 4x + k$ leaves the same remainder when divided by $(x - 3)$.

$p(x) = kx^3 + 4x^2 + 3x - 4$

From $x - 3 = 0;\ x = 3$

$p(3) = k(3)^3 + 4(3)^2 + 3(3) - 4$

$= 27k + 36 + 9 - 4$

$= 27k + 41 \quad (R_1)$

Now $q(x) = x^3 - 4x + k$

and $q(3) = (3)^3 - 4(3) + k$

$= 27 - 12 + k$

$= 15 + k \qquad (R_2)$

Now $R_1 = R_2$

$27k + 41 = 15 + k$

$27k - k = 15 - 41$

$26k = -26$

thus $k = -1$

Q.6. The remainder after dividing the polynomial $p(x) = x^3 + 9x^2 + 7$ by $(x + 1)$ is $2b$. Calculate the value of a and b if this expression leaves a remainder of $(b + 5)$ on being divided by $(x - 2)$.

Solution:-

$p(x) = x^3 + ax^2 + 7$

From $x + 1 = 0;\ x = -1$

$p(-1) = (-1)^3 + a(-1)^2 + 7$

$= -1 + a + 7$

$= a + 6$

$a + 6 = 2b \qquad$ (1st Condition)

Now $p(x) = x^3 + ax^2 + 7$

From $(x - 2) = 0;\ x = 2$

and $p(2) = (2)^3 + a(2)^2 + 7$

$= 8 + 4a + 7$

$= 4a + 15$

$4a + 15 = b + 5 \qquad$ (2nd Condition)

$4a - b = 5 - 15$

$4a - b = -10 \qquad$ (B)

$a - 2b = -6 \qquad$ (i) From A. B

$8a - 2b = 20 \qquad$ (ii) multiplying B by 2

$7a = -14 \qquad$ subtracting (i) from (ii)

$a = -\frac{14}{7}$

$a = -2$

$a - 2b = -6 \qquad$ From A

$$-2-2b = -6 \quad \text{(Putting } a = -6\text{)}$$
$$-2b = -6+2$$
$$-2b = -4$$
$$\therefore \quad b = 2$$
Thus $a = -2, b = 2$

Q.7. *The polynomial $x^3+lx^2+mx+24$ has a factor $(x+4)$ and it leaves a remainder of 36 when divided by $(x-2)$. Find the values of l and m.*

Solution:-

$$p(x) = x^3 + lx^2 + mx + 24$$
From $x+4 = 0; x = -4$
$$p(-4) = (-4)^3 + l(-4)^2 + m(-4) + 24$$
$$= -64 + 16l - 4m + 24$$
$$= +16l - 4m - 40$$
$$+16l - 4m - 40 = 0 \quad \text{(1st Condition)}$$
or $4l - m - 10 = 0$
or $4l - m = 10$ (A)
Now $p(x) = x^3 + lx^2 + mx + 24$
From $x - 2 = 0; x = 2$
$$p(2) = (2)^3 + l(2)^2 + m(2) + 24$$
$$= 8 + 4l + 2m + 24$$
$$= 4l + 2m + 32$$
$$4l + 2m + 32 = 36 \quad \text{(2nd Condition)}$$
or $4l + 2m = 36 - 32$
or $4l + 2m = 4$
or $2l + m = 2$ (B)
$4l - m = 10$ From (A)
$6l = 12$ From $A + B$
$l = 2$
Putting $l = 2$ in A
$$4l - m = 10$$
$$4(2) - m = 10$$
$$8 - m = 10$$

$$-m = 10 - 8$$
$$-m = 2$$
$$m = -2$$
thus $l = 2,\ m = -2$

Q.8. ***The expression $lx^3 + mx^2 - 4$ leaves remainder of -3 and 12 when divided by $(x - 1)$ and $(x + 2)$ respectively. Calculate the values of l and m.***

Solution:-

$$p(x) = lx^3 + mx^2 - 4$$
From $x - 1 = 0;\ x = 1$
$$p(1) = l(1)^3 + m(1)^2 - 4$$
$$= l + m - 4$$
$$l + m - 4 = -3 \qquad (1^{st}\text{ Condition})$$
$$l + m = -3 + 4$$
$$l + m = 1 \qquad (A)$$
$$p(x) = lx^3 + mx^2 - 4$$
From $x + 2 = 0;\ x = -2$
$$p(-2) = l(-2)^3 + m(-2)^2 - 4$$
$$= -8l + 4m - 4$$
$$-8l + 4m - 4 = 12 \qquad (2^{nd}\text{ Condition})$$
$$-8l + 4m = 12 + 4$$
$$-8l + 4m = 16$$
$$2l - m = -4 \qquad (B)$$
$$l + m = 1 \qquad \text{From } A$$
$$3l = -3 \qquad \text{From } A + B$$
$$l = -1$$
$$l + m = 1 \qquad (\text{From } A)$$
Putting $l = -1$
$$-1 + m = 1$$
$$m = 1 + 1$$
$$m = 2$$
Thus $l = -1,\ m = 2$

Q.9. ***The expression $ax^3 - 9x^2 + bx + 3a$ is exactly divisible by $x^2 - 5x + 6$. Find the values of a and b.***

Solution:-

Factorizing: $x^2 - 5x + 6$

$= x^2 - 2x - 3x + 6$

$= x(x-2) - 3(x-2)$

$= (x-2)(x-3)$

If $x^2 - 5x + 6$ divides the expression $ax^3 - 9x^2 + bx + 3a$ then $x + 2$ and $x - 3$ will also divide it.

$p(x) = ax^3 - 9x^2 + bx + 3a$

From $x - 2 = 0;\ x = 2$

$p(2) = a(2)^3 - 9(2)^2 + b(2) + 3a$

$= 8a - 36 + 2b + 3a$

$= 11a + 2b - 36$

$11a + 2b - 36 = 0$ (1st Condition)

$11a + 2b = 36$ (A)

$p(x) = ax^3 - 9x^2 + bx + 3a$

$p(3) = a(3)^3 - 9(3)^2 + b(3) + 3a$

$= 27a - 81 + 3b + 3a$

$= 30a + 3b - 81$

$30a + 3b - 81 = 0$ (2nd Condition)

$30a + 3b = 81$

$10a + b = 27$ (B) Dividing by 3

$11a + 2b = 36$ A

$10a + b = 27$ B

$20a + 2b = 54$ (i) Multiplying B by 3

and $11a + 2b = 36$ (ii) Subtracting ii from i

$9a = 18$

$a = 2$

$10a + b = 27$ From B

$10(2) + b = 27$ Putting $a = 2$

$20 + b = 27$

$b = 27 - 20$

Thus $a = 2,\ b = 7$

Exercise 5.4

Factorize each of the following cubic polynomials by factor theorem.

1.	$x^3 - 2x^2 - x + 2$	2.	$x^3 - x^2 - 22x + 40$
3.	$x^3 - 6x^2 + 3x + 10$	4.	$x^3 + x^2 - 10x + 8$
5.	$x^3 - 2x^2 - 5x + 6$	6.	$x^3 + 5x^2 - 2x - 24$
7.	$3x^3 - x^2 - 12x + 4$	8.	$2x^3 + x^2 - 2x - 1$

Solutions:-

1. $P(x) = x^3 - 2x^2 - x + 2$

$p = 2$ and possible factors of 2 are $\pm 1 \pm 2$

$q = 1$ and possible factor of 1 are 1 possible factors of $P(x)$ will be from

$$\frac{p}{q} = \pm 1, \pm 2$$

$$P(x) = x^3 - 2x^2 - x + 2$$

$$P(1) = (1)^3 - 2(1)^2 - (1) + 2$$

$$= 1 - 2 - 1 + 2 = 0 \quad \text{(Remainder)}$$

$x - 1$ is one factor

$$P(1) = (-1)^3 - 2(-1)^2 - (-1) + 2$$

$$= -1 - 2 + 1 + 2 = 0 \quad \text{(Remainder)}$$

$x + 1$ is another factor

$$P(2) = (2)^3 - 2(2)^2 - (2) + 2$$

$$= 8 - 8 - 2 + 2 = 0 \quad \text{(R)}$$

$x - 2$ is third factor

Thus

$$x^3 - 2x^2 - x + 2 = (x - 1)(x + 1)(x - 2)$$

2. $P(x) = x^3 - x^2 - 22x + 40$

$p = 40$, possible factors of 40 are $\pm 1, \pm 2, \pm 4, \pm 5, \pm 8, \pm 10, \pm 20$

$q = 1$, possible factor of 1 are ± 1, $P(x) = 0$; possible factors of $P(x)$ will be from

$$\frac{p}{q} = \pm 1, \pm 2, \pm 4, \pm 5, \pm 8, \pm 10, \pm 20$$

$$P(x) = x^3 - x^2 - 22x + 40$$

Put $x = 2$

$$P(2) = (2)^3 - (2)^2 - 22(2) + 40$$
$$= 8 - 4 - 44 + 40 = 0 \quad (R)$$

$x - 2$ is factor of $P(x)$

Put $x = 4$

$$P(4) = (4)^3 - (4)^2 - 22(4) + 40$$
$$= 64 - 16 - 88 + 40$$
$$= 104 - 104 = 0 \quad (R)$$

$x - 4$ is also factor of P(x)

Put $x = 5$

$$P(5) = (5)^3 - (5)^2 - 22(5) + 40$$
$$= 125 - 25 - 110 + 40$$
$$= 165 - 135 \neq 0 \quad (R)$$

$(x - 5)$ is not factor of $P(x)$

Put $x = -5$

$$P(-5) = (-5)^3 - (-5)^2 - 22(-5) + 40$$
$$= -125 - 25 + 110 + 40$$
$$= -150 + 150 = 0 \quad (R)$$

$x + 5$ is also factor of $P(x)$

Thus

$$x^3 - x^2 - 22x + 40 = (x - 2)(x - 4)(x + 5)$$

3. $$P(x) = x^3 - 6x^2 + 3x + 10$$

$P = 10$; possible factor of 10 are $\pm 1, \pm 2, \pm 5, \pm 10$

$q = 1$ factor of 1 are ± 1 possible factors of $P(x)$ can be found from

$$\frac{p}{q} = \pm 1, \pm 2, \pm 5, \pm 10$$

$$P(x) = x^3 - 6x^2 + 3x + 10$$

Put $x = 1$

$$P(x) = (1)^3 - (1)^2 - 6(1)^2 - 10$$

$$1 - 6 + 3 + 10$$

$$= 14 - 6 - 8 \neq 0 \qquad (R)$$

$x - 1$ is not factor

Put $x = -1$

$$P(x) = (-1)^3 - 6(-1)^2 + 3(-1) + 10$$

$$= -1 - 6 - 3 + 10 = 0 \qquad (R)$$

$(x + 1)$ is factor

$$P(x) = x^3 - 6x^2 + 3x + 10$$

Put $x = 2$

$$P(2) = (2)^3 - 6(2)^2 + 3(2) + 10$$

$$= 8 - 24 + 6 + 10$$

$$= 24 - 24 = 0 \qquad (R)$$

$x - 2$ factor

Put $x = 5$

$$P(5) = (5)^3 - 6(5)^2 + 3(5) + 10$$

$$= 125 - 150 + 15 + 10$$

$$150 - 150 = 0 \qquad (R)$$

$x - 5$ is also factor

Thus

$$x^3 - 6x^2 + 3x + 10 = (x + 1)(x - 2)(x - 5)$$

4. $$P(x) = x^3 + x^2 - 10x + 8$$

$P = 8$; factor of 8 are $\pm 1, \pm 2, \pm 4, \pm 8$

$q = 1$. factor of 1 are ± 1 possible factors can be found from

$$\frac{p}{q} = \pm 1, \pm 2, \pm 4, \pm 8$$

$$P(x) = x^3 + x^2 - 10x + 8$$

Put $x = 1$

$$P(1) = (1)^3 + (1)^2 - 10(1) + 8$$
$$= 1 + 1 - 10 + 8 = 0 \qquad (R)$$

$x - 1$ is factor of $P(x)$

$$P(x) = x^3 + x^2 - 10x + 8$$

Put $x = 2$

$$P(2) = (2)^3 + (2)^2 - 10(2) + 8$$
$$= 8 + 4 - 20 + 8$$
$$= 20 - 20 = 0 \qquad (R)$$

$x - 2$ is also factor

$$P(x) = x^3 + x^2 - 10x + 8$$

Put $x = -4$

$$P(-4) = (-4)^3 + (-4)^2 - 10(-4) + 8$$
$$= -64 + 16 - 40 + 8$$
$$= -64 + 64 = 0 \qquad (R)$$

$x + 4$ is factor

Thus

$x^3 + x^2 - 10 + 8 = (x - 1)(x - 2)(x + 4)$

5. $$P(x) = x^3 - 2x^2 - 5x + 6$$

$P = 6$; factor of 6 are $\pm 1, \pm 2, \pm 3, \pm 6$

$q = 1$, factor of 1 are ± 1 members of factors of $P(x)$ can be found from

$$\frac{p}{q} = \pm 1, \pm 2, \pm 3, \pm 6$$

$$P(x) = x^3 - 2x^2 - 5x + 6$$

Put $x = 1$

$$P(1) = (1)^3 - 2(1)^2 - 5(1) + 6$$
$$= 1 - 2 - 5 + 6$$
$$= 7 - 7 = 0$$

$x - 1$ is factor of $P(x)$

$P(x) = x^3 - 2x^2 - 5x + 6$

Put $x = 2$

$P(2) = (+2)^3 - 2(+2)^2 - 5(+2) + 6$

$= +8 - 8 - 10 + 6$

$= 14 - 18 = -4 \neq 0$ (R)

$x - 2$ is not factor of $P(x)$

$P(x) = x^3 - 2x^2 - 5x + 6$

Put $x = 3$

$P(3) = (3)^3 - 2(3)^2 - 5(3) + 6$

$= 27 - 18 - 15 + 6$

$= 33 - 33 = 0$ (R)

$x - 3$ is also factor of $P(x)$

Thus

$P(x) = x^3 - 2x^2 - 5x + 6$

Put $x = -2$

$P(-2) = (-2)^3 - 2(-2)^2 - 5(-2) + 6$

$= -8 - 8 + 10 + 6$

$= -16 + 16 = 0$ (R)

$x + 2$ is one factor of $P(x)$

Thus

$x^3 - 2x^2 - 5x + 6 = (x - 1)(x - 3)(x + 2)$

6. $P(x) = x^3 + 5x^2 - 2x - 24$

$P = -24$; factor of 6 are $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$

$q = 1$, factor of q are $= \pm 1$ possible factors of $P(x)$ will be found from

$\frac{p}{q}$ $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$

$P(x) = x^3 + 5x^2 - 2x - 24$

Put $x = 2$

$P(2) = (2)^3 + 5(2)^2 - 2(2) - 24$

$= 8 + 20 - 4 - 24$

$= 28 - 28 = 0$ (R)

$(x - 2)$ is factor of $P(x)$

$P(x) = x^3 + 5x^2 - 2x - 24$

Put $x = -3$

$P(-3) = (-3)^3 + 5(-3)^2 - 2(-3) - 24$

$= -27 + 45 + 6 - 24$

$= 51 - 51 = 0$ (R)

$x + 3$ is factor of $P(x)$

$P(x) = x^3 + 5x^2 - 2x - 24$

Put $x = 4$

$P(4) = (4)^3 + 5(4)^2 - 2(4) - 24$

$= 64 + 80 - 8 - 24$

$= 144 - 32 \neq 0$ (R)

$x - 4$ is not factor of $P(x)$

Put $x = -4$

$P(-4) = (-4)^3 + 5(-4)^2 - 2(-4) - 24$

$= -64 + 80 + 8 - 24$

$= -88 - 88 = 0$ (R)

$x + 4$ is factor of $P(x)$

$x^3 + 5x^2 - 2x - 24 = (x - 2)(x + 3)(x + 4)$

7. $P(x) = 3x^3 - x^2 - 12x + 4$

$P = 4$; factor of 4 are $\pm 1, \pm 2, \pm 4$

$q = 3$, factor of 3 are $= \pm 1, \pm 3$ possible factors of $P(x)$ can be found from $\frac{p}{q}$

$$\frac{p}{q} = \pm 1, \pm 2, \pm 3, \pm 4, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{3}{3}$$

$$P(x) = 3x^3 - x^2 - 12x + 4$$

Put $x = 2$

$$P(2) = 3(2)^3 - (2)^2 - 12(2) + 4$$
$$= 24 - 4 - 24 + 4$$
$$= 28 - 28 = 0 \qquad (R)$$

$x - 2$ is factor of $P(x)$

Put $x = -2$ in $P(x)$

$$P(-2) = 3(-2)^3 - (-2)^2 - 12(-2) + 4$$
$$= -24 - 4 + 24 + 4$$
$$= -28 + 28 = 0 \qquad (R)$$

$x + 2$ is factor of $P(x)$

Put $x = \frac{1}{2}$

$$P(\frac{1}{2}) = 3(\frac{1}{2})^3 - (\frac{1}{2})^2 - 12(\frac{1}{2}) + 4$$
$$= \frac{3}{8} - \frac{1}{4} - 6 + 4 \neq 0 \qquad (R)$$

$2x - 1$ is not factor of $P(x)$

Put $x = \frac{1}{3}$

$$P(\frac{1}{3}) = 3(\frac{1}{3})^3 - (\frac{1}{3})^2 - 12(\frac{1}{3}) + 4$$
$$= 3 \times \frac{1}{27} - \frac{1}{9} - 4 + 4 = 0 \qquad (R)$$

$x = \frac{1}{3}$ gives $3x - 1 = 0$

$3x - 1$ is factor of $P(x)$

Thus

$3x^3-x^2-12x+4 = (x-2)(x+2)(3x-1)$

8. $P(x) = 2x^3 + x^2 - 2x - 1$

$P = -1$; factor of -1 are ± 1

$q = 2$, factor of 2 are $= \pm 1, \pm 2$ possible factors of $P(x)$ will be found from $\frac{p}{q}$

$$\frac{p}{q} = \pm 1, \pm\frac{1}{2}$$

$$P(x) = 2x^3 + x^2 - 2x - 1$$

Put $x = 1$

$$P(1) = 2(1)^3 + (1)^2 - 2(1) - 1$$
$$= 2 + 1 - 2 - 1 = 0 \qquad (R)$$

$x + 1$ is factor of $P(x)$

Put $x = -1$

$$P(-1) = 2(-1)^3 + (-1)^2 - 2(-1) - 1$$
$$= -2 + 1 + 2 - 1 \qquad (R)$$

$x + 1$ is factor of $P(x)$

Put $x = -\frac{1}{2}$

$$P\left(-\frac{1}{2}\right) = 2\left(-\frac{1}{2}\right)^3 + \left(-\frac{1}{2}\right)^2 - 2\left(-\frac{1}{2}\right) - 1$$
$$= -2 \times \frac{1}{8} + \frac{1}{4} + 1 - 1$$
$$= -\frac{1}{4} + \frac{1}{4} + 1 - 1 = 0 \qquad (R)$$

From $x = -\frac{1}{2}$, $2x + 1$ is factor of $P(x)$

Thus

... $x^2 - 2x - 1 = (x-1)(x+1)(2x+1)$

Unit 6 ALGEBRAIC MANIPULATION

Highest Common Factor (H.C.F) and Least Common Multiple (L.C.M) of Algebraic Expressions

Highest Common Factor (H.C.F):

If two or more algebraic expressions are given then their common factor of highest power is called the H.C.F. of the expressions.

Least Common Multiple (L.C.M.)

If an algebraic expression p(x) is exactly divisible by two or more expressions, then p(x) is called the Common Multiple of the given expressions.

The Least Common Multiple (L.C.M) is the product of common factor of the given expressions.

Square Root:

Square root of an algebraic expression p(x) will be another algebraic expression q(x) if $[q(x)] \times [q(x)] = p(x)$

Exercise 6.1

Q.1 Find the H.C.F. of the following expressions.

(i) $39x^7y^3z, 91x^5y^6z^7$

(ii) $102xy^2z, 85x^2yz, 187xyz^2$

1(i) $39x^7y^3z, 91x^5y^6z^7$

Solution:

$$39x^7y^3z = 3 \times 13\, x^5.x^2.y^3.z$$
$$91x^5y^6z^7 = 7 \times 13\, x^5.y^3.y^3.z.z^6$$
$$\text{H.C.F.} = 13x^5y^3z$$

1(ii) $102xy^2z, 85x^2yz, 187xyz^2$

Solution:

$$102xy^2z = 2 \times 3 \times 17\, xyyz$$
$$85x^2yz = 5 \times 17xxyz$$

$$187xyz^2 = 11 \times 17xyzz$$
$$H.C.F. = 17xyz$$

Q.2 Find the H.C.F. of the following expressions by factorization.

(i) $x^2 + 5x + 6$, $x^2 - 4x - 12$

(ii) $x^3 - 27$, $x^2 + 6x - 27$, $2x^2 - 18$

(iii) $x^3 - 2x^2 + x$, $x^2 + 2x - 3$, $x^2 + 3x - 4$

(iv) $18(x^3 - 9x^2 + 8x)$, $24(x^2 - 3x + 2)$

(v) $36(3x^4 + 5x^3 - 2x^2)$, $54(27x^4 - x)$

2(i) $x^2 + 5x + 6$, $x^2 - 4x - 12$

Solution:

$$\begin{aligned} x^2 + 5x + 6 &= x^2 + 2x + 3x + 6 \\ &= x(x+2) + 3(x+2) \\ &= (x+2)(x+3 \end{aligned}$$

and $x^2 - 4x - 12$

$$\begin{aligned} x^2 - 4x - 12 &= x^2 - 6x + 2x - 12 \\ &= x(x-6) + 2(x-6) \\ &= (x-6)(x+2) \end{aligned}$$

Now
$$\begin{aligned} x^2 + 5x + 6 &= (x+2)(x+3) \\ x^2 - 4x - 12 &= (x-6)(x+2) \\ H.C.F. &= x + 2 \end{aligned}$$

2(ii) $x^3 - 27$, $x^2 + 6x - 27$, $2x^2 - 18$

Solution:

$$\begin{aligned} x^3 - 27 &= (x)^3 - (3)^3 \\ &= (x-3)[(x)^2 + (x)(3) + (3)^2] \\ &= (x-3)(x^2 + 3x + 9) \end{aligned}$$

$$\begin{aligned} x^2 + 6x - 27 &= x^2 + 9x - 3x - 27 \\ &= x(x+9) - 3(x+9) \\ &= (x-3)(x+9) \end{aligned}$$

$$\begin{aligned} 2x^2 - 18 &= 2(x^2 - 9) \\ &= 2[(x)^2 - (3)^2] \\ &= 2(x-3)(x+3) \end{aligned}$$

Now $x^3 - 27 = (x-3)(x^2+3x+9)$

$x^2 + 6x - 27 = (x-3)(x+9)$

$2x^2 - 18 = 2(x-3)(x+3)$

H.C.F. $= (x-3)$

2(iii) $x^3 - 2x^2 + x, \quad x^2 + 2x - 3, \quad x^2 + 3x - 4$

Solution:

$$\begin{aligned}
x^3 - 2x^2 + x &= x(x^2 - 2x + 1) \\
&= x(x^2 - x - x + 1) \\
&= x[x(x-1) - 1(x-1)] \\
&= x(x-1)(x-1) \\
x^2 + 2x - 3 &= x^2 + 3x - x - 3 \\
&= x(x+3) - 1(x+3) \\
&= (x-1)(x+3) \\
x^2 + 3x - 4 &= x^2 + 4x - x - 4 \\
&= x(x+4) - 1(x+4) \\
&= (x-1)(x+4)
\end{aligned}$$

Now
$$\begin{aligned}
x^3 - 2x^2 + x &= x(x-1)(x-1) \\
x^2 + 2x - 3 &= (x-1)(x+3) \\
x^2 + 3x - 4 &= (x-1)(x+4) \\
\text{H.C.F.} &= x - 1
\end{aligned}$$

2(iv) $18(x^3 - 9x^2 + 8x), \quad 24(x^2 - 3x + 2)$

Solution:

$$\begin{aligned}
18(x^3 - 9x^2 + 8x) &= 2 \times 3 \times 3x(x^2 - 9x + 8) \\
&= 2 \times 3 \times 3x[x^2 - x - 8x + 8] \\
&= 2 \times 3 \times 3x[x(x - 1 - 8(x-1)] \\
&= 2 \times 3 \times 3x(x-1)(x-8) \\
24(x^2 - 3x + 2) &= 2 \times 2 \times 2 \times 3[x^2 - x - 2x + 2] \\
&= 2 \times 2 \times 2 \times 3[x(x-1) - 2(x-1)] \\
&= 2 \times 2 \times 2 \times 3(x-1)(x-2)
\end{aligned}$$

Now
$$\begin{aligned}
18(x^3 - 9x^2 + 8x) &= 2 \times 3 \times 3x(x-1)(x-8) \\
24(x^2 - 3x + 2) &= 2 \times 2 \times 2 \times 3(x-1)(x-2) \\
\text{H.C.F} &= 2 \times 3(x-1) \\
&= 6(x-1)
\end{aligned}$$

2(v) $36(3x^4 + 5x^3 - 2x^2),\quad 54(27x^4 - x)$

Solution: $36(3x4 + 5x3 - 2x2) = 4 \times 9x2(3x2 + 5x - 2)$

$= 4 \times 9x2[3x2 + 6x - x - 2]$

$= 4 \times 9x2[3x(x + 2) - 1(x + 2)]$

$= 2\times2 \times 3 \times 3x2(x + 2)(3x - 1)$

$54(27x4 - x) = 6 \times 9x(27x3 - 1)$

$= 6 \times 9x[(3x)3 - (1)3]$

$= 2\times3\times3\times3x(3x-1)(9x2+3x+1)$

Now $36(3x4 + 5x3 - 2x2) = 2\times2 \times 3 \times 3x2(x + 2)(3x - 1)$

$54(27x4 - x) = 2\times3\times3\times3x(3x-1)(9x2+3x+1)$

H.C.F. $= 2 \times 3 \times 3x(3x - 1)$

$= 18x(3x - 1)$

Q.3 Find the H.C.F. of the following by division method.

(i) $x^3 + 3x^2 - 16x + 12,\quad x^3 + x^2 - 10x + 8$

(ii) $x^4 + x^3 - 2x^2 + x - 3,\quad 5x^3 + 3x^2 - 17x + 6$

(iii) $2x^5 - 4x^4 - 6x,\quad x^5 + x^4 - 3x^3 - 3x^2$

3(i) $x^3 + 3x^2 - 16x + 12,\quad x^3 + x^2 - 10x + 8$

Solution:

$x^3 + x^2 - 10x + 8 \,\big)\, x^3 + 3x^2 - 16x + 12 \,(\, 1$

$\underline{\pm x^3 \pm x^2 \mp 10x \pm 8}$

$2\,\big|\,\underline{2x^2 - 6x + 4}$

$x2 - 3x + 2 \,\big)\, x3 + x2 - 10x + 8 \,(\, x+4$

$\underline{\pm x^3 - 3x^2 \pm 2x \pm 8}$

$4x^2 - 12x + 8$

$4x^2 - 12x + 8$

$\underline{\quad + \quad\quad - \quad}$

$\times$

H.C.F. $= x^2 - 3x + 2$

3(ii) $x^4 + x^3 - 2x^2 + x - 3,\quad 5x^3 + 3x^2 - 17x + 6$

Solution:

$5x^3 + 3x^2 - 17x + 6 \;\big)\; x^4 + x^3 - 2x^2 + x - 3 \;\big(\; x + 2$

$\times 5$

$5x^4 + 5x^3 - 10x^2 + 5x - 15$

$\pm 5x^4 \pm 3x^3 \mp 17x^2 \pm 6x$

$2x^3 + 7x^2 - x - 15$

$\times 5$

$10x^3 + 35x^2 - 5x - 75$

$\pm 10x^3 \pm 6x^2 - 34x \pm 12$

$29 \;\big)\; 29x^2 + 29x - 87$

$x^2 + x - 3 \;\big)\; 5x^3 + 3x^2 - 17x + 6 \;\big(\; 5x - 2$

$\pm 5x^3 \pm 5x^2 \mp 15x$

$-2x^2 - 2x + 6$

$\mp 2x^2 \mp 2x \pm 6$

$\times$

$\text{H.C.F} = x^2 + x - 3$

3(iii) $2x^5 - 4x^4 - 6x,\quad x^5 + x^4 - 3x^3 - 3x^2$

Solution: $2x^5 - 4x^4 - 6x = x(2x^4 - 4x^3 - 6)$

$x^5 + x^4 - 3x^3 - 3x^2 = x^2(x^3 + x^2 - 3x - 3)$

$x^3 + x^2 - 3x - 3 \;\big)\; 2x^4 - 4x^3 \quad - 6 \;\big(\; 2x - 6$

$\pm 2x^4 \pm 2x^3 - 6x^2 \pm 6x$

$-6x^3 + 6x^2 + 6x - 6$

$\mp 6x^3 \mp 6x^2 \pm 18x \pm 18$

$12 \;\big)\; 12x^2 - 12x - 24$

$x^2 - x - 2 \;\big)\; x^3 + x^2 - 3x - 3 \;\big(\; x + 2$

$\pm x^3 - x^2 - 2x$

$2x^2 - x - 3$

$\pm 2x^2 \mp 2x \mp 4$

$x + 1 \;\big)\; x^2 - x - 2 \;\big(\; 5x - 2$

$\pm x^2 \pm x$

$$\begin{array}{r} -2x-2 \\ \underline{-2x-2} \\ \times \end{array}$$

H.C.F. of x and x^2 is x

$\therefore$ H.C.F. $= x(x+1)$

Q.4 ***Find the L.C.M. of the following expression by factorization.***

(i) $39x^7y^3z,\quad 91x^5y^6z^7$

(ii) $102xy^2z,\quad 85x^2yz,\quad 187xyz^2$

4(i) $39x^7y^3z,\quad 91x^5y^6z^7$

Solution: $39x^7y^3z = 3 \times 13x^7y^3z \quad = 3 \times 13x^5x^2y^3z$

$91x^5y^6z^7 = 7 \times 13x^5y^6z^7 \quad = 7 \times 13x^5y^3y^2zz^6$

L.C.M. = (Common factor)×(non-common factor)

$= (13x^5y^3z)(3 \times 7x^2y^3z^6)$

$= 13 \times 3 \times 7x^{5+2}y^{3+3}z^{1+6}$

$= 273x^7y^6z^7$

4(ii) $102xy^2z,\quad 85x^2yz,\quad 187xyz^2$

Solution: $102xy^2z = 2\times3\times17xyyz$

$85x^2yz = 5 \times 17\ x.x.y.z$

$187xyz^2 = 11 \times 17xyzz$

L.C.M. = (Common factor)×(non-common factor)

$= (17xyz)(2 \times 5 \times 11xyz)$

$= 17 \times 2 \times 3 \times 5 \times 11x^2y^2z^2$

$= 5610x^2y^2z^2$

Q.5 ***Find the L.C.M. of the following expression by factorization.***

(i) $x^2 - 25x + 100,\ x^2 - x - 20$

(ii) $x^2 + 4x + 4,\ x^2 - 4,\ 2x^2 + x - 6$

(iii) $2(x^4 - y^4),\ 3(x^3 + 2x^2 - xy^2 - 2y^3)$

(iv) $4(x^4 - 1),\ 6(x^3 - x^2 - x + 1)$

5(i) $\mathbf{x^2 - 25x + 100, x^2 - x - 20}$

Solution:

$$
\begin{aligned}
x^2 - 25x + 100 &= x^2 - 5x - 20x + 100 \\
&= x(x-5) - 20(x-5) \\
&= (x-5)(x-20) \\
x^2 - x - 20 &= x^2 - 5x + 4x - 20 \\
&= x(x-5) + 4(x-5) \\
&= (x-5)(x+4)
\end{aligned}
$$

Now $x^2 - 25x + 100 = (x-5)(x-20)$

and $x^2 - x - 20 = (x-5)(x+4)$

$$\text{L.C.M.} = (x-5)(x-20)(x+4)$$

5(ii) $\mathbf{x^2 + 4x + 4, x^2 - 4, 2x^2 + x - 6}$

Solution:

$$
\begin{aligned}
x^2 + 4x + 4 &= x^2 + 2x + 2x + 4 \\
&= x(x+2) + 2(x+2) \\
&= (x+2)(x+2) \\
x^2 - 4 &= (x)^2 - (2)^2 \\
&= (x-2)(x+2) \\
2x^2 + x - 6 &= 2x^2 + 4x - 3x - 6 \\
&= 2x(x+2) - 3(x+2) \\
&= (x+2)(2x-3)
\end{aligned}
$$

Now

$$
\begin{aligned}
x^2 + 4x + 4 &= (x+2)(x+2) \\
x^2 - 4 &= (x-2)(x+2) \\
2x^2 + x - 6 &= (x+2)(2x-3) \\
\text{L.C.M.} &= (x+2)(x+2)(x-2)(2x-3) \\
&= (x+2)^2(x-2)(2x-3)
\end{aligned}
$$

5(iii) $\mathbf{2(x^4 - y^4), 3(x^3 + 2x^2 - xy^2 - 2y^3)}$

Solution:

$$
\begin{aligned}
2(x^4 - y^4) &= 2[(x^2)^2 - (y^2)^2] \\
&= 2(x^2 + y^2)(x^2 - y^2) \\
&= 2(x^2 + y^2)(x+y)(x-y) \\
3(x^3 + 2x^2 - xy^2 - 2y^3) &= 3[(x^3 + 2x^2y) = (xy^2 - 2y^3)] \\
&= 3[x^2(x+2y) - y^2(x+2y)]
\end{aligned}
$$

$$= 3(x + 2y)(x^2 - y^2)$$
$$= 3(x + 2y)(x + y)(x - y)$$
$$2(x^4 - y^4) = 2(x^2 + y^2)(x + y)(x - y)$$
$$3(x^3 + 2x^2 - xy^2 - 2y^3) = 3(x + 2y)(x + y)(x - y)$$
$$\text{L.C.M.} = 2\times3(x + y)(x - y)(x + 2y)(x^2+y^2)$$
$$= 6[(x + y)(x - y)](x + 2y)(x^2+y^2)$$
$$= 6(x^2 - y^2)(x^2 + y^2)(x + 2y)$$
$$= 6[(x^2 - y^2)(x^2 + y^2)](x + 2y)$$
$$= 6(x^4 - y^4)(x + 2y)$$

5(iv) $4(x^4 - 1), 6(x^3 - x^2 - x + 1)$

Solution:
$$4(x^4 - 1) = 2 \times 2[(x^2)^2 - (1)^2]$$
$$= 2 \times 2(x^2 + 1)(x^2 - 1)$$
$$= 2 \times 2(x^2 + 1)(x + 1)(x - 1)$$
$$6(x^3 - x^2 - x + 1) = 2 \times 3[x^3 - x^2 - x + 1]$$
$$= 2 \times 3[x^2(x - 1) - 1(x - 1)]$$
$$= 2 \times 3(x - 1)(x^2 - 1)$$
$$4(x^4 - 1) = 2 \times 2(x^2 + 1)(x + 1)(x - 1)$$
$$6(x^3 - x^2 - x + 1) = 2 \times 3(x - 1)(x - 1)(x + 1)$$
$$\text{L.C.M.} = 2\times2\times3(x-1)(x + 1)(x - 1)(x^2+1)$$
$$= 12(x - 1)[(x + 1)(x - 1)](x^2 + 1)$$
$$= 12(x - 1)(x^2 - 1)(x^2 - 1)$$
$$= 12(x - 1)[(x^2)^2 + (1)^2]$$
$$= 12(x - 1)(x^4 - 1)$$

Q.6 **For what value of k is (x + 4) the H.C.F. of $x^2 + x - (2k + 2)$ and $2x^2 + kx - 12$?**

Solution: $(x + 4)$ will divide completely $x^2 + x - (2k + 2)$ and $2x^2 + kx - 12$.

$$p(x) = x^2 + x - 2k - 2$$

Put $x = -4$

$$p(-4) = (-4)^2 + (-4) - 2k - 2$$

$= 16 - 4 - 2k - 2$

$= 10 - 2k = R$

R must be zero

$\therefore \quad 10 - 2k = 0$

$-2k = -10$

$k = 5$

$q(x) = 2x^2 + kx - 12$

Put $x = -4$

$q(-4) = 2(-4)^2 - 4k - 12$

$= 32 - 4k - 12$

$= 20 - 4k = R$

R must be zero

$20 - 4k = 0$

$-4k = -20$

$\therefore \quad k = 5$

Q.7 If $(x + 3)(x - 2)$ is the H.C.F. of
$p(x) = (x + 3)(2x^2 - 3x + k)$ and
$q(x) = (x - 2)(3x^2 + 7x - l)$, find k and l.

Solution: $(x - 2)(x + 3)$ will divide $p(x) = (x + 3)(2x^2 - 3x + k)$

We put $x = 2$ in it.

$p(2) = (2 + 3)[2 \times 2^2 - 3(2) + k]$

$= 5(8 - 6 + k)$

$= 5(2 + k) = R$

R must be zero

$\therefore \quad 5(2 + k) = 0$

$2 + k = 0$

$k = -2$

$q(x) = (x-2)(3x^2 + 7x - l)$

$(x - 2)(x + 3)$ will divide $q(x) = (x - 2)(3x^2 + 7x - l)$

Completely. We put $x = -3$ in it

$$q(-3) = (-3-2)[3(-3)^2 + 7(-3) - l]$$
$$= (-5)(27 - 21 - l)$$
$$= (-5)(6 - l) = R$$

R must be zero

$$\therefore \quad (-5)(6 - l) = 0$$
$$6 - l = 0$$
$$l = 6$$

Q.8 The L.C.M. and H.C.F of two polynomials p(x) and $2(x^4 - 1)$ and $(x + 1)(x^2 + 1)$ respectively.
If $p(x) = x^3 + x^2 + x + 1$, find q(x) are

Solution: Formula $p(x) \times q(x) = \text{L.C.M.} \times \text{H.C.F}$

$$p(x) \times q(x) = 2(x^4 - 1)(x + 1)(x^2 + 1)$$
$$(x^3 + x^2 + x + 1) \times q(x) = 2(x^4 - 1)(x + 1)(x^2 + 1)$$
$$q(x) = \frac{2(x^4-1)(x+1)(x^2+1)}{x^3+x^2+x+1}$$
$$= \frac{2(x^4-1)(x+1)(x^2+1)}{x^2(x+1)+1(x+1)}$$
$$= \frac{2(x^4-1)(x+1)(x^2+1)}{(x+1)(x^2+1)}$$
$$qx = 2(x^4 - 1)$$

Q.9 **Let $p(x) = 10(x^2 - 9)(x^2 - 3x + 2)$ and $q(x) = 10x(x + 3)(x - 1)^2$. If the H.C.F. of p(x), q(x) is $10(x + 3)(x - 1)$, find their L.C.M.**

Solution: Formula $p(x) \times q(x) = \text{L.C.M.} \times \text{H.C.F}$

Putting values in it.

$$10(x+3)(x-1) \times q(x) = 10(x^2-9)(x^2-3x+2)(10)(x)(x+3)(x-1)^2$$
$$q(x) = \frac{10(x^2-9)(x^2-3x+2)(10)(x)(x+3)(x-1)^2}{10(x+3)(x-1)}$$

$$= (x^2-9)(x^2-3x+2)(10)(x)(x-1)$$
$$= 10x(x^2-9)(x-1)(x^2-3x+2)$$
$$= 10x(x^2-9)(x-1)[(x^2-2x-x+2)]$$
$$= 10x(x^2-9)(x-1)[x(x-2)-1(x-2)]$$
$$= 10x(x^2-9)(x-1)(x-1)(x-2)$$
$$= 10x(x^2-9)(x-1)^2(x-2)$$

Q.10 **Let the product of L.C.M. and H.C.F. of two polynomials be $(x+3)^2(x-2)(x+5)$. If one polynomial is $(x+3)(x-2)$ and the second polynomial is $x^2+kx+15$, find the value of k.**

Solution:
$$(\text{L.C.M.}) \times (\text{H.C.F.}) = (x+3)^2(x-2)(x+5)$$
$$p(x) = (x+3)(x-2)$$
$$q(x) = x^2+kx+15$$
$$p(x) \times q(x) = \text{H.C.F} \times \text{L.C.M.}$$

Putting value in it

$$(x^2+kx+15)(x+3)(x-2) = (x+3)^2(x-2)(x+5)$$
$$x^2+kx+15 = \frac{(x+3)^2(x-2)(x+5)}{(x+3)(x-2)}$$
$$x^2+kx+15 = (x+3)(x+5)$$
$$x^2+kx+15 = x^2+8x+15$$
$$kx = x^2+8x+15-x^2-15$$
$$kx = 8x$$
$$\therefore \quad k = 8$$

Q.11 ***Waqas wishes to distribute 128 bananas and also 176 apples among a certain number of children. Find the highest number of children who can get the fruit in this way.***

Solution: Number of children will be H.C.F of 176 and 128

```
          1
128 | 176
      128      2
       48 | 128
             96     1
             32 | 48
                  32     2
                  16 | 32
                       32
                        ×
```

Number of children = 16

Q.2 $\left[\frac{x+1}{x-1}-\frac{x-1}{x+1}-\frac{4x}{x^2+1}\right]+\frac{4x}{x^4-1}$

Solution:
$$=\left(\frac{x+1}{x-1}-\frac{x-1}{x+1}\right)-\frac{4x}{x^2+1}+\frac{4x}{x^4-1}$$
$$=\frac{(x+1)^2-(x-1)^2}{(x-1)(x+1)}-\frac{4x}{x^2+1}+\frac{4x}{x^4-1}$$
$$=\frac{x^2+2x+1-x^2+2x-1}{x^2-1}-\frac{4x}{x^2+1}+\frac{4x}{x^4-1}$$
$$=\left[\frac{4x}{x^2-1}-\frac{4x}{x^2+1}\right]+\frac{4x}{x^4-1}$$
$$=(4x)\left[\frac{1}{x^2-1}-\frac{1}{x^2+1}\right]+\frac{4x}{x^4-1}$$
$$=(4x)\left[\frac{x^2+1-x^2+1}{(x^2-1)(x^2+1)}\right]+\frac{4x}{x^4-1}$$
$$=(4x)\left[\frac{2}{x^4-1}\right]+\frac{4x}{x^4-1}$$
$$=(4x)\left[\frac{2}{x^4-1}+\frac{1}{x^4-1}\right]$$
$$=(4x)\left[\frac{2+1}{(x^4-1)}\right]$$
$$=\frac{12x}{x^4-1}$$

Q.3 $\frac{1}{x^2-8x+15}+\frac{1}{x^2-4x+3}-\frac{2}{x^2-6x+5}$

Solution: =
$$\frac{1}{x^2-3x-5x+15}+\frac{1}{x^2-x-3x+3}-\frac{2}{x^2-x-5x+5}$$

$$= \frac{1}{x(x-3)-5(x-3)} + \frac{1}{x(x-1)-3(x-1)} - \frac{2}{x(x-1)-5(x-1)}$$

$$= \frac{1}{(x-3)(x-5)} + \frac{1}{(x-1)(x-3)} - \frac{2}{(x-1)(x-5)}$$

$$= \frac{(x-1)+(x-5)-2(x-3)}{(x-3)(x-5)(x-1)}$$

$$= \frac{x-1+x-5-2x+6}{(x-3)(x-5)(x-1)}$$

$$= \frac{2x-2x+6-6}{(x-3)(x-5)(x-1)}$$

$$= 0$$

Q.4 $\frac{(x+2)(x+3)}{x^2-9} + \frac{(x+2)(2x^2-32)}{(x-4)(x^2-x-6)}$

Solution:

$$= \frac{(x+2)(x+3)}{(x)^2-(3)^2} + \frac{(x+2)(2)(x^2-16)}{(x-4)(x^2-3x+2x-6)}$$

$$= \frac{(x+2)(x+3)}{(x-3)(x+3)} + \frac{2(x+2)(x^2-4^2)}{(x-4)[x(x-3)+2(x-3)]}$$

$$= \frac{(x+2)}{(x-3)} + \frac{2(x+2)(x-4)(x+4)}{(x-4)(x-3)(x+2)}$$

$$= \frac{x+2}{x-3} + \frac{2(x+4)}{x-3}$$

$$= \frac{x+2+2x+8}{x-3}$$

$$= \frac{3x+10}{x-3}$$

Q.5 $\dfrac{x+3}{2x^2+9x+9}+\dfrac{1}{2(2x-3)}-\dfrac{4x}{4x^2-9}$

Solution:

$$= \frac{x+3}{2x^2+6x+3x+9}+\frac{1}{2(2x-3)}-\frac{4x}{(2x)^2-(3)^2}$$

$$= \frac{x+3}{2x(x+3)+3(x+3)}+\frac{1}{2(2x-3)}-\frac{4x}{(2x+3)(2x-3)}$$

$$= \frac{x+3}{(2x+3)(x+3)}+\frac{1}{2(2x-3)}-\frac{4x}{(2x+3)(2x-3)}$$

$$= \frac{1}{(2x+3)}+\frac{1}{2(2x-3)}-\frac{4x}{(2x+3)(2x-3)}$$

$$= \frac{2(2x-3)+(2x+3)-(4x)(2)}{2(2x+3)(2x-3)}$$

$$= \frac{4x-6+2x+3-8x}{2(2x+3)(2x-3)}$$

$$= \frac{-2x-3}{2(2x+3)(2x-3)}$$

$$= \frac{-(2x+3)}{2(2x+3)(2x-3)}$$

$$= \frac{-1}{2(2x-3)}$$

Q.6 $A-\dfrac{1}{A}$ if $A=\dfrac{a+1}{a-1}$

Solution:

$$A-\frac{1}{A} = \frac{a+1}{a-1}-\frac{1}{\frac{a+1}{a-1}}$$

$$= \frac{a+1}{a-1}-\frac{a-1}{a+1}$$

$$= \frac{(a+1)^2-(a-1)^2}{(a-1)(a+1)}$$

$$= \frac{(a^2+2a+1)-(a^2-2a+1)}{(a-1)(a+1)}$$

$$= \frac{a^2+2a+1-a^2+2a-1}{(a-1)(a+1)}$$

$$= \frac{4a}{a^2-1}$$

Q.7 $\left[\frac{x-1}{x-2}+\frac{2}{2-x}\right]-\left[\frac{x+1}{x+2}+\frac{4}{4-x^2}\right]$

Solution: $= \left[\frac{x-1}{x-2}-\frac{2}{x-2}\right]-\left[\frac{x+1}{x+2}-\frac{4}{x^2-4}\right]$ N.T.S

$$= \left[\frac{x-1-2}{x-2}\right]-\left[\frac{x+1}{x+2}-\frac{4}{(x+2)(x-2)}\right]$$

$$= \left[\frac{x-3}{x-2}\right]-\left[\frac{(x+1)(x-2)(4)}{(x+2)(x-2)}\right]$$

$$= \frac{x-3}{x-2}-\frac{x^2-x-2-4}{(x+2)(x-2)}$$

$$= \frac{x-3}{x-2}-\frac{x^2-x-6}{(x+2)(x-2)}$$

$$= \frac{x-3}{x-2}-\frac{x^2-3x+2x-6}{(x+2)(x-2)}$$

$$= \frac{x-3}{x-2}-\frac{x(x-3)+2(x-3)}{(x+2)(x-2)}$$

$$= \frac{x-3}{x-2}-\frac{(x-3)(x+2)}{(x+2)(x-2)}$$

$$= \frac{x-3}{x-2} - \frac{x-3}{x-2}$$

$$= 0$$

Q.8 ***What rational expression should be subtracted from*** $\frac{2x^2+2x-7}{x^2+x-6}$ ***to get*** $\frac{x-1}{x-2}$***?***

Let A be the required expression.

$$\frac{x^2+2x-7}{x^2+x-6} - A = \frac{x-1}{x-2}$$

Solution:Thus $A = \frac{2x^2+2x-7}{x^2+x-6} - \frac{x-1}{x-2}$

$$= \frac{2x^2+2x-7}{x^2+3x-2x-6} - \frac{x-1}{x-2}$$

$$= \frac{2x^2+2x-7}{x(x+3)-2(x+3)} - \frac{x-1}{x-2}$$

$$= \frac{2x^2+2x-7}{(x+3)(x-2)} - \frac{x-1}{x-2}$$

$$= \frac{2x^2+2x-7-(x-1)(x+3)}{(x+3)(x-2)}$$

$$= \frac{2x^2+2x-7-(x^2+2x-3)}{(x+3)(x-2)}$$

$$= \frac{2x^2+2x-7-x^2-2x+3}{(x+3)(x-2)}$$

$$= \frac{x^2-4}{(x+3)(x-2)}$$

$$= \frac{(x)^2-(2)^2}{(x+3)(x-2)}$$

$$= \frac{(x+2)(x-2)}{(x+3)(x-2)}$$

$$= \frac{x+2}{x+3}$$

Perform the indicated operations and simplify to the lowest forms. $\frac{x^2+x-6}{x^2-x-6} \times \frac{x^2-4}{x^2-9}$

Q.9 Simplify to the lowest form

$$\frac{x^2+x-6}{x^2-x-6} \times \frac{x^2-2^2}{x^2-3^2}$$

Solution:

$$= \frac{x^2+3x-2x-6}{x^2-3x+2x-6} \times \frac{x^2-2^2}{x^2-3^2}$$

$$= \frac{x(x+3)-2(x+3)}{x(x-3)+2(x-3)} \times \frac{(x+2)(x-2)}{(x+3)(x-3)}$$

$$= \frac{(x+3)(x-2)}{(x-3)(x+2)} \times \frac{(x+2)(x-2)}{(x+3)(x-3)}$$

$$= \frac{(x-2)(x-2)}{(x-3)(x-3)}$$

$$= \frac{(x-2)^2}{(x-3)^2}$$

Q.10 $\frac{x^3-8}{x^2-4} + \frac{x^2+6x+8}{x^2-2x+1}$

$$\frac{x^3-2^3}{x^2-2^2} \times \frac{x^2+4x+2x+8}{x^2-x-x+1}$$

Solution:

$$= \frac{x^3-2^3}{x^2-2^2} + \frac{x(x+4+2(x+4)}{x(x-1)-1(x-1)}$$

$$= \frac{(x-2)(x^2+2x+4)}{(x-2)(x+2)} + \frac{x(x+4)+2(x+4)}{x(x-1)-1(x-1)}$$

$$= \frac{x^2+2x+4}{x+2} \div \frac{(x+4)(x+2)}{(x-1)(x-1)}$$

$$= \frac{(x^2+2x+4)(x+4)}{(x-1)(x-1)}$$

$$= \frac{(x+4)(x^2+2x+4)}{(x-1)^2}$$

Q.11. $\frac{x^4-8x}{2x^2+5x-3} \times \frac{2x-1}{x^2+2x+4} \times \frac{x+3}{x^2-2x}$

Solution:

$$= \frac{x(x^3-8)}{2x^2+6x-x-3} \times \frac{2x-1}{x^2+2x+4} \times \frac{x+3}{x(x-2)}$$

$$= \frac{x(x^3-2^3)}{2x(x+3)-1(x+3)} \times \frac{2x-1}{x^2+2x+4} \times \frac{x+3}{x(x-2)}$$

$$= \frac{x(x-2)(x^2+2x+4)}{(x+3)(2x-1)} \times \frac{2x-1}{x^2+2x+4} \times \frac{x+3}{x(x-2)}$$

$$= \frac{x(x-2)(x^2+2x+4)(2x-1)(x+3)}{x(x-2)(x^2+2x+4)(2x-1)(x+3)}$$

$$= 1$$

Q.12 $\frac{2y^2+7y-4}{3y^2-13y+4} \div \frac{2y^2-1}{6y^2+y-1}$

$\frac{2y^2+7y-4}{3y^2-13y+4} \div \frac{4y^2-1}{6y^2+y-1}$

Solution:

$$= \frac{2y^2+8y-y-4}{3y^2-12y-y+4} \div \frac{(2y)^2-(1)^2}{6y^2+3y-2y-1}$$

$$= \frac{2y(y+4)-1(y+4)}{3y(y-4)-1(y-4)} \div \frac{(2y-1)(2y+1)}{3y(2y+1)-1(2y+1)}$$

$$= \frac{(y+4)(2y-1)}{(y-4)(3y-1)} \div \frac{(2y-1)(2y+1)}{(2y+1)(3y-1)}$$

$$= \frac{(y+4)(2y-1)}{(y-4)(3y-1)} \div \frac{2y-1}{3y-1}$$

$$= \frac{(y+4)(2y-1)}{(y-4)(3y-1)} \times \frac{3y-1}{2y-1}$$

$$= \frac{y+4}{y-4}$$

Q.13 $\left[\frac{x^2+y^2}{x^2-y^2} - \frac{x^2-y^2}{x^2+y^2}\right] \div \left[\frac{x+y}{x-y} - \frac{x-y}{x+y}\right]$

Solution:$= \left[\frac{(x^2+y^2)^2-(x^2-y^2)^2}{(x^2-y^2)(x^2+y^2)}\right] \div \left[\frac{(x+y)^2-(x-y)^2}{(x-y)(x+y)}\right]$

$$\frac{(x^4+y^4+2x^2y^2)-(x^4+y^4-2x^2y^2)}{(x^2-y^2)(x^2+y^2)} \div \frac{x^2+y^2+2xy-(x^2+y^2-2xy)}{(x-y)(x+y)}$$

$$\frac{x^4+y^4+2x^2y^2-x^4-y^4+2x^2y^2}{(x^2-y^2)(x^2+y^2)} \div \frac{x^2+y^2+2xy-x^2-y^2+2xy}{x^2-y^2}$$

$$= \frac{4x^2y^2}{(x^2-y^2)(x^2+y^2)} \div \frac{4xy}{x^2-y^2}$$

$$= \frac{4x^2y^2}{(x^2-y^2)(x^2+y^2)} \times \frac{x^2-y^2}{4xy}$$

$$= \frac{xy}{x^2+y^2}$$

Exercise 6.3

Q.1 ***Use factorization to find the square root of the following expressions.***

(i) $4x^2 - 12xy + 9y^2$

(ii) $x^2 - 1 + \frac{1}{4x^2}, (x \neq 0)$

(iii) $\frac{1}{16}x^2 - \frac{1}{12}xy + \frac{1}{36}y^2$

(iv) $4(a+b)^2 - 12(a^2 - b^2) + 9(a-b)^2$

(v) $\frac{4x^6 - 12x^3y^3 + 9y^6}{9x^4 + 24x^2y^2 + 16y^4}, (x \neq 0)$

(vi) $\left(x + \frac{1}{x}\right)^2 - 4\left(x - \frac{1}{x}\right), (x \neq 0)$

(vii) $\left(x^2 + \frac{1}{x^2}\right)^2 - 4\left(x + \frac{1}{x}\right)^2 + 12\ (x \neq 0)$

(viii) $(x^2 + 3x + 2)(x^2 + 4x + 3)(x^2 + 5x + 6)$

(ix) $(x^2 + 8x + 7)(2x^2 - x - 3)(2x^2 + 11x - 21)$

1(i) $4x^2 - 12xy + 9y^2$

Solution:
$$4x^2 - 12xy + 9y^2 = 4x^2 - 6xy - 6xy + 9y^2$$
$$= 2x(2x - 3y) - 3y(2x - 3y)$$
$$= (2x - 3y)(2x - 3y)$$
$$= (2x - 3y)^2$$
$$\sqrt{4x^2 - 12xy + 9y^2} = \pm(2x - 3y)$$

1(ii) $x^2 - 1 + \frac{1}{4x^2}, (x \neq 0)$

Solution:
$$x^2 - 1 + \frac{1}{4x^2} = (x)^2 - 1 + \left(\frac{1}{2x}\right)^2$$

$$= (x)^2 - 2(x)\left(\frac{1}{2x}\right) + \left(\frac{1}{2x}\right)^2$$

$$= \left(x - \frac{1}{2x}\right)^2$$

$$\sqrt{x^2 - 1 + \frac{1}{4x^2}} = \pm\left(x - \frac{1}{2x}\right)$$

I(iii) $\frac{1}{16}x^2 - \frac{1}{12}xy + \frac{1}{36y^2}$

Solution: $\frac{1}{16}x^2 - \frac{1}{12}xy + \frac{1}{36y^2} = \left(\frac{1}{4}x\right)^2 - \frac{1}{12}xy + \left(\frac{1}{6y}\right)^2$

$$= \left(\frac{1}{4}x\right)^2 - 2\left(\frac{1}{4}x\right)\left(\frac{1}{6y}\right) + \left(\frac{1}{6y}\right)^2$$

$$= \left(\frac{1}{4}x - \frac{1}{6y}\right)^2$$

$$\sqrt{\frac{1}{16}x^2 - \frac{1}{12}xy + \frac{1}{36y^2}} = \pm\left(\frac{x}{4} - \frac{1}{6y}\right)$$

I(iv) $4(a + b)^2 - 12(a^2 - b^2) + 9(a - b)^2$

Solution: $= [2(a + b)]^2 - 12(a + b)(a - b) + [3(a - b)]^2$

$= [2(a + b)]^2 - (2)(2)(a + b)(3)(a - b) + [3(a - b)]^2$

$= [2(a + b) - 3(a - b)]^2$

$= (2a + 22b - 3a + 3b)^2$

$= [5b - a]^2$

$$\sqrt{4(a+b)^2 - 12(a^2 - b^2) + 9(a - b)^2} = \pm(5b - a)$$

I(v) $\frac{4x^6 - 12x^3y^3 + 9y^6}{9x^4 + 24x^2y^2 + 16y^4}, (x = 0)$

Solution:

$$= \frac{(2x^3)^2 - 2(2x^3)(3y^3) + (3y^3)^2}{(3x^2)^2 + 2(3x^2)(4y^2) + (4y^2)^2}$$

$$= \frac{(2x^3 - 3y^3)^2}{(3x^2 + 4y^2)^2}$$

$$\frac{4x^6 - 12x^3y^3 + 9y^6}{9x^4 + 24x^2y^2 + 16y^4} = \left(\frac{2x^3 - 3y^3}{3x^2 + 4y^2}\right)^2$$

$$\sqrt{\frac{4x^6 - 12x^3y^3 + 9y^6}{9x^4 + 24x^2y^2 + 16y^4}} = \pm\frac{2x^3 - 3y^3}{3x^2 + 4y^2}$$

I(vi) $\left(x + \frac{1}{x}\right)^2 - 4\left(x - \frac{1}{x}\right), (x \neq 0)$

Solution: Let $x + \frac{1}{x} = a$

$$\text{Now } \left(x + \frac{1}{x}\right)^2 = \left(x - \frac{1}{x}\right)^2 + 4$$

$$= a^2 + 4$$

$= (a^2 + 4) - 4a$ (Given expression)

$= a^2 - 4a + 4$

$= (a - 2)^2$

$$= \left[\left(x - \frac{1}{x}\right) - 2\right]^2$$

$$\left(x + \frac{1}{x}\right)^2 - 4\left(x - \frac{1}{x}\right) = \left[\left(x - \frac{1}{x}\right) - 2\right]^2$$

$$\sqrt{\left(x + \frac{1}{x}\right)^2 - 4\left(x - \frac{1}{x}\right)} = \pm\left[\left(x - \frac{1}{x}\right) - 2\right]$$

1(vii) $\left(x^2+\frac{1}{x^2}\right)^2-4\left(x+\frac{1}{x}\right)^2+12$

Solution: Let $x+\frac{1}{x} = a$ (A)

$$\left(x+\frac{1}{x}\right)^2 = a^2 \quad \text{(Squaring)}$$

$$x^2+\frac{1}{x^2}+2 = a^2$$

$$x^2+\frac{1}{x^2} = a^2-2 \quad \text{(B)}$$

$\left(x^2+\frac{1}{x^2}\right)^2-4\left(x+\frac{1}{x}\right)^2+12 = (a^2-2)^2-4a^2+12$ (From A,B)

$$= a^4-4a^2+4-4a^2+12$$
$$= a^4-8a^2+16$$
$$= (a^2)^2-2(a^2)(4)+(4)^2$$
$$= (a^2-4)^2$$

$\sqrt{\left(x^2+\frac{1}{x^2}\right)^2-4\left(x+\frac{1}{x}\right)^2+12} = \sqrt{(a^2-4)^2}$ (Taking square root)

$$= \pm(a^2-4)$$

$= \pm\left[\left(x+\frac{1}{x}\right)^2-4\right]$ (Putting value of a)

$= \pm\,[x2+\frac{1}{x^2}+2-4] = \pm[x^2+\frac{1}{x^2}-2]$

1(viii) $(x^2+3x+2)(x^2+4x+3)(x^2+5x+6)$

Solution: $= [x^2+2x+x+2][x^2+x+3x+3][x^2+2x+3x+6]$

$= [x(x+2)+1(x+2)][x(x+1)+3(x+1)][x(x+2)+3(x+2)$

$= (x+2)(x+1)(x+1)(x+3)(x+2)(x+3)$

$= (x+1)^2(x+2)^2(x+3)^2$

Taking square root

$\sqrt{(x+1)^2(x+2)^2(x+3)^2} = (x+1)(x+2)(x+3)$

1(ix) $(x^2+8x+7)(2x^2-x-3)(2x^2+11x-21)$

Solution: $= [x^2+x+7x+7][2x^2-3x+2x-3][2x^2+14x-3x-21]$

$= [x(x+1)+7(x+1)][x(2x-3)+1(2x-3)][2x(x+7)-(x+7)$

$= (x+1)(x+7)(2x-3)(x+1)(x+7)(2x-3)$

$= (x+1)^2(x+7)^2(2x-3)^2$

Taking square root

$\sqrt{(x+1)^2(x+7)^2(2x-3)^2} = (x+1)(x+7)(2x-3)$

Q.2 ***Use division method to find the square root of the following expression.***

(i) $4x^2+12xy+9y^2+16x+24y+16$

(ii) $x^4-10x^3+37x^2-60x+36$

(iii) $9x^4-6x^3+7x^2-2x+1$

(iv) $4+25x^2-12x-24x^3+16x^4$

(v) $\frac{x^2}{y^2}-10\frac{x}{y}+27-\frac{10y}{x}+\frac{y^2}{x^2}, (x \neq 0, y \neq 0)$

2(i) $4x^2+12xy+9y^2+16x+24y+16$

Solution:

$$\begin{array}{r|l}
 & \quad 2x+3y+4 \\ \hline
2x & 4x^2+12xy+9y^2+16x+24y+16 \\
 & \pm 4x^2 \\ \hline
4x+3y & \qquad 12xy+9y^2 \\
 & \qquad \pm 12xy \pm 9y^2 \\ \hline
4x+6y+4 & \qquad\qquad 16x+24y+16 \\
 & \qquad\qquad \pm 16x \pm 24y \pm 16 \\ \hline
 & \qquad\qquad\qquad \times
\end{array}$$

Square root $= 2x+3y+4$

2(ii) $x^4 - 10x^3 + 37x^2 - 60x + 36$

Solution:

$$
\begin{array}{r|l}
 & \quad x^2 - 5x + 6 \\
\hline
x^2 & x^4 - 10x^3 + 37x^2 - 60x + 36 \\
 & \underline{\pm 4x^4} \\
2x^2 - 5x & \quad -10x^3 + 37x^2 \\
 & \quad \underline{-10x^3 \pm 25x^2} \\
2x^2 - 10x + 6 & \qquad\qquad 12x^2 - 60x + 36 \\
 & \qquad\quad \underline{\pm 12x^2 - 60x \pm 36} \\
 & \qquad\qquad\qquad \times
\end{array}
$$

Square root $= x^2 - 5x + 6$

2(iii) $9x^4 - 6x^3 + 7x^2 - 2x + 1$

Solution:

$$
\begin{array}{r|l}
 & \quad 3x^2 - x + 1 \\
\hline
3x^2 & 9x^4 - 6x^3 + 7x^2 - 2x + 1 \\
 & \underline{\pm 9x^4} \\
6x^2 - x & \quad -6x^3 + 7x^2 \\
 & \quad \underline{-6x^3 \pm x^2} \\
6x^2 - 2x + 1 & \qquad\qquad 6x^2 - 2x + 1 \\
 & \qquad\quad \underline{\pm 6x^2 - 2x \pm 1} \\
 & \qquad\qquad\qquad \times
\end{array}
$$

Square root $= 3x^2 - x + 1$

2(iv) $4 + 25x^2 - 12x - 24x^3 + 16x^4$

Solution:

$$
\begin{array}{r|l}
 & \quad 4x^2 - 3x + 2 \\
\hline
4x^2 & 16x^4 - 24x^3 + 25x^2 - 12x + 4 \\
 & \underline{\pm 16x^4} \\
8x^2 - 3x & \quad -24x^3 + 25x^2 \\
 & \quad \underline{-24x^3 \pm 9x^2} \\
8x^2 - 6x + 2 & \qquad\qquad 16x^2 - 12x + 4 \\
 & \qquad\quad \underline{\pm 16x^2 - 12x \pm 4} \\
 & \qquad\qquad\qquad \times
\end{array}
$$

Square root $= 4x^2 - 3x + 2$

2(v) $\dfrac{x^2}{y^2} - 10\dfrac{x}{y} + 27 - \dfrac{10y}{x} + \dfrac{y^2}{x^2}, (x \neq 0, y \neq 0)$

Solution: Writing in descending order

$$
\begin{array}{r|l}
 & \quad \frac{x}{y} - 5 + \frac{y}{x} \\
\hline
\frac{x}{y} & \frac{x^2}{y^2} - \frac{10x}{y} + 27 - \frac{10y}{x} + \frac{y^2}{x^2} \\
 & \frac{x^2}{y^2} \\
\hline
\frac{2x}{y} - 5 & \quad -\frac{10x}{y} + 27 \\
 & \quad -\frac{10x}{y} + 25 \\
 & \quad + \qquad - \\
\hline
\frac{2x}{y} - 10 + \frac{y}{x} & \qquad 2 - \frac{10y}{x} + \frac{y^2}{x^2} \\
 & \qquad 2 - \frac{10y}{x} + \frac{y^2}{x^2} \\
 & \qquad - \quad + \qquad - \\
\hline
 & \qquad\qquad x
\end{array}
$$

Square root $= \frac{x}{y} - 5 + \frac{y}{x}$

Q.3 ***Find the value of k for which the following expressions will become a perfect square.***

(i) $4x^4 - 12x^3 + 37x^2 - 42x + k$

(ii) $x^4 - 4x^3 + 10x^2 - kx + 9$

3(i) $4x^4 - 12x^3 + 37x^2 - 42x + k$

Solution: Taking square root

$$
\begin{array}{r|l}
 & 2x^2 - 3x + 7 \\
\hline
2x^2 & 4x^4 - 12x^3 + 37x^2 - 42x + k \\
 & \pm 4x^4 \\
\hline
4x^2 - 3x & \quad - 12x^3 + 37x^2 \\
 & \quad - 12x^3 \pm 9x^2 \\
\hline
4x^2 - 6x + 7 & \qquad\qquad 28x^2 - 42x + k \\
 & \qquad\qquad \pm 28x^2 \mp 42x \pm 49 \\
\hline
 & \qquad\qquad\qquad k - 49
\end{array}
$$

In case of perfect square, remainder is zero

Therefore $k - 49 = 0$

and $k = 49$

3(ii) $x^4 - 4x^3 + 10x^2 - kx + 9$

Solution: Taking square root

$$
\begin{array}{r|l}
 & \quad x^2 - 2x + 3 \\
\hline
x^2 & x^4 - 4x^3 + 10x^2 - kx + 9 \\
 & \underline{\pm x^4} \\
2x^2 - 2x & \quad -4x^3 + 10x^2 \\
 & \quad \underline{\mp 4x^3 \pm 4x^2} \\
2x^2 - 4x + 3 & \qquad\quad 6x^2 - kx + 9 \\
 & \qquad\quad \underline{\pm 6x^2 \mp 12x \pm 9} \\
 & \qquad\qquad -kx + 12x
\end{array}
$$

In case of perfect square

$-kx + 12 = 0$

$-x(k - 12) = 0$

$k - 12 = 0$

$\therefore \quad k = 12$

Q.4 ***Find the value of l and m for which the following expressions will become perfect square.***

(i) $x^4 + 4x^3 + 16x^2 + lx + m$

(ii) $49x^4 - 70x^3 + 109x^2 - lx - m$

4(i) $x^4 + 4x^3 + 16x^2 - lx + m$

Solution: Taking square root

$$
\begin{array}{r|l}
 & \quad x^2 + 2x + 6 \\
\hline
x^2 & x^4 + 4x^3 + 16x^2 + lx + m \\
 & \underline{\pm x^4} \\
2x^2 + 2x & \quad 4x^3 + 16x^2 \\
 & \quad \underline{\pm 4x^3 \pm 4x^2} \\
2x^2 - 4x + 3 & \qquad\quad 12x^2 + lx + m \\
 & \qquad\quad \underline{\pm 12x^2 \pm 24x \pm 36} \\
 & \qquad (lx - 24x) + (m - 36)
\end{array}
$$

In case of perfect square, remainder is zero

$$lx - 24x = 0$$
$$x(l - 24) = 0$$
$$l - 24 = 0 \qquad x = 0$$
$$l = 24$$

and $m - 36 = 0$
$$m = 36$$

4(ii) $49x^4 - 70x^3 + 109x^2 - lx - m$

Solution: Taking square root

$$\begin{array}{r|l}
 & \quad x^2 + 2x + 6 \\
\hline
7x^2 & 49x^4 - 70x^3 + 109x^2 - lx - m \\
 & \pm 49x^4 \\
\hline
14x^2 - 5x & \quad -70x^3 + 109x^2 \\
 & \quad -70x^3 \pm 25x^2 \\
\hline
14x^2 - 10x + 6 & \qquad\quad 84x^2 + lx - m \\
 & \qquad\quad \pm 84x^2 - 60x \pm 36 \\
\hline
 & \quad (lx - 60x) + (-m - 36)
\end{array}$$

In case of perfect square, remainder is zero

$$lx + 60x = 0$$
$$x(l + 60) = 0$$
$$l + 60 = 0 \qquad x = 0$$
$$l = -60$$

and $-m - 36 = 0$
$$-m = 36$$
$$m = -36$$

Q.5 ***To make the expression*** $9x^4 - 12x^3 + 22x^2 - 13x + 12$, ***a perfect square***

(i) ***What should be added to it?***

(ii) ***What should be subtracted from it?***

(iii) ***What should be the value of x?***

Solution: Taking square root

$$3x^2 - 2x + 3$$

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$$
\begin{array}{r|l}
3x^2 & 9x^4 - 12x^3 + 22x^2 - 13x + 12 \\
 & \underline{\pm 9x^4} \\
14x^2 - 5x & \quad -12x^3 + 22x^2 \\
 & \quad \underline{\pm 12x^3 \pm 4x^2} \\
14x^2 - 10x + 6 & \qquad 18x^2 - 13x + 12 \\
 & \qquad \underline{\pm 18x^2 - 12x \pm 9} \\
 & \qquad\qquad -x + 3 \ (R)
\end{array}
$$

(i) What should be subtracted; remainder should be subtracted. i.e. $-x + 3$

(ii) What should be added, Negative of remainder should be added. i.e. $x - 3$

(iii) To find x, put remainder equal to zero.

$x - 3 = 0$

$x = 3$

Unit 7 LINEAR EQUATIONS AND INEQUALITIES

Linear Equations

A linear equation is one unknown variable x is an equation of the form $ax + b = 0$, where $a, b \in R$ and $a \neq 0$.

Radical Equations

When the variable in an equation occurs under a radical the equation is called a radical equation.

Exercise 7.1

Q.1 Solve the following equations.

(i) $\frac{2}{3}x - \frac{1}{2}x = x + \frac{1}{6}$ ***(ii)*** $\frac{x-3}{3} - \frac{x-2}{2} = -1$

(iii) $\frac{1}{2}\left(x - \frac{1}{6}\right) + \frac{2}{3} = \frac{5}{6} + \frac{1}{3}\left(\frac{1}{2} - 3x\right)$

(iv) $x + \frac{1}{3} = 2\left(x - \frac{2}{3}\right) - 6x$

(v) $\frac{5(x-3)}{6} - x = 1 - \frac{x}{9}$

(vi) $\frac{x}{3x-6} = 2 - \frac{2x}{x-2}, x \neq 2$

(vii) $\frac{2x}{2x+5} = \frac{2}{3} - \frac{5}{4x+10}, x \neq -\frac{5}{2}$

(viii) $\frac{2x}{x-1} + \frac{1}{3} = \frac{5}{6} + \frac{2}{x-1}, x \neq 1$

(ix) $\frac{2}{x^2-1} - \frac{1}{x+1} = \frac{1}{x+1}, x \neq \pm 1$

(x) $\dfrac{2}{3x+6} = \dfrac{1}{6} - \dfrac{1}{2x+4}, x \neq -2$

1(i) $\dfrac{2}{3}x - \dfrac{1}{2}x = x + \dfrac{1}{6}$

L.C.M. of 2, 3, 6 is 6 multiplying by 6

Solution: $(6)\dfrac{2}{3}x - (6)\dfrac{1}{2}x = 6x + 6 \times \dfrac{1}{6}$

$$4x - 3x = 6x + 1$$
$$x = 6x + 1$$
$$x - 6x = 1$$
$$-5x = 1$$
$$x = -\frac{1}{5}$$

Solve $x = \left\{-\dfrac{1}{5}\right\}$

1(ii) $\dfrac{x-3}{3} - \dfrac{x-2}{2} = -1$

Solution: $\left(\dfrac{x-3}{3}\right)\times 6 - \left(\dfrac{x-2}{2}\right)\times 6 = (-1)\cdot(6)$

L.C.M. of 2, 3, 6 multiplying by 6

$$(x-3)\cdot 2 - (x-2)\cdot 3 = -6$$
$$2x - 6 - 3x + 6 = -6$$
$$2x - 3x = -6$$
$$-x = -6$$
$$x = 6$$

Solve $x = \{6\}$

1(iii) $\dfrac{1}{2}\left(x - \dfrac{1}{6}\right) + \dfrac{2}{3} = \dfrac{5}{6} + \dfrac{1}{3}\left(\dfrac{1}{2} - 3x\right)$

Solution: $\dfrac{x}{2} - \dfrac{1}{12} + \dfrac{2}{3} = \dfrac{5}{6} + \dfrac{1}{6} - x$

L.C.M. of 2, 12, 3, 6 is 12 multiplying by 12

$$12\left(\frac{x}{2}\right)-12\left(\frac{1}{12}\right)+12\left(\frac{2}{3}\right) = 12\left(\frac{5}{6}\right)+12\left(\frac{1}{6}\right)-12(x)$$

$$6x - 1 + 8 = 10 + 2 - 12x$$

$$6x + 12x = 10 + 2 + 1 - 8$$

$$18x = 5$$

$$x = \frac{5}{18}$$

Solve $\quad x = \left\{\frac{5}{18}\right\}$

I(iv) $\quad x+\frac{1}{3}=2\left(x-\frac{2}{3}\right)-6x$

Solution: $\quad x+\frac{1}{3} = 2x-\frac{4}{3}-6x$

Multiplying by 3

$$3x+3\left(\frac{1}{3}\right) = 3(2x)-3\left(\frac{4}{3}\right)-3(6x)$$

$$3x + 1 = 6x - 4 - 18x$$

$$3x - 6x + 18x = -4 - 1$$

$$15x = -5$$

$$x = -\frac{5}{15}$$

$$x = -\frac{1}{3}$$

Solve set $\quad x = \left\{-\frac{1}{3}\right\}$

I(v) $\quad \frac{5(x-3)}{6}-x=1-\frac{x}{9}$

Solution: L.C.M. of 6, 9, is 18 multiplying by 18

$$\frac{18(5)(x-3)}{6} - 18x = 18 - 18\left(\frac{x}{9}\right)$$

$$15(x-3) - 18x = 18 - 2x$$

$$15x - 45 - 18x = 18 - 2x$$

$$15x - 18x + 2x = 18 + 45$$

$$-x = 63$$

$$x = -63$$

Solve set $x = \{-63\}$

I(vi) $\frac{x}{3x-6} = 2 - \frac{2x}{x-2}, x \neq 2$

Solution: $\frac{x}{3x-6} + \frac{2x}{x-2} = 2$

$$\frac{x+3(2x)}{3(x-2)} = 2$$

$$\frac{7x}{3(x-2)} = 2$$

Multiplying by $2(x-2)$

$$7x = 2(3)(x-2)$$

$$7x = 6(x-2)$$

$$7x = 6x - 12$$

$$7x - 6x = -12$$

$$x = -12$$

Solve Set $x = \{-12\}$

I(vii) $\frac{2x}{2x+5} = \frac{2}{3} - \frac{5}{4x+10}, x \neq -\frac{5}{2}$

Solution: $\frac{2x}{2x+5} + \frac{5}{4x+10} = \frac{2}{3}$

$$\frac{2x}{2x+5} + \frac{5}{2(2x+5)} = \frac{2}{3}$$

$$\frac{2(2x)+5}{2(2x+5)} = \frac{2}{3}$$

$$\frac{4x+10}{2(2x+5)} = \frac{2}{3}$$

Multiplying by $2 \times 3(2x + 5)$

$$2 \times 3(2x+5)\left(\frac{4x+10}{2(2x+5)}\right) = \frac{2}{3} \times 2 \times 3 \times (2x+5)$$

$$3(4x+5) = 4(2x+5)$$

$$12x + 15 = 8x + 20$$

$$12x - 8x = 20 - 15$$

$$4x = 5$$

$$x = \frac{5}{4}$$

Solve Set $x = \left\{\frac{5}{4}\right\}$

1(viii) $\frac{2x}{x-1} + \frac{1}{3} = \frac{5}{6} + \frac{2}{x-1}, x \neq 1$

Solution: $\frac{2x}{x-1} - \frac{2}{x-1} = \frac{5}{6} - \frac{1}{3}$

$$\frac{2x-2}{x-1} = \frac{5-2}{6}$$

$$\frac{2(x-1)}{x-1} = \frac{3}{6}$$

$$2 = \frac{3}{6}$$

Solution is not possible.

1(ix) $\dfrac{2}{x^2-1}-\dfrac{1}{x+1}=\dfrac{1}{x+1}, x \neq \pm 1$

Solution: $\dfrac{2}{(x+1)(x-1)}-\dfrac{1}{x+1}=\dfrac{1}{x+1}$

Multiplying by $(x+1)(x-1)$

$$\frac{2(x+1)(x-1)}{(x+1)(x-1)}-(x+1)(x-1)\times\frac{1}{x+1}=(x+1)(x-1)\times\frac{1}{x+1}$$

$$2-(x-1)=x-1$$
$$2-x+1=x-1$$
$$-x-x=-1-2-1$$
$$-2x=-4$$
$$x=\frac{-4}{-2}$$
$$x=2$$

Solve Set $x=\{2\}$

1(x) $\dfrac{2}{3x+6}=\dfrac{1}{6}-\dfrac{1}{2x+4}, x \neq -2$

Solution: $\dfrac{2}{3(x+2)}=\dfrac{1}{6}-\dfrac{1}{2(x+2)}$

$$\frac{2}{3(x+2)}+\frac{1}{2(x+2)}=\frac{1}{6}$$
$$\frac{2\times2+1\times3}{6(x+2)}=\frac{1}{6}$$
$$\frac{4+3}{6(x+2)}=\frac{1}{6}$$

Multiplying by $6(x+2)$

$$7=x+2$$
$$x=7-2$$
$$x=5$$

Q.2 Solve each equation and check for extraneous solution, if any:

(i) $\sqrt{3x+4}=2$ *(ii)* $\sqrt[3]{2x-4}-2=0$

(iii) $\sqrt{x-3}-7=0$ *(iv)* $2\sqrt{t+4}=5$

(v) $\sqrt[3]{2x+3}=\sqrt[3]{x-2}$ *(vi)* $\sqrt[3]{2-t}=\sqrt[3]{2t-28}$

(vii) $\sqrt{2t+6}-\sqrt{2t-5}=0$ *(viii)* $\sqrt{\dfrac{x+1}{2x+5}}=2, x\neq\dfrac{-5}{2}$

2(i) $\sqrt{3x+4}=2$

Solution:

$$\left(\sqrt{3x+4}\right)^2=(2)^2 \quad \text{(Squaring)}$$
$$3x+4=4$$
$$3x=4-4$$
$$3x=0$$
$$x=0$$

2(ii) $\sqrt[3]{2x-4}-2=0$

Solution:

$$\sqrt[3]{2x-4}=2$$
$$(2x-4)^{1/3}=2$$
$$[(2x-4)^{1/3}]^3=(2)^3 \quad \text{(Cubing)}$$
$$(2x-4)^{1/3\cdot 3}=8$$
$$2x-4=8$$
$$2x=8+4$$
$$2x=12$$
$$x=\frac{12}{2}$$
$$x=6$$

2(iii) $\sqrt{x-3} - 7 = 0$

Solution:

$$\sqrt{x-3} = 7$$
$$(x-3)^{1/2} = 7$$
$$[(x-3)^{1/2}]^2 = (7)^2 \quad \text{(Squaring)}$$
$$(x-3)^{1/2\times 2} = 49$$
$$x - 3 = 49$$
$$x = 49 + 3$$
$$x = 52$$

2(iv) $2\sqrt{t+4} = 5$

Solution:

$$2\sqrt{t+4} = 5$$
$$2(t+4)^{1/2} = 5$$
$$4(t+4)^{1/2\times 2} = (5)^2 \quad \text{(Squaring)}$$
$$4(t+4) = 25$$
$$4t + 16 = 25$$
$$4t = 25 - 16$$
$$4t = 9$$
$$t = \frac{9}{4}$$

2(v) $\sqrt[3]{2x+3} = \sqrt[3]{x-2}$

Solution:

$$\sqrt[3]{2x+3} = \sqrt[3]{x-2}$$

or

$$(2x+3)^{1/3} = (x-2)^{1/3}$$
$$[(2x+3)^{1/3}]^3 = [(x-2)^{1/3}]^3 \quad \text{(Cubing)}$$
$$(2x+3)^{1/3\times 3} = (x-2)^{1/3\times 3}$$
$$(2x+3) = x - 2$$
$$2x - x = -2 - 3$$
$$x = -5$$

Solve Set $x = \{-5\}$

2(vi) $\sqrt[3]{2-t} = \sqrt[3]{2t-28}$

Solution:

$$\sqrt[3]{2-t} = \sqrt[3]{2t-28}$$

or

$$(2-t)^{1/3} = (2t-28)^{1/3}$$

$$[(2-t)^{1/3}]^3 = [(2t-28)^{1/3}]^3 \quad \text{(Cubing)}$$

$$(2-t)^{1/3 \cdot 3} = (2t-28)^{1/3 \cdot 3}$$

$$2-t = 2t-28$$

$$-t-2t = -28-2$$

$$-3t = -30$$

$$t = \frac{-30}{-3}$$

$$t = 10$$

2(vii) $\sqrt{2t-6} - \sqrt{2t-5} = 0$

Solution:

$$\sqrt{2t-6} = \sqrt{2t-5}$$

or

$$(2t-6)^{1/2} = (2t-5)^{1/2}$$

$$[(2t-6)^{1/2}]^2 = [(2t-5)^{1/2}]^2$$

$$2t-6 = 2t-5$$

$$2t-2t = -5+6$$

$$0 = 1$$

Sol. set is not possible

Sol. set = Ø

2(viii) $\sqrt{\frac{x+1}{2x+5}} = 2$

Solution:

$$\sqrt{\frac{x+1}{2x+5}} = 2$$

or

$$\left(\frac{x+1}{2x+5}\right)^{\frac{1}{2}} = 2$$

$$\left[\left(\frac{x+1}{2x+5}\right)^{\frac{1}{2}}\right]^2 = (2)^2 \qquad \text{(Squaring)}$$

$$\left(\frac{x+1}{2x+5}\right) = 4$$

Multiplying 2x + 5

$$\left(\frac{x+1}{2x+5} \times 2x+5\right) = 4 \times (2x+5)$$

$$x + 1 = 4(2x + 5)$$

$$x + 1 = 8x + 20$$

$$x - 8x = 20 - 1$$

$$-7x = 19$$

$$x = -\frac{19}{7}$$

Equation Involving Absolute Value

Another type of linear equation is the one that contains absolute value. To solve equations involving absolute value we first give the following definition.

Definition

The absolute of a real number 'a' denoted by $|a|$, is defined as

$$|a| = a, \quad \text{if } a \geq 0$$
$$= -a, \quad \text{if } a < 0$$

e.g., $|6| = 6$, $|0| = 0$ and $|-6| = 6$

Some properties of absolute value

If $a, b \in R$, then

(i) $|a| \geq 0$ (ii) $|-a| = |a|$

(iii) $|ab| = |a| . |b|$ (iv) $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}, b \neq 0$

Exercise 7.2

Q.1 ***Identify the following statements as True of False.***

(i) $|x| = 0$ has only one solution.

(ii) All absolute value equations have two solution.

(iii) The equation $|x| = 2$ is equivalent to $x = 2$ or $x = -2$

(iv) The equation $|x - 4| = -4$ has no solution.

(v) The equation $|2x - 3| = 5$ is equivalent to $2x - 3 = 5$ or $2x + 3 = 5$.

Answers:

(i)	T	*(ii)*	F	*(iii)*	T	*(iv)*	T	*(v)*	F

Q.2 ***Solve for x:***

(i) $|3x - 5| = 4$ *(ii)* $\frac{1}{2}|3x + 2| - 4 = 11$

(iii) $|2x + 5| = 11$ *(iv)* $|3 + 2x| = |6x - 7|$

(v) $|x + 2| - 3 = 5 - |x + 2|$ *(vi)* $\frac{1}{2}|x + 3| + 21 = 9$

(vii) $\left|\frac{3-5x}{4}\right| - \frac{1}{3} = \frac{2}{3}$ *(viii)* $\left|\frac{x+5}{2-x}\right| = 2$

2(i) $|3x - 5| = 4$

Solution:

$$|3x - 5| = 4$$
$$3x = 4 + 5$$
$$3x = 9$$
$$x = \frac{9}{3}$$
$$x = 3$$

$$-(3x - 5) = 4$$
$$3x - 5 = -4$$
$$3x = -4 + 5$$
$$3x = 1$$
$$x = \frac{1}{3}$$

$$= \left\{3, \frac{1}{3}\right\}$$

2(ii) $\frac{1}{2}|3x+2| - 4 = 11$

Solution:

$$\frac{1}{2}|3x+2| = 11 + 4$$

$$\frac{1}{2}|3x+2| = 15$$

$$|3x+2| = 30 \qquad \text{(Multiplying by 2)}$$

Now

$$3x + 2 = 30$$
$$3x = 30 - 2$$
$$3x = 28$$
$$x = \frac{28}{3}$$

$$-(3x+2) = 30$$
$$3x + 2 = -30$$
$$3x = -30 - 2$$
$$3x = -32$$
$$x = \frac{-32}{3}$$

$$\text{Sol. Set} = \left\{\frac{28}{3}, \frac{-32}{3}\right\}$$

2(iii) $|2x+5| = 11$

Solution:

$$2x + 5 = 11$$
$$2x = 11 - 5$$
$$2x = 6$$
$$x = \frac{6}{2}$$
$$x = 3$$

$$-(2x+5) = 11$$
$$2x + 5 = -11$$
$$2x = -11 - 5$$
$$2x = -16$$
$$x = \frac{-16}{2}$$
$$x = -8$$

$$\text{Sol. Set} = \{3, -8\}$$

2(iv) $|3+2x| = |6x-7|$

Solution:

$$3 + 2x = 6x - 7$$
$$3 + 7 = 6x - 2x$$
$$10 = 4x$$

$$3 + 2x = -(6x-7)$$
$$3 + 2x = -6x + 7$$
$$2x + 6x = 7 - 3$$

$$x = \frac{10}{4} \qquad\qquad 8x = 4$$

$$x = \frac{5}{2} \qquad\qquad x = \frac{4}{8}$$

$$x = \frac{1}{2}$$

$$\text{Sol. Set } \left\{\frac{5}{2}, \frac{1}{2}\right\}$$

2(v) $|x + 2| - 3 = 5 - |x + 2|$

Solution: $|x + 2| + |x + 2| = 5 + 3$

$$2|x + 2| = 8$$

Dividing by 2

$$|x + 2| = 4$$

$$x + 2 = 4 \qquad\qquad -(x + 2) = 4$$

$$x = 4 - 2 \qquad\qquad x + 2 = -4$$

$$x = 2 \qquad\qquad x = -4 - 2$$

$$x = -6$$

$$\text{Sol. Set} = \{2, -6\}$$

2(vi) $\frac{1}{2}|x + 3| + 21 = 9$

Solution: $\frac{1}{2}|x + 3| = 9 - 21$

$$\frac{1}{2}|x + 3| = -12$$

$$|x + 3| = -24 \qquad \text{(Multiplying by 2)}$$

$$\text{Sol. Set} = \phi$$

(Absolute value is never negative)

2(vii) $\left|\frac{3 - 5x}{4}\right| - \frac{1}{3} = \frac{2}{3}$

Solution: $\left|\frac{3 - 5x}{4}\right| - \frac{1}{3} = \frac{2}{3}$

$$\left|\frac{3 - 5x}{4}\right| = \frac{2}{3} + \frac{1}{3}$$

$$\left|\frac{3-5x}{4}\right| = 1$$

Now

$$\frac{3-5x}{4} = 1$$

$$3-5x = 4$$
$$-5x = 4-3$$
$$-5x = 1$$
$$x = -\frac{1}{5}$$

and

$$\left(\frac{3-5x}{4}\right) = -1$$

$$\frac{3-5x}{4} = -1$$
$$3-5x = -4$$
$$-5x = -4-3$$
$$-5x = -7$$
$$x = \frac{7}{5}$$

$$\text{Sol. Set} = \left\{-\frac{1}{5}, \frac{7}{5}\right\}$$

2(viii) $\left|\dfrac{x+5}{2-x}\right| = 6$

Solution:

$$\frac{x+5}{2-x} = 6$$
$$x+5 = 6(2-x)$$
$$x+5 = 12-6x$$
$$x+6x = 12-5$$
$$7x = 7$$
$$x = 1$$

$$\left(\frac{x+5}{2-x}\right) = -6$$
$$\frac{x+5}{2-x} = -6$$
$$x+5 = -6(2-x)$$
$$x+5 = -12+6x$$
$$x-6x = -12-5$$
$$-5x = -17$$
$$x = \frac{17}{5}$$

$$\text{Sol. Set} = \left\{1, \frac{17}{5}\right\}$$

Defining Inequalities

Let a, b be real numbers. Then a is greater than b if the difference $a-b$ is positive and we denote this order relation by the inequality $a > b$. An equivalent statement is that b is less

than a, symbolized by $b > a$. Similarly, if $a - b$ is negative, then a is less than b and expressed in symbols as $a < b$.

Properties of Inequalities

1. Law of Trichotomy

For any $a, b \in R$, one and only one of the following statements is true.

$$a < b \quad \text{or} \quad a = b, \quad \text{or} \quad a > b$$

An important special case of this property is the case for $b = 0$; namely,

$$a < 0 \text{ or } a = 0 \text{ or } a > 0 \text{ for any } a \in R.$$

2. Transitive Property

Let $a, b, c \in R$.

(i) If $a > b$ and $b > c$, then $a > c$

(ii) If $a < b$ and $b < c$, then $a < c$

3. Additive Closure Property

For $a, b, c \in R$.

(i) If $a > b$, then $a + c > b + c$
If $a < b$, then $a + c < b + c$

(ii) If $a > 0$ and $b > 0$, then $a + b > 0$
If $a < 0$ and $b < 0$, then $a + b < 0$

4. Multiplicative Property

Let $a, b, c, d \in R$.

(i) If $a > 0$ and $b > 0$, then $ab > 0$,
whereas $a < 0$ and $b < 0 \Rightarrow ab > 0$

(ii) If $a > b$ and $c > 0$, then $ac > bc$
or if $a < b$ and $c > 0$, then $ac < bc$

(iii) If $a > b$ and $c < 0$, then $ac < bc$
or if $a < b$ and $c < 0$, then $ac > bc$

The above property (iii) states that the sign of inequality is reversed if each side is multiplied by a negative real number.

(iv) If $a > b$ and $c > d$, then $ac > bd$.

Exercise 7.3

Q.1 ***Solve the following inequalities.***

(i) $3x + 1 < 5x - 4$ *(ii)* $4x - 10.3 \le 21x - 1.8$

(iii) $4 - \frac{1}{2}x \ge -7 + \frac{1}{4}x$ *(iv)* $x - 2(5 - 2x) \ge 6x - 3\frac{1}{2}$

(v) $\frac{3x+2}{9} - \frac{2x+1}{3} > -1$

(vi) $3(2x + 1) - 2(2x + 5) < 5(3x - 2)$

(vii) $3(x - 1) - (x - 2) > -2(x + 4)$

(viii) $2\frac{2}{3}x + \frac{2}{3}(5x - 4) > -\frac{1}{3}(8x + 7)$

1(i) $3x + 1 > 5x - 4$

Solution:

$$3x + 1 < 5x - 4$$
$$3x + 1 - 5x < 5x - 4 - 5x$$
$$-2x + 1 < -4$$
$$-2x + 1 - 1 < -4 - 1$$
$$-2x < -5 \quad \text{Dividing by } -2$$
$$x > \frac{5}{2} \quad \therefore \text{ L.S change of sign}$$

Sol. Set = $\{x | x > \frac{5}{2}\}$

1(ii) $4x - 10.3 \le 21x - 1.8$

Solution: (Subtracting 21x)

$$4x - 10.3 - 21x \le 21x - 1.8 - 21x$$
$$-17x - 10.3 \le -1.8$$
$$-17x - 10.3 + 10.3 \le -1.8 + 10.3 \quad \text{(Adding 10.3)}$$
$$-17x \le 8.5$$

N.T.S Dividing by -17 (change of sign)

$$x \geq \frac{8.5}{-17}$$

$$x \geq -.5$$

$$\text{Sol. Set} = \{x|x \geq -.5\}$$

I(iii) $4 - \frac{1}{2}x \geq -7 + \frac{1}{4}x$

Solution: $4 - \frac{1}{2}x \geq -7 + \frac{1}{4}x$

Multiplying by 4

$$16 - 2x \geq -28 + x$$

Subtracting by x

$$16 - 2x - x \geq -28 + x - x$$

$$16 - 3x \geq -28$$

Subtracting by 16

$$16 - 3x - 16 \geq -28 - 16$$

$$-3x \geq -44$$

Subtracting by -3

$$x \leq \frac{-44}{-3}$$ N.T.S (change of sign)

$$x \leq \frac{44}{3}$$

$$\text{Sol. Set} = \{x|x < \frac{44}{3}\}$$

I(iv) $x - 2(5 - 2x) \geq 6x - 3\frac{1}{2}$

Solution: $x - 2(5 - 2x) \geq 6x - 3\frac{1}{2}$

$$x - 10 + 4x \geq 6x - \frac{7}{2}$$

Multiplying by 2

$$2x - 20 + 8x \geq 12x - 7$$

$$10x - 20 \geq 12x - 7$$

Subtracting 12x

$$10x - 20 - 12x \geq 12x - 7 - 12x$$

$$-2x - 20 \geq -7$$

Adding 20

$$-2x - 20 + 20 \geq -7 + 20$$

$$-2x \geq 13$$

Dividing by –2

$$x \leq -\frac{13}{2} \qquad \text{(change of sign)}$$

$$x \leq -6.5$$

$$\text{Sol. Set} = \{x | x \leq 6.5\}$$

i(v) $\frac{3x+2}{9} - \frac{2x+1}{3} > -1$

Solution: Multiplying by 9

$$\frac{9(3x+2)}{9} - \frac{9(2x+1)}{3} > -1(9)$$

$$3x + 2 - 3(2x + 1) > -9$$

$$3x + 2 - 6x - 3 > -9$$

$$-3x - 1 > -9$$

$$-3x - 1 + 1 > -9 + 1 \text{ (Adding 1)}$$

$$-3x > -8$$

Dividing by –3 (change of sign)

$$x < \frac{-8}{-3}$$

$$x < \frac{8}{3}$$

1(vi) $3(2x + 1) - 2(2x + 5) < 5(3x - 2)$

Solution:

$$6x + 3 - 4x - 10 < 15x - 10$$
$$2x - 7 < 15x - 10$$

Subtracting 15x

$$2x - 7 - 15x < 15x - 10 - 15x$$
$$-13x - 7 < -10$$
$$-13x - 7 + 7 < -10 + 7 \quad \text{(Adding 7)}$$
$$-13x < -3$$

Dividing by -13 (change of sign)

$$x > \frac{-3}{-13}$$
$$x > \frac{3}{13}$$

$$\text{Sol. Set } = \{x | x > \frac{3}{13}\}$$

1(vii) $3(x - 1) - (x - 2) > -2(x + 4)$

Solution:

$$3x - 3 - x + 2 > -2x - 8$$
$$2x - 1 > -2x - 8$$
$$2x + 2x - 1 > -2x + 2x - 8 \quad \text{(Adding 2x)}$$
$$4x - 1 + 1 > -8 + 1 \quad \text{(Adding 1)}$$
$$4x > -7$$
$$x > \frac{-7}{4} \quad \text{(Dividing by 4)}$$

1(viii) $2\frac{2}{3}x + \frac{2}{3}(5x - 4) > -\frac{1}{3}(8x + 7)$

Solution:

$$\frac{8x}{3} + \frac{2}{3}(5x - 4) > -\frac{1}{3}(8x + 7)$$
$$8x + 2(5x - 4) > -(8x + 7) \quad \text{(Multiplying by 3)}$$
$$8x + 10x - 8 > -8x - 7$$
$$18x - 8 > -8x - 7$$

Adding by 8x

$$18x - 8 + 8x > -8x - 7 + 8x$$
$$26x - 8 > -7 \quad \text{(Adding by 8)}$$
$$26x - 8 + 8 > -7 + 8$$
$$26x > 1$$
$$x > \frac{1}{26} \quad \text{(Dividing by 26)}$$
$$\text{Sol. Set} = \{x|x > \frac{1}{26}\}$$

Q.2 Solve the following inequalities.

(i) $-4 < 3x + 5 < 8$ *(ii)* $-5 \leq \frac{4-3x}{2} < 1$

(iii) $-6 < \frac{x-2}{4} < 6$ *(iv)* $3 \geq \frac{7-x}{2} \geq 1$

(v) $3x - 10 \leq 5 < x + 3$ *(vi)* $-3 \leq \frac{x-4}{-5} < 4$

(vii) $1 - 2x < 5 - x \leq 25 - 6x$

(viii) $3x - 2 < 2x + 1 < 4x + 17$

2(i) $-4 < 3x + 5 < 8$

Solution:
$$-4 < 3x + 5 < 8$$
$$-4 - 5 < 3x + 5 - 5 < 8 - 5 \quad \text{(Subtracting 5)}$$
$$-9 < 3x < 3$$
$$-3 < x < 1 \quad \text{(Dividing by 3)}$$

2(ii) $-5 \leq \frac{4-3x}{2} < 1$

Solution:
$$-10 \leq 4 - 3x < 2 \quad \text{(Multiplying by 2)}$$
$$-10 - 4 \leq 4 - 3x - 4 < 2 - 4 \quad \text{(Subtracting 4)}$$
$$-14 \leq -3x < -2$$

Dividing by -3 (change of sign)

$$\frac{-14}{-3} \geq \frac{-3x}{-3} > \frac{-2}{-3}$$
$$\frac{14}{3} \geq x > \frac{2}{3}$$
$$\frac{2}{3} > x \leq \frac{14}{3}$$

2(iii) $-6 < \dfrac{x-2}{4} < 6$

Solution: $-24 < x - 2 < 24$ (Multiplying by 4)

$-24 + 2 < x - 2 + 2 < 24 + 2$ (Adding 2)

$-22 < x < 26$

2(iv) $3 \geq \dfrac{7-x}{2} \geq 1$

Solution: $6 \geq 7 - x \geq 2$ (Multiplying by 2)

$6 - 7 \geq 7 - x - 7 \geq 2 - 7$ (Subtracting 7)

$-1 \geq -x \geq -5$

Multiplying by -1 (change of sign)

$1 \leq x \leq 5$

2(v) $3x - 10 \leq 5 < x + 3$

Solution: Here

$3x - 10 < 5$	$5 < x + 3$
$3x - 10 + 10 \leq 5 + 10$	or $x + 3 > 5$
$3x \leq 15$	$x + 3 - 3 > 5 - 3$
$\dfrac{3x}{3} \leq \dfrac{15}{3}$	$x > 2$ (ii)
$x \leq 5$ (i)	or $2 < x$

From (i) and (ii)

$2 < x \leq 15$

2(vi) $-3 \leq \dfrac{x-4}{-5} < 4$

Solution: Multiplying by -5 (Change of sign)

$15 \geq x - 4 > -20$

$15 + 4 \geq x - 4 + 4 > -20 + 4$ (Adding + 4)

$19 \geq x > -16$

or $-16 < x \leq 19$

2(vii) $1 - 2x < 5 - x \leq 25 - 6x$

Solution:

$$1 - 2x < 5 - x$$
$$1 - 2x + x < 5 - x + x$$
$$1 - x < 5$$
$$-x < 5 - 1$$
$$-x < 4$$
or $x > -4 \quad \text{(i)}$

$$5 - x \leq 25 - 6x$$
$$5 - x + 6x \leq 25 - 6x + 6x$$
$$5 + 5x \leq 25$$
$$5 + 5x - 5 \leq 25 - 5$$
$$5x \leq 20$$
$$\frac{5x}{5} \leq \frac{20}{5}$$
$$x \leq 4 \quad \text{(ii)}$$

From (i) and (ii)

$$4 < x \leq 4$$

2(viii) $3x - 2 < 2x + 1 < 4x + 17$

Solution: Here

$$3x - 2 < 2x + 1$$
$$3x - 2 - 1 < 2x + 1 - 1$$
$$3x - 3 < 2x$$
$$3x - 2x - 3 < 2x - 2x$$
$$x - 3 < 0$$
$$x - 3 + 3 < 0 + 3$$
$$x < 3 \quad \text{(i)}$$

$$2x + 1 < 4x + 17$$
$$2x + 1 - 1 < 4x + 17 - 1$$
$$2x < 4x + 16$$
$$2x - 4x < 4x + 16 - 4x$$
$$-2x < 16$$

Dividing by 2 (Change of sign)

$$x > \frac{16}{-2}$$
$$x > -8 \quad \text{(ii)}$$

From (i) and (ii)

$$-8 < x < 3$$

Review Exercise 7

Q.1 Multiple choice question, choose the correct answers.

(i) Which of the following is the solution of the inequality $3 - 4x - 11$? ___

(a) 8 (b) 2

(c) $\frac{14}{4}$ (d) None of these

(ii) A statement involving any of the symbols <, >, ≤ or ≥ is called ___

(a) equation (b) identity

(c) inequality (d) linear equation

(iii) x ___ is a solution of the inequality $2 < x < \frac{3}{2}$.

(a) 5 (b) 3

(c) 0 (d) $\frac{3}{2}$

(iv) If x is no larger than 10, then

(a) $x \geq 8$ (b) $x \cdot 10$

(c) $x \cdot 10$ (d) $x \quad 10$

(v) If the capacity c of an elevator is at most 1600 pounds, then ___

(a) $c < 1600$ (b) $c > 1600$

(c) $c \leq 1600$ (d) $c \geq 1600$

(vi) $x \quad 0$ is a solution of the inequality

(a) $x \cdot 0$ (b) $3x - 5 \cdot 0$

(c) $x + 2 < 0$ (d) $x \quad 2 \cdot 0$

Answers:

(i)	b	*(ii)*	c	*(iii)*	c	*(iv)*	b
(v)	c	*(vi)*	d				

Q.2 ***Identify the following statements as True or False.***

(i) The equation $3x - 5 = 7 - x$ is a linear equation.

(ii) The equation $x - 0.3x = 0.7x$ is an identity.

(iii) The equation $-2x + 3 = 8$ is equivalent to $-2x = 11$.

(iv) To eliminate fractions, we multiply each side of an equation by the L.C.M. of denominators.

(v) $4(x + 3) = x + 3$ is a conditional equation.

(vi) The equation $2(3x + 5) = 6x + 12$ is an inconsistent equation.

(vii) To solve $\frac{2}{3}x = 12$, we should multiply each side by $\frac{2}{3}$.

(viii) Equations having exactly the same solution are called equivalent equations.

(ix) A solution that does not satisfy the original equation is called extraneous solution.

Answers:

(i)	True	*(ii)*	True	*(iii)*	False	*(iv)*	True
(v)	True	*(vi)*	True	*(vii)*	False	*(viii)*	True
(ix)	True						

Q.3 **Answer the following short question.**

(i) ***Define a linear inequality in one variable.***

(ii) ***State the trichotomy and transitive properties of inequalities.***

(iii) ***The formula relating degrees Fahrenheit to degrees Celsius is $F = \frac{9}{5}C + 32$. For what value of C is $F < 0$?***

(iv) ***Seven times the sum of an integer and 12 is at least 50 and at most 60. Write and solve the inequality that expresses this relationship.***

Answer:

(i) A linear inequality in one variable x is $ax + b < 0, a \neq 0$

(ii) ***Trichotomy Property:***

$\forall a, b \in R$, one and only one of the following statement is true

$a < b$ or $a = b$ or $a > b$

If $b = 0$ then $a < 0$ or $a = 0$ or $a > 0$ for any $a \in R$.

Transitive Property:

$\forall a, b, c \in R$

(i) If $a > b$ and $b > c$ then $a > c$

(ii) If $a < b$, and $b < c$ then $a < c$

(iii) $F = \frac{9}{5}C + 32 < 0$

$C < -32 \times \frac{5}{9}$

$C < -\frac{160}{9}$

(iv) **Let the integer = x**

According to the conditions

$50 \leq 7(x + 12) \leq 60$

Solution: $50 \leq 7x + 84 \leq 60$

$50 - 84 \leq 7x + 84 - 84 < 60 - 84$ (Subtracting 84)

$-34 \leq 7x \leq -24$

$-\frac{34}{7} \leq \frac{7x}{7} \leq \frac{-24}{7}$

Q.4 Solve each of the following and check for extraneous solution if any

(i) $\sqrt{2t + 4} = \sqrt{t - 1}$

(ii) $\sqrt{3x - 1} - 2\sqrt{8 - 2x} = 0$

4(i) $\sqrt{2t+4} = \sqrt{t-1}$

Solution: $2t + 4 = t - 1$ (Squaring)

$2t - t = -1 - 4$

$t = -5$

Check Put $t = -5$

$\sqrt{2t+4}$ $\quad\quad$ $\sqrt{t-1}$

$\sqrt{2(-5)+4}$ $\quad\quad$ $\sqrt{-5-1}$

$= \sqrt{-10+4}$ $\quad\quad$ $\sqrt{-6}$

$= \sqrt{-6}$

Solution in integers in null set (ϕ)

4(ii) $\sqrt{3x-1} - 2\sqrt{8-2x} = 0$

Solution: $\sqrt{3x-1} - 2\sqrt{8-2x} = 0$

$\sqrt{3x-1} = 2\sqrt{8-2x}$

Squaring

$3x - 1 = 4(8 - 2x)$

$3x - 1 = 32 - 8x$

$3x + 8x = 32 + 1$

$11x = 33$

$x = \frac{33}{11}$

$x = 3$

Check $= \sqrt{3x-1} - 2\sqrt{8-2x}$

Putting $x = 3$

$= \sqrt{3(3)-1} - 2\sqrt{8-2(3)}$

$= \sqrt{9-1} - 2\sqrt{8-6}$

$= \sqrt{8} - 2\sqrt{2}$

$$2\sqrt{2} - 2\sqrt{2} = 0 = \text{R.H.S.}$$

Q.5 ***Solve for x***

(i) $|3x + 14| - 2 = 5x$ *(ii)* $\frac{1}{3}|x - 3| = \frac{1}{2}|x + 2|$

5(i) $|3x + 14| - 2 = 5x$

Solution:

$$|3x + 14| - 2 = 5x$$
$$|3x + 14| = 5x + 2$$

$3x + 14 = 5x + 2$
$3x - 5x = 2 - 14$
$-2x = -12$
$x = \frac{-12}{-2}$
$x = 6$

$-(3x + 14) = 5x + 2$
$-3x - 14 = 5x + 2$
$-3x - 5x = 2 + 14$
$-8x = 16$
$x = \frac{16}{-8}$
$x = -2$

Check: L.H.S.
$= |3x + 14| - 2$
$= |3(6) + 14| - 2$
$= |18 + 14| - 2$
$= |32| - 2$
$= 32 - 2$
$= 30$

R.H.S.
$x = 6$
$= 5(x)$
$= 5 \times 6$
$= 30$

True

Now check

$x = -2$

$|3x + 14| - 2 = 5x + 2$

L.H.S.
$|3(-2) + 14| - 2$
$|-6 + 14|$
$|8|$

R.H.S
$5x + 2$
$= 5(-2) + 2$
$= -10 + 2$
$= -8$

$|8|$ $= -8$ False

Sol. Set = {6}

5(ii) $\frac{1}{3}|x-3| = \frac{1}{2}|x+2|$

$\frac{1}{3}(x-3) = \frac{1}{2}(x+2)$

Divide on 6

$2(x-3) = 3(x+2)$

$2x - 6 = 3x + 6$

$2x - 3x = 6 + 6$

$-x = 12$

$x = -12$

$\frac{1}{3}(x-3) = -\frac{1}{2}(x+2)$

Divide on 6

$2(x-3) = 3(x+2)$

$2x - 6 = -3x - 6$

$2x - 5x = -6 + 6$

$7x = 0$

$x = 0$

Set. {−12, 0}

Q.6 ***Solve the following inequality.***

(i) $-\frac{1}{3}x + 5 \leq 1$ *(ii)* $-3 < \frac{1-2x}{5} < 1$

6(i) $-\frac{1}{3}x + 5 \leq 1$

Solution: $-\frac{1}{3}x + 5 - 5 \leq 1 - 5$

$-\frac{1}{3}x + 5 - 5 \leq 1 - 5$

$-\frac{1}{3}x \leq -4$

Multiplying by −3 (Change of sign)

$$(-3)\left(-\frac{1}{3}x\right) \geq (-3)(-4)$$

$x \geq 12$

6(ii) $-3 < \frac{1-2x}{5} < 1$

Solution: $-15 < 1-2x < 5$ Multiplying by 5

$-15-1 < 1-2x-1 < 5-1$ Subtracting 1

$-16 < -2x < 4$

Dividing by -2 (change of sign)

$\frac{-16}{-2} > \frac{-2x}{-2} > \frac{4}{-2}$

$8 > x > -2$

Unit 8 LINEAR GRAPHS & THEIR APPLICATION

An Ordered Pair of Real Numbers

An ordered pair of real numbers x and y is a pair (x, y) in which elements are written in specific order.i.e.,

(i) (x, y) is an ordered pair in which first element is x and second is y, such that $(x,y) \neq (y, x)$ where.

(ii) (2, 3) and (3, 2) are two different ordered pairs.

(iii) $(x, y) = (m, n)$ only if $x = m$ and $y = n$.

Cartesian Plane

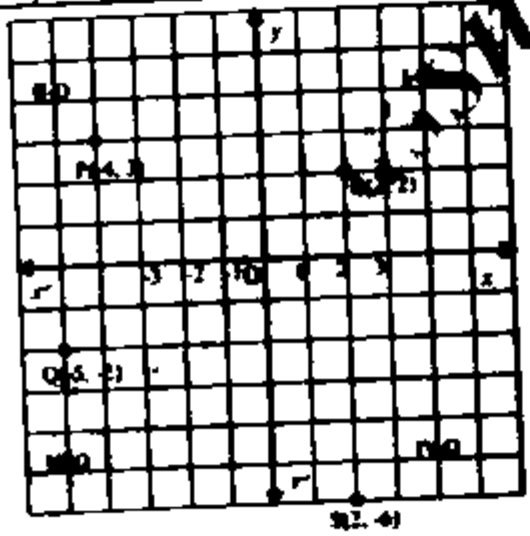
The Cartesian plane establishes one-to-one correspondence between the set of ordered pairs $R \times R = \{(x, y) : x, y \in R\}$ and the points of the Cartesian plane.

In plane two mutually perpendicular straight lines are drawn. The lines are called the coordinate axes. The point O, where the two lines meet is called origin. This plane is called the coordinate plane or the Cartesian plane.

EXERCISE 8.1

Q.1 ***Determine the quadrant of the coordinate plane in which the following points lie: P(–4, 3), Q(–5, –2), R(2, 2) and S(2, –6).***

Ordered Pair	*Quadrant*
S(2, –6)	IV – Q
R(2, 2)	I – Q
Q(–5, –2)	III – Q
P(–4, 3)	II – Q



Q.2 ***Draw the graph of each of the following:***

(i) $x = 2$ *(ii)* $x = -3$ *(iii)* $y = -1$

(iv) $y = 3$ *(v)* $y = 0$ *(vi)* $x = 0$

(vii) $y = 3x$ *(viii)* $-y = 2x$ *(ix)* $\frac{1}{2} = x$

(x) $3y = 5x$ *(xi)* $2x - y = 0$ *(xii)* $2x - y = 2$

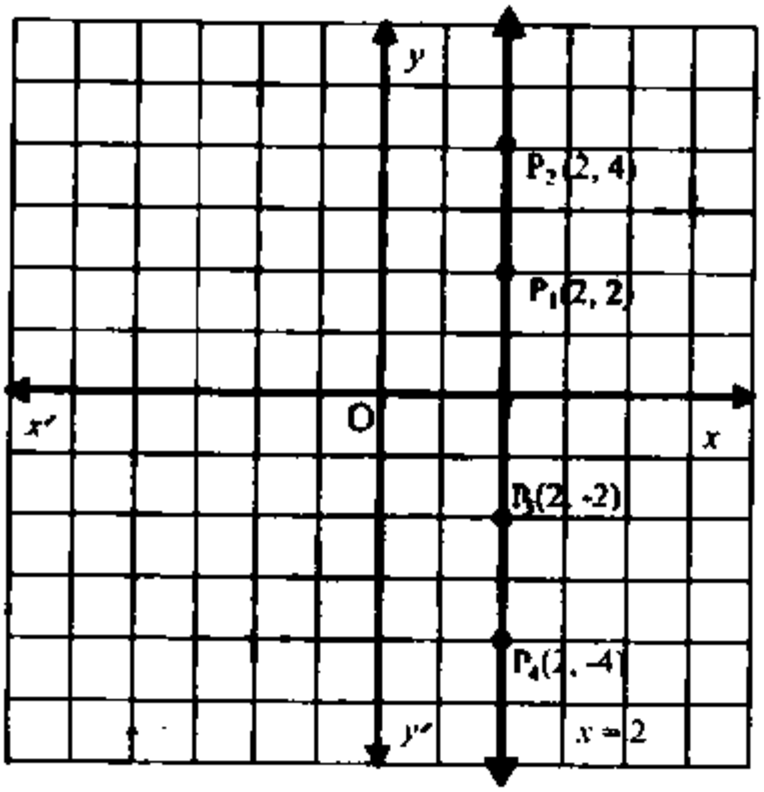
(xiii) $x - 3y + 1 = 0$ *(xiv)* $3x - 2y + 1 = 0$

1(i) $x = 2$

1(i) $x = 2$

Solution: Table ordered pairs that lie on the graph of x = 2.

x =	2	2	2	2
y =	2	4	–2	–4



Graph x = 2

2(ii) $x = -3$

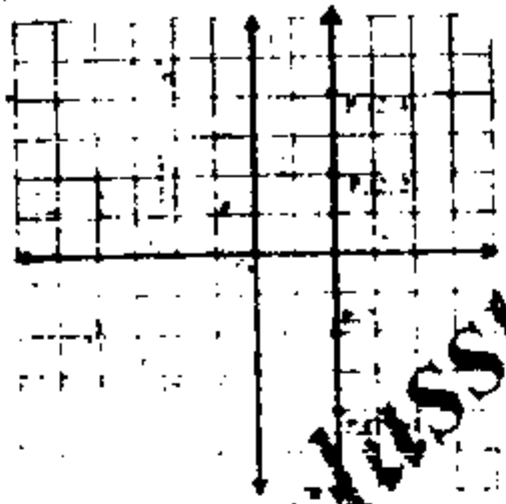
Solution: Table ordered pairs that lie on the graph of x = –3.

x =		–3	–3	–3	–3	–3
y =	...	2	5	–2	–4	...

2(ii) $x = -3$

Solution: Table ordered pairs that lie on the graph of $x = -3$.

x	-3	-3	-3	-3	-3	-3
y	[illegible]	2	5	-2	-4	[illegible]

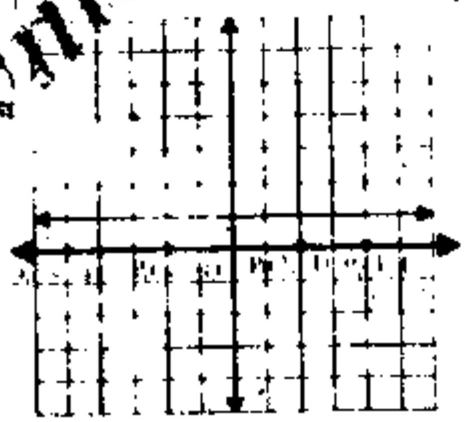


Graph x = -3

2(iii) $y = 1$

Solution: Table ordered pairs that lie on the graph of $y = 1$

x	[illegible]	[illegible]	-4	2	4	6
y	1	1	1	1	1	1

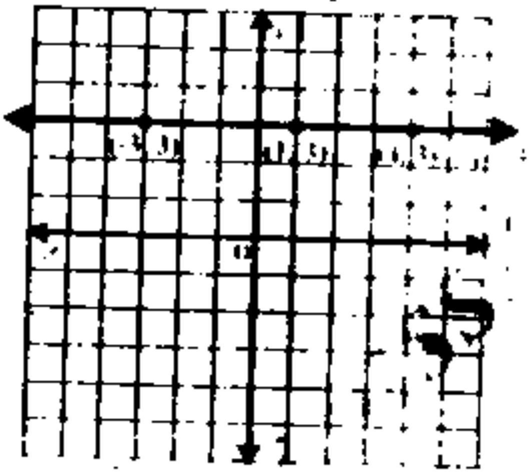


Graph y = 1

2(iv) $y = 3$

Solution: Table ordered pairs that lie on the graph of y = 3

x	...	1	4	5	6
y	3	3	3	3	3

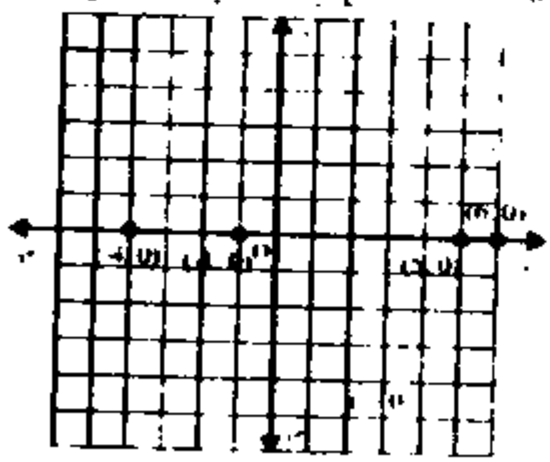


Graph y = 3

2(v) $y = 0$

Solution: Table ordered pairs that lie on the graph of y = 0

x	...	-4	1	5	6	
y	0	0	0	0	0	0



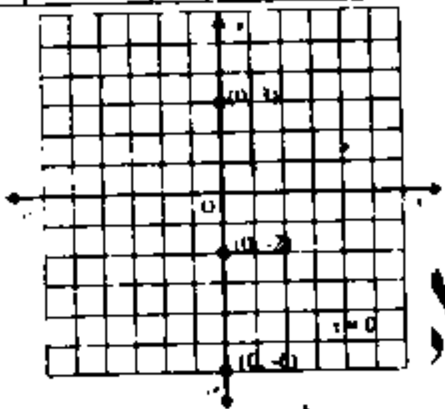
Graph y = 0

This is x-axis

2(vi) $x = 0$

Solution: Table ordered pairs that lie on the graph of $x = 0$.

X =	0	0	0	0	0	0
y =	...	3	5	−2	−6	...



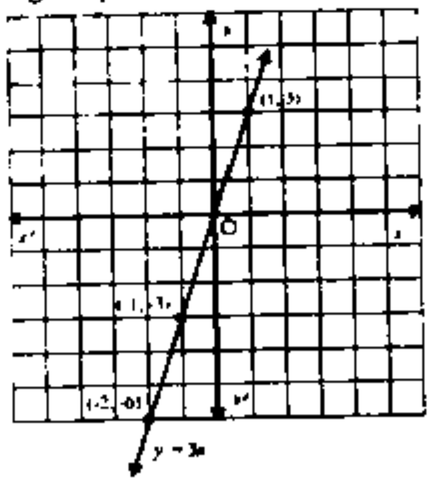
Graph x = 0 **This is y-axis**

2(vii) $y = 3x$

Solution: Table ordered pairs that lie on the graph of $y = 3x$.

x =	1	2	−1	−2
y =	3	6	−3	−6

Plotting the points on the graph paper

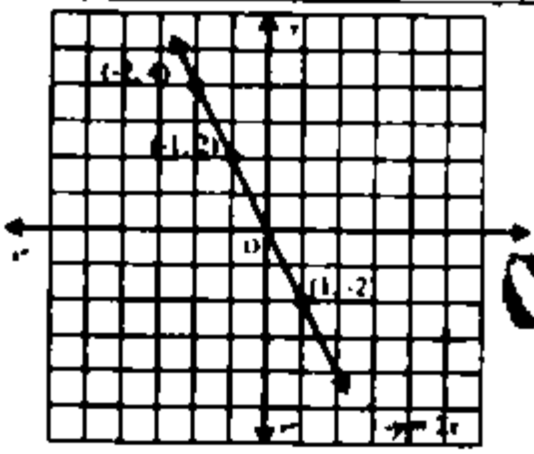


Graph of y = 3x

2(viii) $-y = 2x$

Solution: Table ordered pairs that lie on the graph of $-y = 2x$.

x =	–1	–2	1	4
y =	2	4	–2	–8

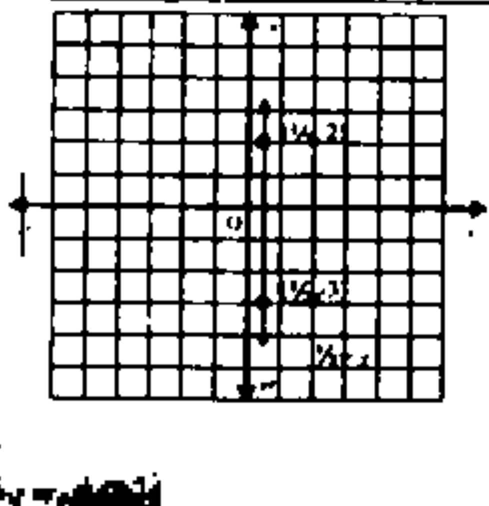


Graph $-y = 2x$

2(ix) $\frac{1}{2} = x$ or $x = \frac{1}{2}$

Solution: Table ordered pairs that lie on the graph of $x = \frac{1}{2}$.

x =	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
y =	....	2	–3	....



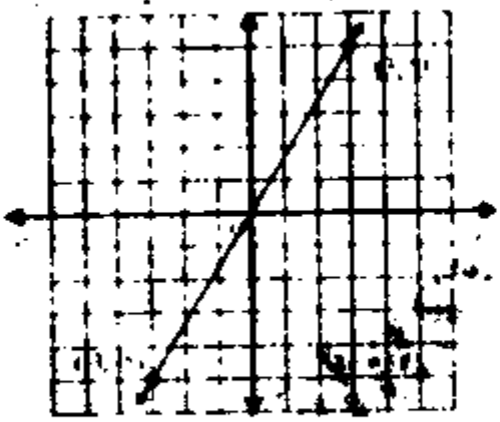
Graph $x = \frac{1}{2}$

2(x) $3y = 5x$

Solution: $y = \frac{5}{3}x$

Table ordered pairs that lie on the graph of $3y = 5x$.

x	3		3
y	5		5



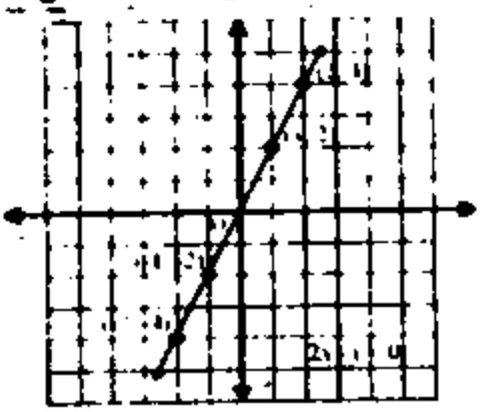
Graph $3y = 5x$

2(xi) $2x - y = 0$

Solution: $y = 2x$

Table ordered pairs that lie on the graph of $2x - y = 0$.

x	1	1	2	2
y	2	2	4	4



Graph $2x - y = 0$

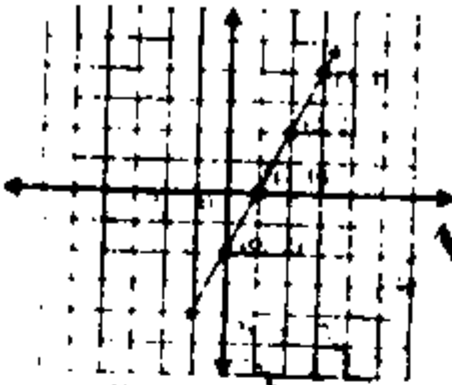
2(xii) $2x - y = 2$

Solution: $y = 2 + 2x$ [illegible]

$y = 2x - 2$

Table ordered pairs that lie on the graph of $2x - y = 2$.

x	[illegible]	2	3	1	2
y	[illegible]	2	4	0	[illegible]



Graph $y = 2x - 2$

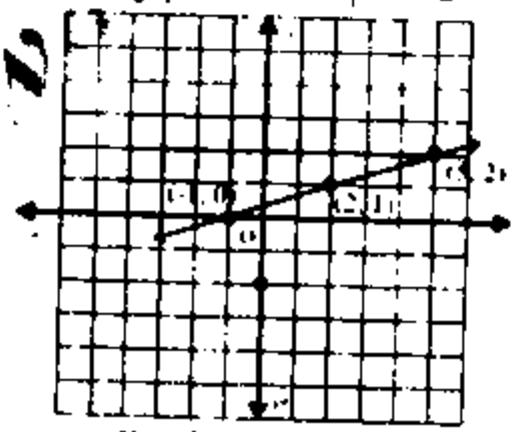
2(xiii) $x - 3y + 1 = 0$

Solution: $3y = 1 + x$

$y = \frac{1+x}{3}$

Table ordered pairs that lie on the graph of $x - 3y + 1 = 0$

x	-1	[illegible]	2	5
y	0	[illegible]	1	2



Graph $x - 3y + 1 = 0$

2(xiv) $3x - 2y + 1 = 0$

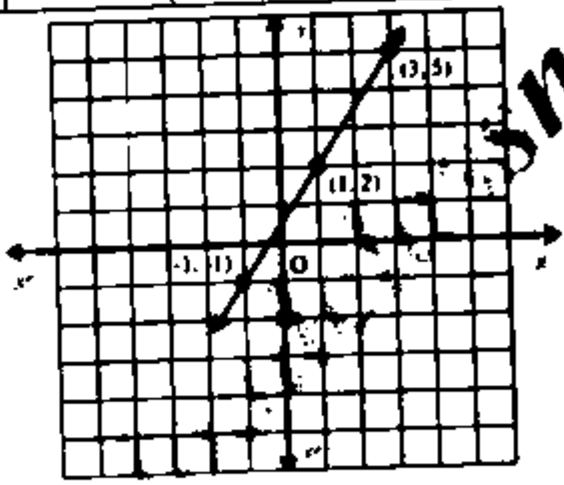
Solution: $-2y = -1 - 3x$

or $y = \dfrac{1+3x}{2}$

Table ordered pairs that lie on the graph of

$3x - 2y + 1 = 0.$

x =	1	−1	3
y =	2	−1	5



Graph $3x - 2y + 1 = 0$

Q.3 ***Are the following lines (i) parallel to x-axis (ii) parallel to y-axis***

(i) $2x - 1 = 3$ *(ii)* $x + 2 = -1$ *(iii)* $2y + 3 = 2$

(iv) $x + y = 0$ *(v)* $2x - 2y = 0$

3(i) $x - 1 = 3$

Solution:

$$2x - 1 = 3$$
$$2x = 3 + 1$$
$$2x = 4$$

$$x = \frac{4}{2} = 2$$

Parallel to y-axis

3(ii) $x + 2 = -1$

Solution:

$$x + 2 = -1$$
$$x = -1 - 2$$
$$x = -3$$

Parallel to y-axis

3(iii) $2y + 3 = 2$

Solution:

$$2y + 3 = 2$$
$$2y = 2 - 3$$
$$2y = -1$$
$$y = -\frac{1}{2}$$

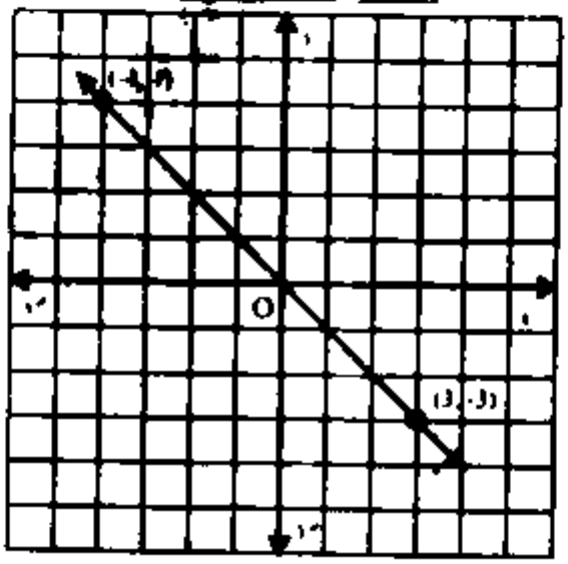
Parallel to x-axis

3(iv) $x + y = 0$

Solution:

$$x + y = 0$$
$$y = -x$$

x =	3	-4
y =	-3	4



$x + y = 0$

It is neither parallel to x-axis, not to y-axis

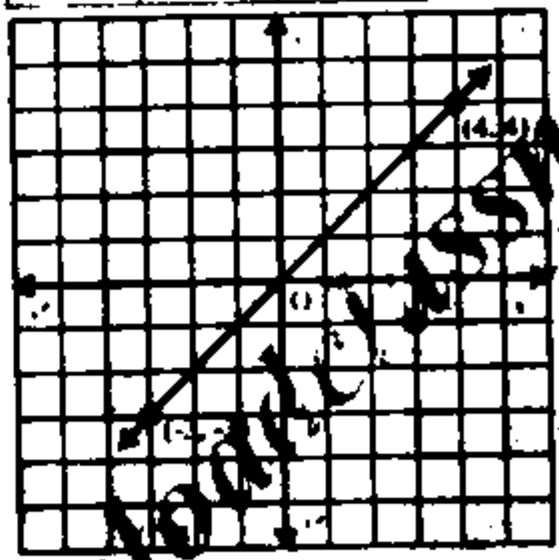
3(v) $2x - 2y = 0$

Solution: $2x - 2y = 0$

$2y = 2x$

$y = x$

x =	4	3
y =	4	−3



It is neither parallel to x-axis, not to y-axis

Q.4 ***Find the value of m and c of the following lines by expressing them in the form y = mx + c***

(i)	$2x + 3y - 1 = 0$	*(ii)*	$x - 2y = -2$
(iii)	$3x + y - 1 = 0$	*(iv)*	$2x - y = 7$
(v)	$3 - 2x + y = 0$	*(vi)*	$2x = y + 3$

4(i) $2x + 3y - 1 = 0$

Solution: $2x + 3y - 1 = 0$

or $3y = -2x + 1$

or $y = -\frac{2}{3}x + \frac{1}{3}$ (i) (Dividing by 3)

$y = mx + c$ (ii) Comparing (i) and (ii)

$m = -\frac{2}{3}$

$c = \frac{1}{3}$

4(ii) $x - 2y = -2$

Solution: $x - 2y = -2$

$-2y = -x - 2$

$\frac{-2}{-2}y = \frac{-x}{-2} - \frac{2}{-2}$ (Dividing by -2)

$y = \frac{1}{2}x + 1$ (Comparing it with)

$y = mx + c$

$m = \frac{1}{2},\ c = 1$

4(iii) $3x + y - 1 = 0$

Solution: $3x + y - 1 = 0$

or $y = -3x + 1$ (i)

$y = mx + c$ (ii) Comparing (i) and (ii)

$m = -3,\ c = 1$

4(iv) $2x - y = 7$

Solution: $2x - y = 7$

or $-y = -2x + 7$

or $y = 2x - 7$ (i)

$y = mx + c$ (ii) Comparing (i) and (ii)

$m = 2,\ c = -7$

4(v) $3 - 2x + y = 0$

Solution: $3 - 2x + y = 0$

or $y = 2x - 3$ (i)

$y = mx + c$ (ii) Comparing (i) and (ii)

$m = 2, \; c = -3$

4(vi) $2x = y + 3$

Solution: $2x = y + 3$

or $y + 3 = 2x$

or $y = 2x - 3$ (i)

$y = mx + c$ (ii) Comparing (i) and (ii)

$m = 2, \; c = -3$

Q.5 ***Verify whether the following points lie on the line*** **$2x - y + 1 = 0$ or not.**

(i) (2, 3) *(ii)* (0,0) *(iii)* (–1, 1)

(iv) (2, 5) *(v)* (5, 3)

5(i) (2, 3)

Solution: $2x - y + 1 = 0$

$-y = -2x + 1$

$y = 2x + 1$ (i) (2, 3)

Putting $x = 2, y = 3$ in the equation

$3 = 2(2) + 1$

$3 = 4 + 1$

$3 = 5$ (Impossible)

The point (2, 3) does not lie on the line.

5(ii) (0,0)

Solution: $2x - y + 1 = 0$

$y = 2x + 1$

Putting $x = 0$, $y = 0$ in the equation

$0 = 2(0) + 1$

$0 = 0 + 1$ (Impossible)

The point (0, 0) does not lie on the line.

5(iii) **(–1, 1)**

Solution:

$$2x - y + 1 = 0$$
$$y = 2x + 1$$

Putting $x = -1$, $y = 1$ in the equation

$$1 = 2(-1) + 1$$
$$1 = -2 + 1$$
$$1 = -1 \qquad \text{(Impossible)}$$

The point (–1, 1) does not lie on the line.

5(iv) **(2, 5)**

Solution:

$$2x - y + 1 = 0$$
$$y = 2x + 1$$

Putting $x = 2$, $y = 5$ in the equation

$$5 = 2(2) + 1$$
$$5 = 4 + 1$$
$$5 = 5 \qquad \text{(True)}$$

The point (2, 5) lie on the line.

5(v) **(5, 3)**

Solution:

$$2x - y + 1 = 0$$
$$y = 2x + 1$$

Putting $x = 5$, $y = 3$ in the equation

$$3 = 2(5) + 1$$
$$3 = 10 + 1$$
$$3 = 11 \qquad \text{(Impossible)}$$

The point (5, 3) does not lie on the graph of the line.

EXERCISE 8.2

Q.1 ***Draw the conversion graph between litres and gallons using the relation 9 litres = 2 gallons (approximately), and taking litres along horizontal axis and gallons along vertical axis. Form the graph, read***

(i) The number of gallons in 18 litres.

(ii) The number of litres in 8 gallons.

Solution: (Litre) 9 = 2 (gallon)

$$y = \frac{2}{9}x$$

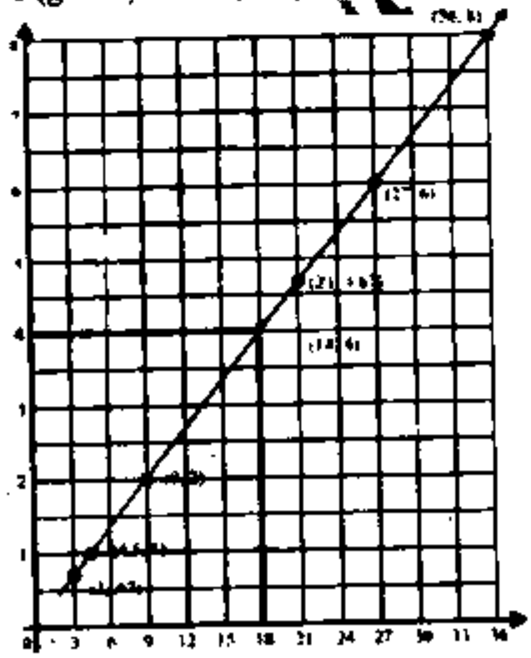
Value table for (x, y)

x =	3	4.5	21	27
y =	.67	1	4.67	6

From graph

18 (litre) = 4 (gallon)

8 (gallon) = 26 (litre)



Scale on y-axis 2 sq. = 1

Scale on x-axis 1 sq. = 3

Q.2 ***On 15.03.2008 the exchange rate of Pakistani currency and Saudi Riyal was as under:***

1 S. Riyal = 16.70 Rupees

If Pakistani currency y is an expression of S. Riyal x, expressed under the rule y = 16.70x, then draw the conversion graph between these two currencies by taking S. Riyal along x-axis.

Solution:-

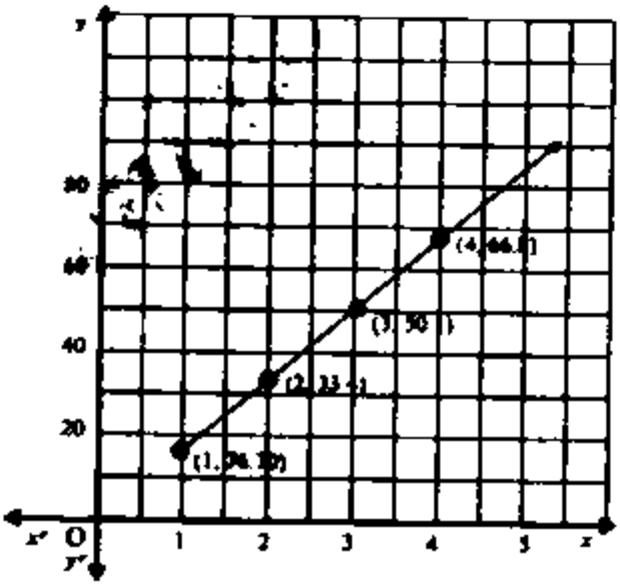
y = 16.70x where y is Pak. Rupee and x is S. Riyal

Table value for x and y.

x =	1	2	3	4
y =	16.70	33.40	50.10	66.8

From graph

$$y = 16.70 \times 2 = 33.40$$
$$y = 16.70 \times 3 = 50.10$$
$$y = 16.70 \times 4 = 66.8$$



Graph: y = 16.70

Q.3 Sketch the graph of each of the following lines.

(a)	$x - 3y + 2 = 0$	*(b)*	$3x - 2y - 1 = 0$
(c)	$2y - x + 2 = 0$	*(d)*	$y - 2x = 0$
(e)	$3y - 1 = 0$	*(f)*	$y + 3x = 0$
(g)	$2x + 6 = 0$		

3(a) $x - 3y + 2 = 0$

Solution:

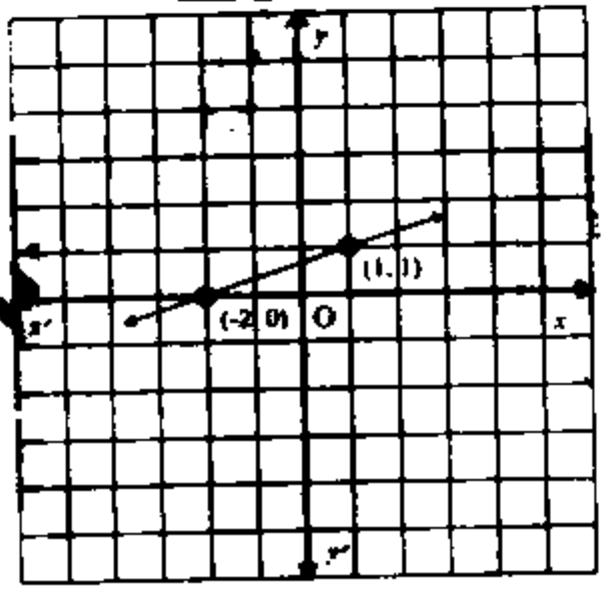
$$-3y = -x - 2$$

$$y = \frac{-x-2}{-3}$$

$$y = \frac{x+2}{3}$$

Table value for x and y

x =	1	−2
y =	1	0



Graph: $x - 3y + 2 = 0$

3(b) $3x - 2y - 1 = 0$

Solution:

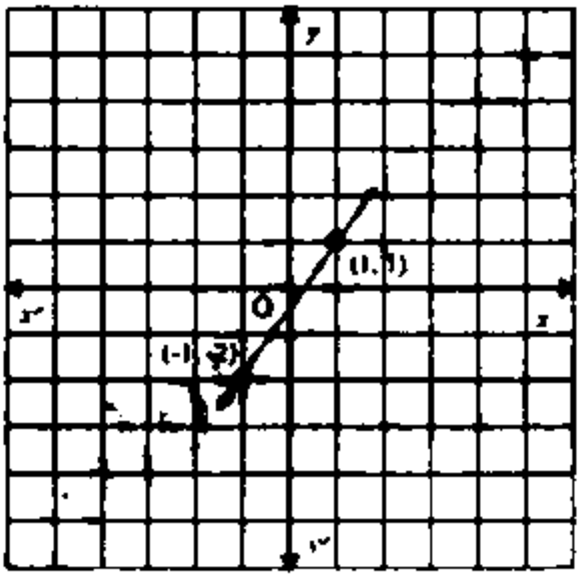
$$-2y = -3x + 1$$

$$y = \frac{-3x+1}{-2}$$

$$y = \frac{3x-1}{2}$$

Table of ordered pairs for (x, y)

x =	1	−1
y =	1	−2



Graph: $3x - 2y - 1 = 0$

3(c) $2y - x + 2 = 0$

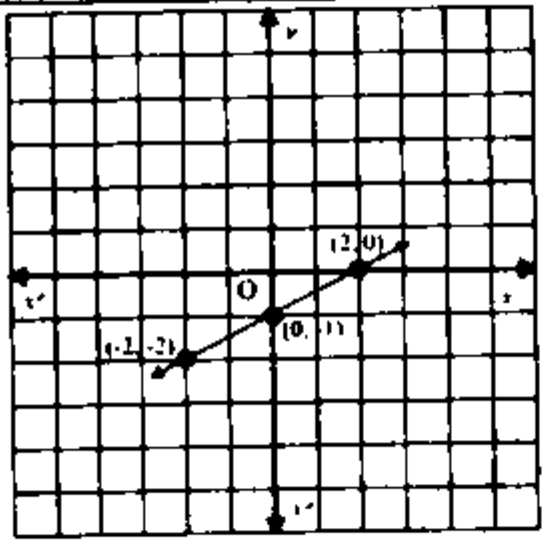
Solution:

$$2y = x - 2$$

$$y = \frac{x-2}{2}$$

Table of ordered pairs for (x, y)

x =	2	−2	0
y =	0	−2	−1



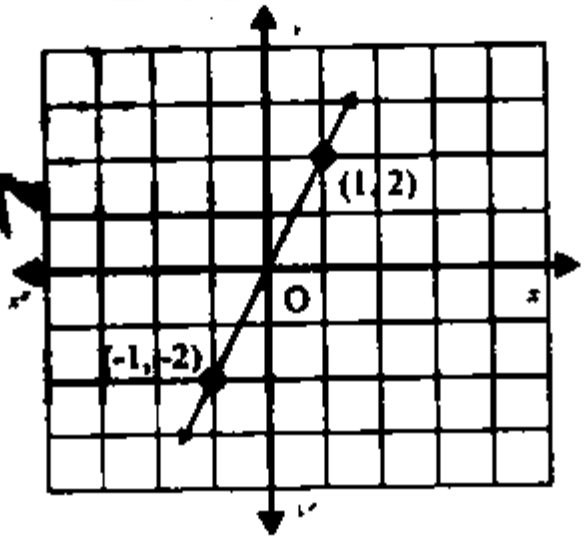
Graph: $2y - x + 2 = 0$

3(d) $y - 2x = 0$

Solution: $y = 2x$

Table of ordered pairs for (x, y)

x =	1	−1
y =	2	−2



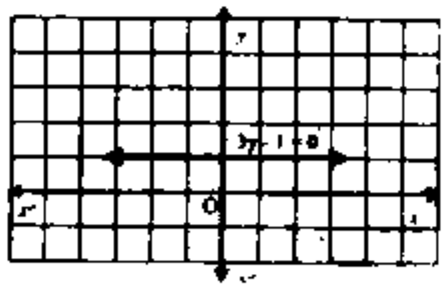
Graph: $y - 2x = 0$

3(e) $y - 1 = 0$

Solution: $3y = 1$ $y = \frac{1}{3}$

Table of ordered pairs for (x, y)

x =	...	1	2	3	–2	...
y =	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	......



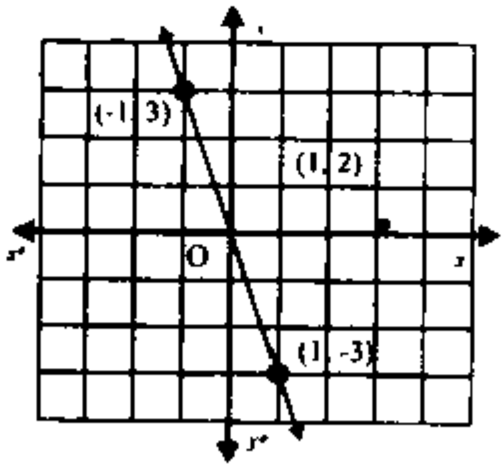
Graph: $3y - 1 = 0$

3(f) $y + 3x = 0$

Solution: $y = -3x$

Table of ordered pairs for (x, y)

x =	–1	1
y =	3	–3



Graph: $y + 3x = 0$

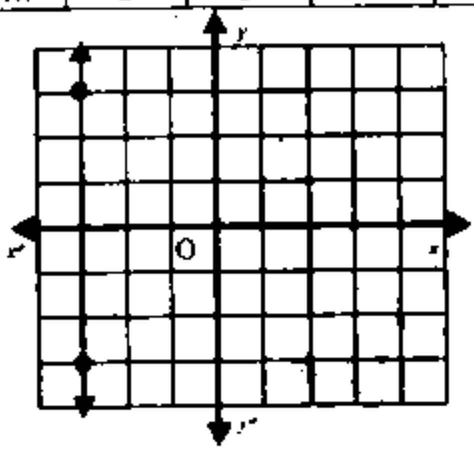
3(g) $2x + 6 = 0$

Solution: $2x = -6$

or $x = -3$

Table of ordered pairs for (x, y)

x =	−3	−3	−3	−3	−3
y =	....	2	3	−3	....



Graph: $2x + 6 = 0$

Q.4 *Draw the graph for following relations.*

(i) One mile = 1.6 km

(ii) One Acre= 0.4 Hectare

(iii) $F = \frac{9}{5}C + 32$

(iv) One Rupee = $\frac{1}{86}$ \$

4(i) One mile = 1.6 km

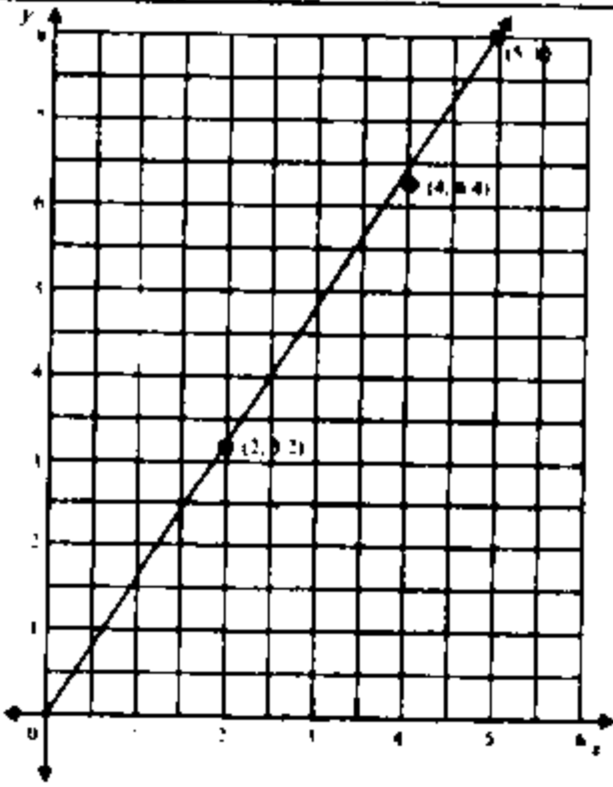
Solution: one mile = 1.6 km

Let us show it by an equation

$y = 1.6x$

We prepare a table in form of ordered pairs in x, y

x =	0	2	5	4
y =	0	3.2	8.0	6.4



Graph: Scale 2 sq. = 1

4(ii) **One Acre= 0.4 Hectare**

Solution: One Acre = 0.4 Hector

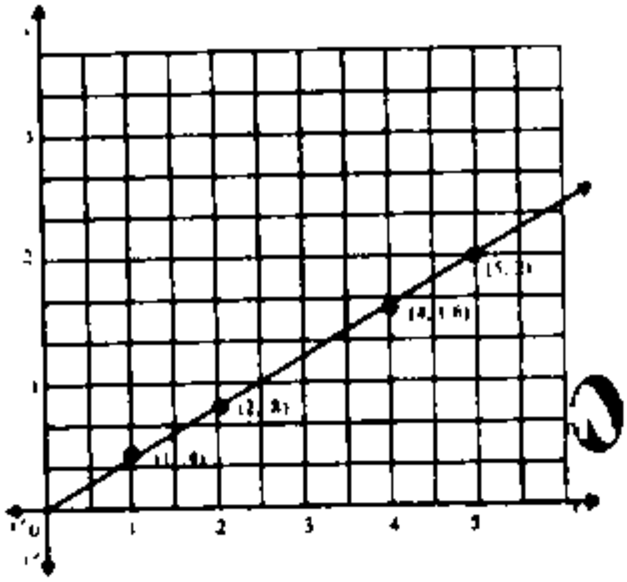
Let us show it by an equation

$$y = 0.4x$$

Table of ordered pairs in (x, y)

x =	0	1	2	4	5
y =	0	0.4	.8	1.6	2.0

Putting the ordered pair on the graph and joining them, we get a straight line.



Graph: Scale 3 sq. = 1, Scale 2 sq. = 1

4(iii) $F = \frac{9}{5}C + 32$

Solution:

Table value for F and C in form of (x, y)

x =	C	0	10	20	50	100
y =	F	32	50	68	122	212

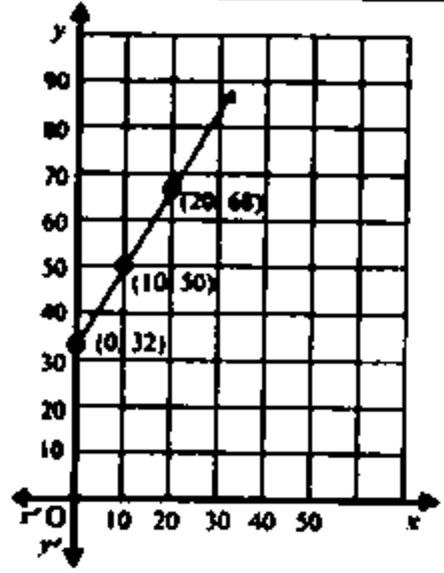
$F = \frac{9}{5}C + 32$

$F = \frac{9}{5}(0) + 32 = 32$

$F = \frac{9}{5}(10) + 32 = 50$

$F = \frac{9}{5}(20) + 32 = 68$

$F = \frac{9}{5}(100) + 32 = 212$



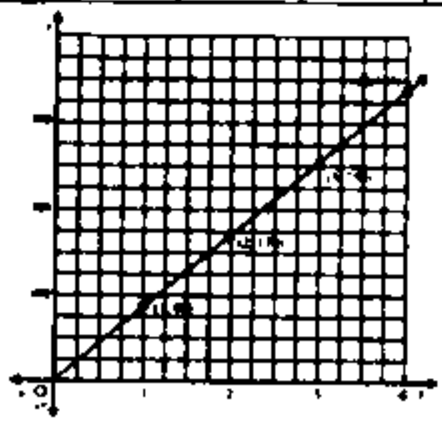
4(iv) **One Rupee = $\frac{1}{86}$ \$**

Solution: One rupee $= \frac{1}{86}$ \$

We show the relation with the help of an equation.

$$y = \frac{1}{86}x$$

x =	0	86	172	258	344
y =	0	1	2	3	4



EXERCISE 8.3

Solve the following pair of equations in x and y graphically.

1. $x + y = 0, 2x - y + 3 = 0$
2. $x - y + 1 = 0, x - 2y = -1$
3. $2x + y = 0, x + 2y = 2$
4. $x + y - 1 = 0, x - y + 1 = 0$
5. $2x + y - 1 = 0, x = -y$

1. $x + y = 0, 2x - y + 3 = 0$

Solution:

$$x + y = 0$$
$$y = -x$$

$$2x - y + 3 = 0$$
$$-y = -2x - 3$$
$$y = 2x + 3$$

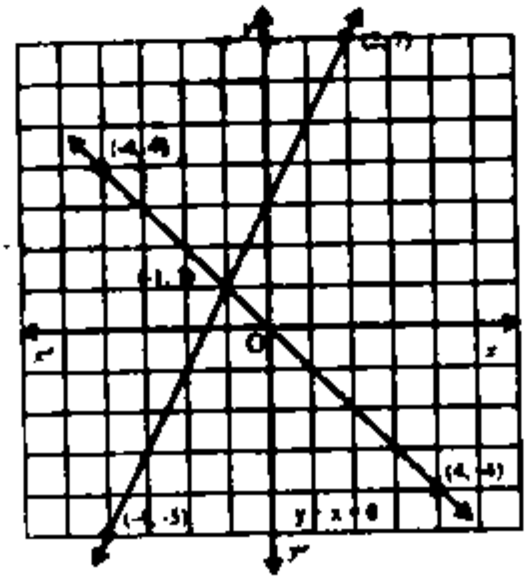
Table for x and y values

x =	4	–4
y =	–4	4

Table for x and y value

x =	2	–4
y =	7	–5

Plotting points on the graph paper



Point of intersection (–1, 1)

Sol. Set = {(–1, 1)}

2. $x - y + 1 = 0, x - 2y = -1$

Solution:

$$x - y + 1 = 0$$
$$-y = -x - 1$$
$$y = x + 1$$

Table for x and y values

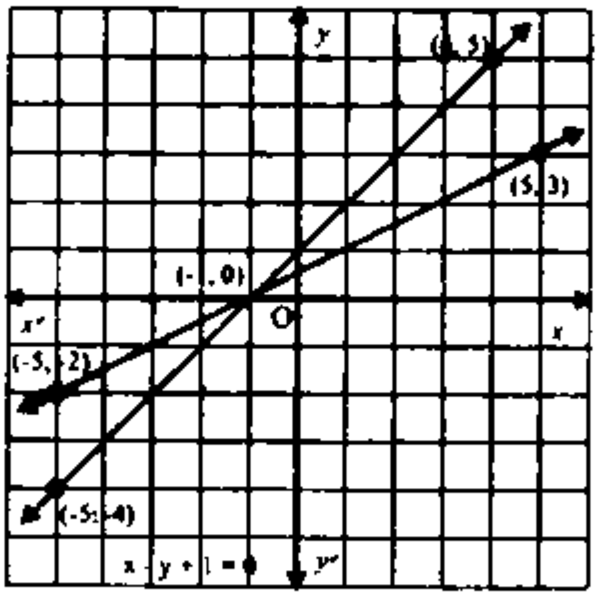
x =	4	–5
y =	5	–4

$$x - 2y = -1$$
$$-2y = -1 - x$$
$$y = \frac{-1 - x}{-2}$$
$$y = \frac{1 + x}{2}$$

Table for x and y value

x =	5	–5
y =	3	–2

Plotting the pair of points of both the equations.



Point of intersection (–1, 0)

Sol. Set = {(–1, 0)}

3. $2x + y = 0, x + 2y = 2$

Solution:

$$2x + y = 0$$
$$y = -2x$$

Table value for x, y

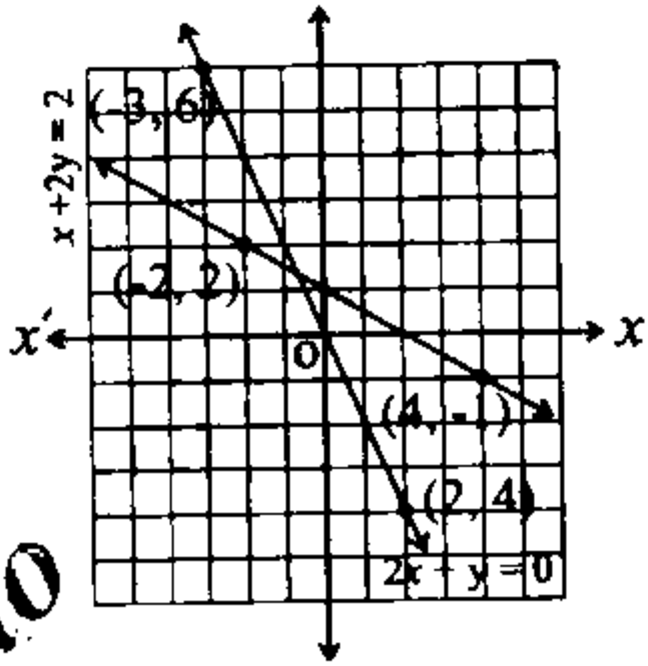
x =	2	–3
y =	–4	6

$$x + 2y = 2$$
$$2y = 2 - x$$
$$y = \frac{2-x}{2}$$

Table value for x, y

x =	4	–2
y =	–1	2

Plotting points on the graph paper



Point of intersection $\left(-\frac{2}{3}, \frac{4}{3}\right)$

Sol. Set = $\left\{\left(-\frac{2}{3}, \frac{4}{3}\right)\right\}$

4. $x + y - 1 = 0,\ x - y + 1 = 0$

Solution:

$$x + y - 1 = 0$$
$$y = 1 - x$$

Table value of x, y

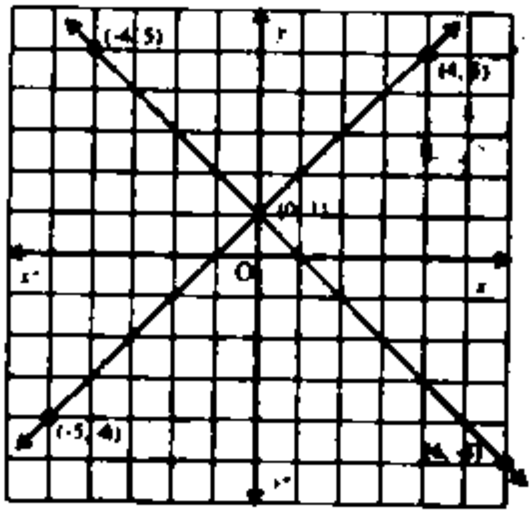
x =	–4	6
y =	5	–5

$$x - y + 1 = 0$$
$$-y = -x - 1$$
$$y = x + 1$$

Table value of x, y

x =	4	–5
y =	5	–4

Plotting points on the graph paper



Point of intersection (0, 1)

Sol. Set = {(0, 1)}

5. $2x + y - 1 = 0,\ x = -y$

Solution:

$$2x + y - 1 = 0$$
$$y = 1 - 2x$$

Table values for x, y

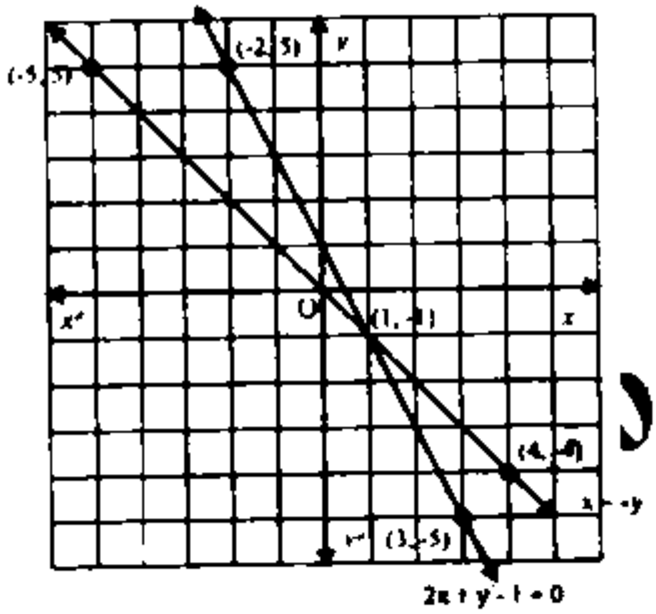
x =	–2	3
y =	5	–5

$$x = -y$$
$$y = -x$$

Table value for x, y

x =	4	–5
y =	–4	5

Plotting points on the graph paper



Point of intersection (1, −1)

Sol. Set = {(1, −1)}

Review Exercise 8

Q.1 Choose the correct answers.

(i) If (x − 1, y + 1) = (0, 0), then (x, y) is

(a) (1, −1) (b) (−1, 1)

(c) (1, 1) (d) (−1, −1)

(ii) If (x, 0) = (0, y), then (x, y) is

(a) (0, 1) (b) (1, 0)

(c) (0, 0) (d) (1, 1)

(iii) Point (2, −3) lies in quadrant.

(a) I (b) II

(c) III (d) IV

(iv) Point (−3, −3) lies in quadrant

(a) I (b) II

(c) III (d) IV

(v) If $y = 2x + 1$, $x = 2$ then y is

(a) 2 (b) 3

(c) 4 (d) 5

(vi) Which ordered pair satisfy the equation $y = 2x$

(a) (1, 2) (b) (2, 1)

(c) (2, 2) (d) (0, 1)

Answers:

(i)	a	*(ii)*	C	*(iii)*	d	*(iv)*	a
(v)	d	*(vi)*	A				

Q.2 Identify the following statements as True or False.

(i) The point O(0, 0) is a vertical line.

(ii) The point P(2, 0) lies on x-axis.

(iii) The graph of $x = -2$ is a vertical line.

(iv) $3 - y = 0$ is a horizontal line.

(v) The point Q(−1, 2) is in quadrant III.

(vi) The point R(−1, −2) is in quadrant IV.

(vii) $y = x$ is a line on which origin lies.

(viii) The point P(1, 1) lies on the line $x + y = 0$.

(ix) The point S(1, −3) lies in quadrant III.

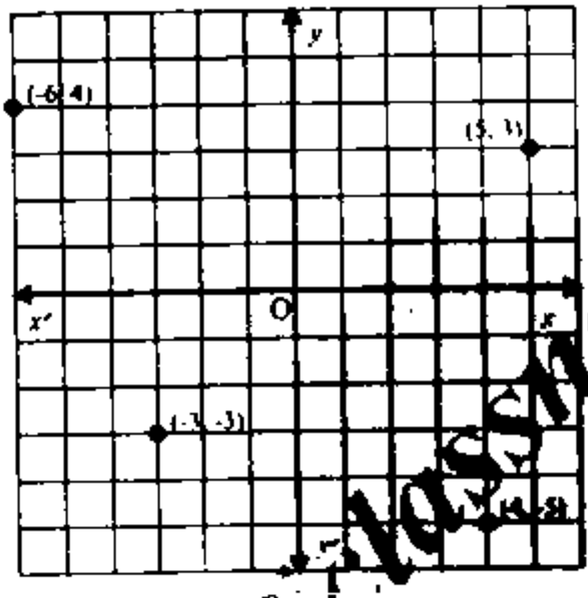
(x) The point R(0, 1) lies on the x-axis.

Answers:

(i)	F	*(ii)*	T	*(iii)*	T	*(iv)*	T
(v)	F	*(vi)*	F	*(vii)*	T	*(viii)*	F
(ix)	F	*(x)*	F				

Q.3 **Draw the following points on the graph paper.**

$(-3, -3), (-6, 4), (4, -5), (5, 3)$



Graph

Q.4 ***Draw the graph of the following equations.***

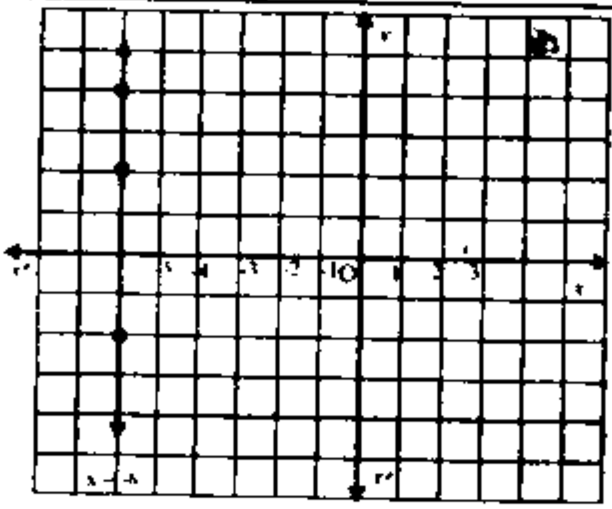
(i) $x = -6$ **(ii)** $y = 7$ **(iii)** $x = \frac{5}{2}$

(iv) $y = \frac{9}{2}$ **(v)** $y = 4x$ **(vi)** $y = -2x + 1$

4(i) $x = -6$

Solution: Table value for x, y

$x =$	-6	-6	-6	-6	-6	-6	……
$y =$	…	2	4	5	-1	-2	….

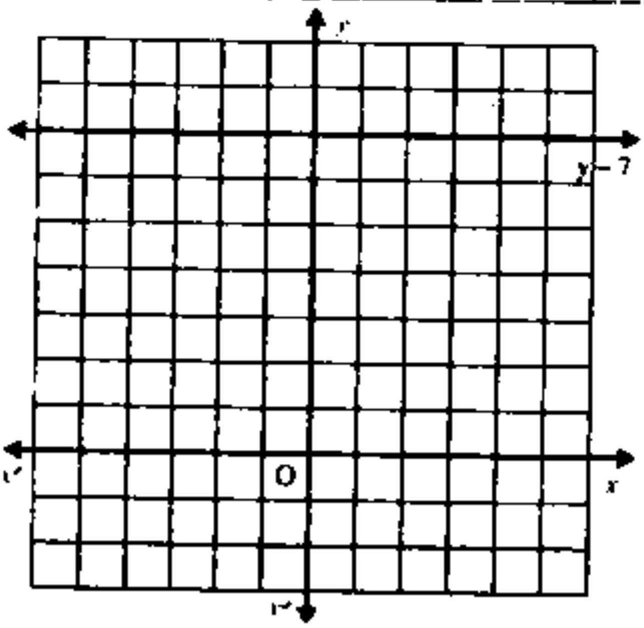


Graph of x = −6 to parallel to y-axis and at the left side of it.

4(ii) y = 7

Solution: Table value for x, y

x =	...	2	3	4	−1	−3	......
y =	7	7	7	7	7	7	7



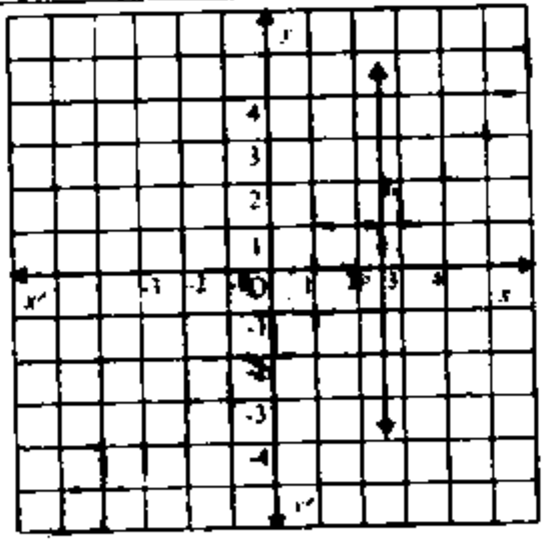
Graph of $y = 7$ is parallel to x-axis and it is box it at a distance of 7 units.

4(iii) $x = \frac{5}{2}$

Solution: $x = 2.5$

Table value for x, y

x =	2.5	2.5	2.5	2.5	2.5	2.5
y =	....	3	4	2	−1	...



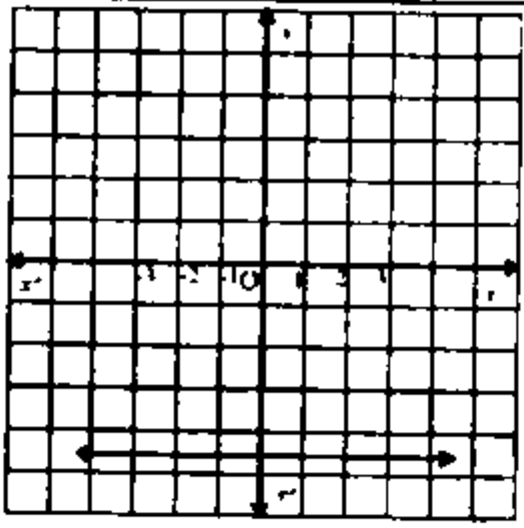
Graph of $x = \frac{5}{2}$ is parallel to y-axis and is on the right of it at a distance of $\frac{5}{2}$ units.

4(iv) $y = -\frac{9}{2}$

Solution: $y = -4.5$

Table value for x, y

x =	...	3	4	−2	....
y =	−4.5	−4.4	−4.5	−4.5	−4.5

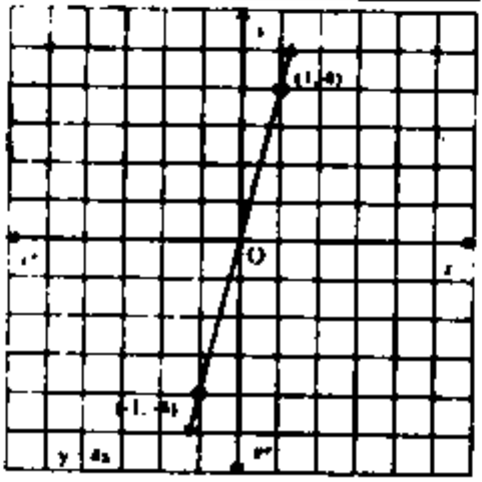


Graph of $y = -\frac{9}{2}$ is parallel to x-axis and it is below at a distance of $-\frac{9}{2}$ units.

4(v) $y = 4x$

Solution: Table value for x, y

x =	0	1	−1
y =	0	4	−4

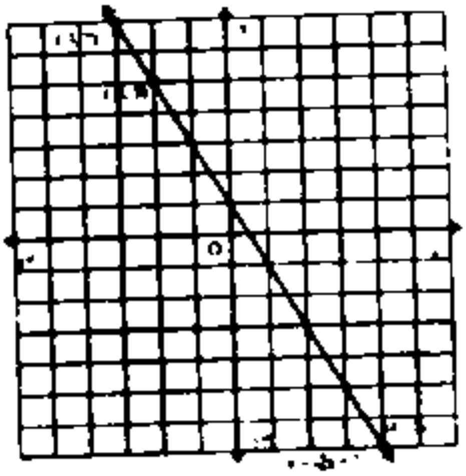


$y = 4x$

4(vi) $y = -2x + 1$

Solution: Table value for x, y

x =	−2	4	−3
y =	5	−7	7



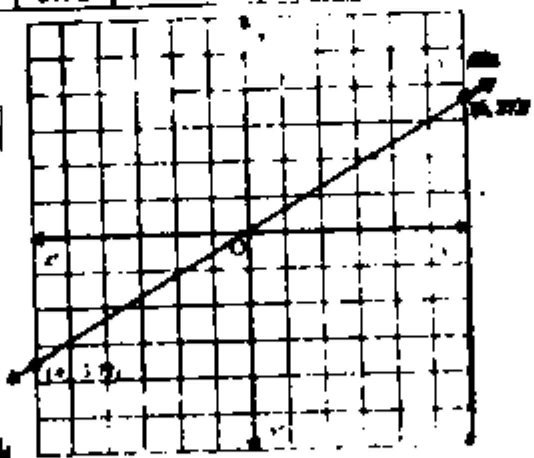
Q.5 Draw the graph of the following equation.

(i) $y = 0.62x$ (ii) $y = 2.5x$

5(i) $y = 0.62x$

Solution: Table value of x, y

x =	2	6	−6	0
y =	1.24	3.72	−3.72	0

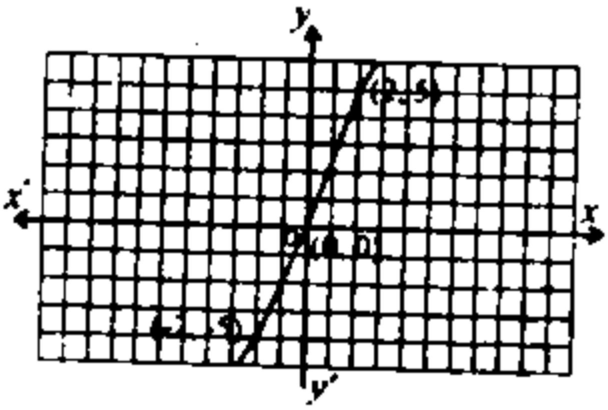


Graph $y = 0.62x$

5(ii) $y = 2.5x$

Solution: Table value for x, y

x =	0	2	−2
y =	0	5	−5



Graph $y = 2.5x$

Q.6 Solve the following equations

(i) $x - y = 1$, $x + y = \frac{1}{2}$

(ii) $x = 3y$, $2x - 3y = -6$

(iii) $\frac{1}{3}(x + y) = $ [illegible] , $\frac{1}{3}(x - y) = -1$

6(i) $x + y = \frac{1}{2}$, $x - y = 1$

Solution:

$x + y = \frac{1}{2}$

$y = \frac{1}{2} - x$

$= \frac{1-2x}{2}$

Table values for x, y

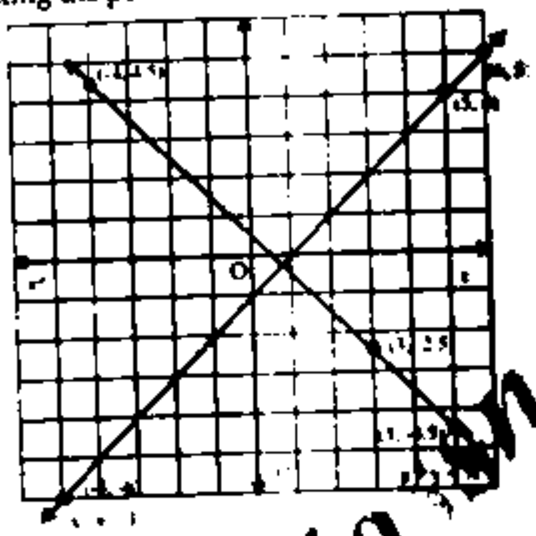
x =	3	−4	5
y =	−2.5	4.5	−4.5

$x - y = 1$
$-y = 1 - x$
$y = x - 1$

Table value for x, y

x =	5	6	−5
y =	4	5	−6

Plotting the points



Point of intersection $\left(\frac{3}{4}, -\frac{1}{4}\right)$

Sol. Set = $\left\{\left(\frac{3}{4}, -\frac{1}{4}\right)\right\}$

6(ii) $2x - 3y = -6$; $x = 3y$

Solution:

$$2x - 3y = -6$$
$$-3y = -6 - 2x$$
$$= \frac{-6-2x}{-3}$$
$$= \frac{2x+6}{3}$$

$$x = 3y$$
$$3y = x$$
$$y = \frac{x}{3}$$

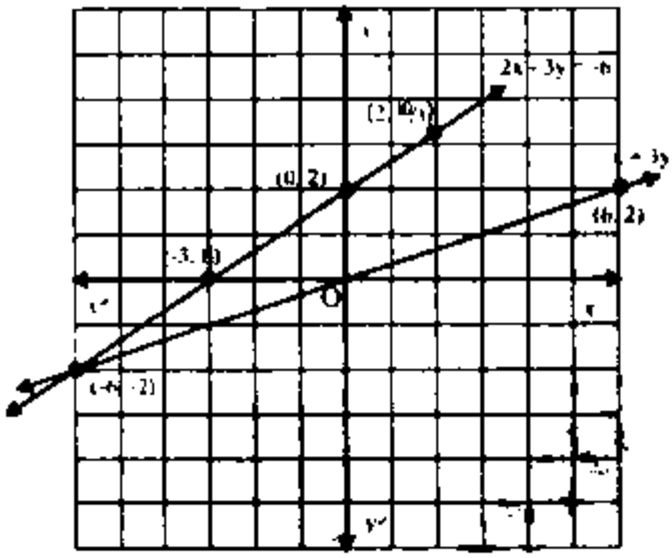
Table values for x, y

x =	0	2	–3
y =	2	10/3	0

Table value for x, y

x =	0	6	–6
y =	0	2	–2

Plotting the points



Point of intersection (–6, –2)

Common Sol. Set = {(–6, –2)}

6(iii) $\frac{1}{2}(x-y)=-1,\quad \frac{1}{3}(x+y)=2$

Solution:

$$\frac{1}{2}(x-y)=-1 \qquad\qquad \frac{1}{3}(x+y)=2$$

$$x-y=-2 \qquad\qquad x+y=6$$

$$-y=-2-x \qquad\qquad y=6-x$$

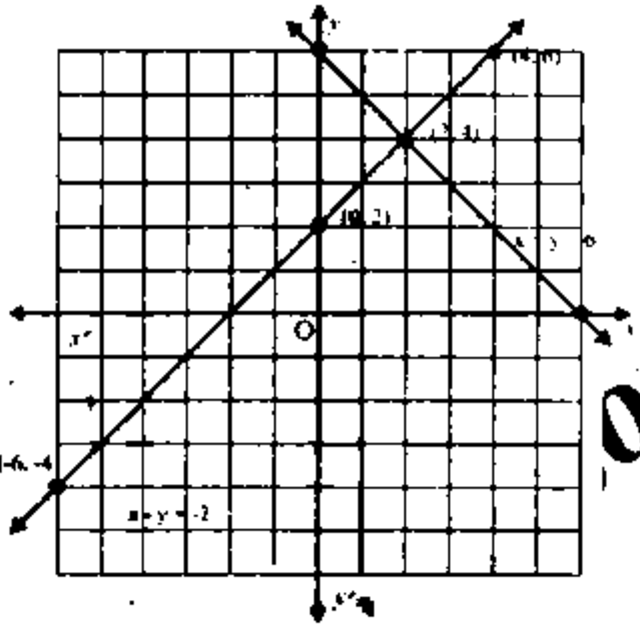
$$y=x+2$$

Table values for x, y

x =	4	–6	0
y =	6	–4	2

Table value for x, y

x =	4	–2	0
y =	2	8	6



Point of intersection (2, 4)
Common Sol. Set = {(2, 4)}

Unit 9 INTRODUCTION TO CO-ORDINATE GEOMETRY

(i) **The distance formula.**

(ii) **Collinear Points.**

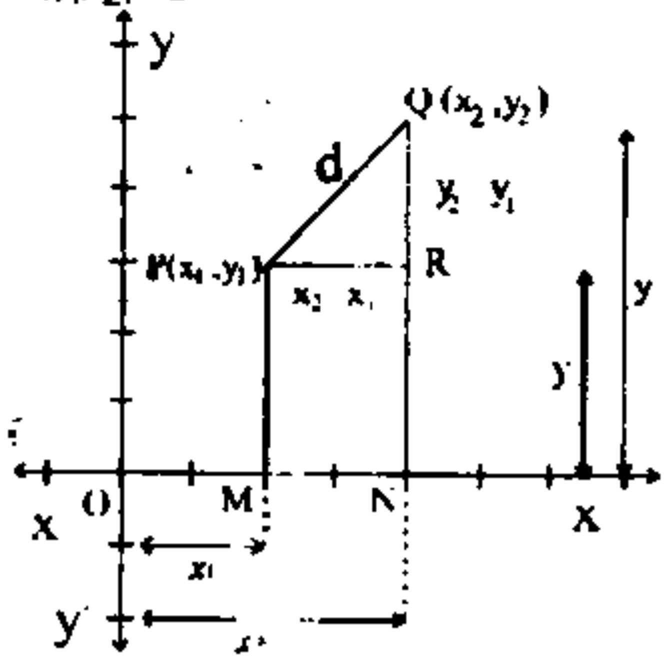
(iii) **Mid Point Formula.**

Distance Formula

(Finding distance between two points).

Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be any two points in the coordinate plane where distance between P and Q is 'd' (a real number).

i.e; $|PQ| = d$



$\overline{PM} \perp X'OX$, $\overline{QN} \perp X'OX$ and $\overline{PR} \perp \overline{QN}$; therefore, $\overline{PM}$, $\overline{QN}$ are parallel to Y'OY and $\overline{PR}$ is parallel to X'OX.

Thus Δ PRQ is a right angled triangle and m ∠PRQ = 90^{O}.

Now $m\ \overline{PR} = |x_2 - x_1|$ and $m\ \overline{RQ} = |y_2 - y_1|$

Using Pythagoras Theorem

$$(m\ \overline{PQ})^2 = (m\ \overline{PR})^2 + (m\ \overline{RQ})^2$$

or $$d^2 = |x_2 - x_1|^2 + |y_2 - y_1|^2$$

or $$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

$$= \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

Corollary:

Distance of any point (x_1, y_1) from the origin $(0, 0)$ = $\sqrt{x_1^2 + y_1^2}$

EXERCISE 9.1

1. Find the distance between the following pairs of points.

(a) $A(9, 2), B(7, 2)$ (b) $A(2, -6), B(3, -6)$

(c) $A(-8, 1), B(6, 1)$ (d) $A(-4, \sqrt{2}), B(-4, -3)$

(e) $A(3, -11), B(3, -4)$ (f) $A(0, 0), B(0, -5)$

Solution:-

1(a) $A(9, 2), B(7, 2)$

$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2} \text{ (Formula)}$$
$$|AB| = \sqrt{(7-9)^2 + (2-2)^2}$$
$$= \sqrt{(-2)^2 + 0}$$
$$= \sqrt{4} = 2$$

1(b) $A(2, -6), B(3, -6)$

$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2} \text{ (Formula)}$$
$$|AB| = \sqrt{(3-2)^2 + [(-6) - (-6)]^2}$$
$$= \sqrt{(1)^2 + (-6+6)^2}$$
$$= \sqrt{1+0} = 1$$

1(c) $A(-8, 1), B(6, 1)$

$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2} \text{ (Formula)}$$
$$|AB| = \sqrt{[6-(-8)]^2 + (1-1)^2}$$
$$= \sqrt{(6+8)^2 + 0}$$
$$= \sqrt{(14)^2} = 14$$

1(d) $A(-4, \sqrt{2}), B(-4, -3)$

$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2} \text{ (Formula)}$$
$$|AB| = \sqrt{[(-4) - (-4)]^2 + [(-3) - \sqrt{2}]^2}$$

$$= \sqrt{(-4+4)^2 + (-3-\sqrt{2})^2}$$
$$= \sqrt{0 + (-1)^2(3+\sqrt{2})^2} =$$

$3+\sqrt{2}$

1(e) $A(3, -1), B(3, -4)$

$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$
(Formula)

$$|AB| = \sqrt{[(3-3)]^2 + [(-4)-(-1)]^2}$$
$$= \sqrt{0 + (-4+1)^2}$$
$$= \sqrt{(-3)^2} = 3$$

1(f) $A(0, 0), B(0, -5)$

$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$
(Formula)

$$|AB| = \sqrt{[(0-0)]^2 + (-5) - 0)^2}$$
$$= \sqrt{0 + (-5)^2}$$
$$= \sqrt{(-5)^2} = 5$$

2. ***Let P be the point on x–axis with x–coordinate 'a' and Q be the point on y–axis with y–coordinate 'b' as given below. Find the distance between P and Q.***

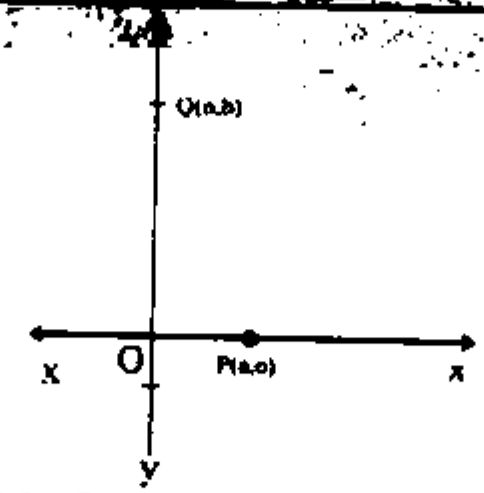
(i) $a = 9, b = 7$ (ii) $a = 2, b = 3$

(iii) $a = -8, b = 6$ (iv) $a = -2, b = -3$

(v) $a = \sqrt{2}, b = 1$ (vi) $a = -9, b = -4$

Solution:-

The points which are on x–axis have y-coordinate 'o' and the points which are on y–axis have x coordinate 'o'.



2(i) $a = 9,\ b = 7$

Points are $P(9, 0)$: $Q(0, 7)$

$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$ (Formula)

$$PQ = \sqrt{[(0-9)]^2 + (7-0)^2}$$
$$= \sqrt{(-9)^2 + (7)^2}$$
$$|PQ| = \sqrt{81 + 49}$$
$$= \sqrt{130}$$

2(ii) $a = 2,\ b = 3$

Points are $P(2, 0)$: $Q(0, 3)$

$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$ (Formula)

$$|PQ| = \sqrt{(0-2)^2 + (3-0)^2}$$
$$= \sqrt{(-2)^2 + (3)^2}$$
$$= \sqrt{4 + 9}$$
$$= \sqrt{13}$$

2(iii) $a = -8,\ b = 6$

Points are $P(-8, 0)$: $Q(0, 6)$

$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$ (Formula)

$$|PQ| = \sqrt{[0-(-8)]^2 + (6-0)^2}$$

$= \sqrt{(8)^2 + (6)^2}$
$= \sqrt{64 + 36}$
$= \sqrt{100} = 10$

2(iv) $a = -2, b = -3$

Points are $P(-2, 0)$; $Q(0, -3)$

$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$ (Formula)

$|PQ| = \sqrt{[0-(-2)]^2 + [(-3-0)]^2}$
$= \sqrt{(2)^2 + (-3)^2}$
$= \sqrt{4 + 9}$
$= \sqrt{13}$

2(v) $a = \sqrt{2}, b = 1$

Points are $P(\sqrt{2}, 0)$; $Q(0, 1)$

$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$ (Formula)

$|PQ| = \sqrt{(0-\sqrt{2})^2 + (1-0)^2}$
$= \sqrt{(-\sqrt{2})^2 + (1)^2}$
$= \sqrt{2 + 1}$
$= \sqrt{3}$

2(vi) $a = -9, b = -4$

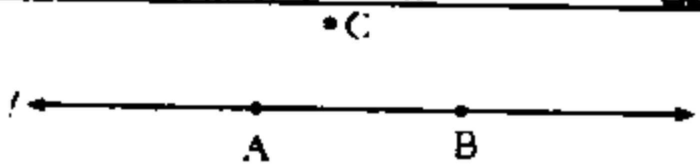
Points are $P(-9, 0)$; $Q(0, -4)$

$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$ (Formula)

$|PQ| = \sqrt{[0-(-9)]^2 + [(-4)-0]^2}$
$= \sqrt{(9)^2 + (-4)^2}$
$= \sqrt{81 + 16}$
$= \sqrt{97}$

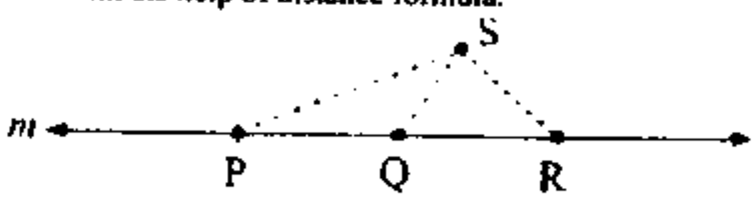
Collinear Points

Two or more than two points which lie on the same straight line are called collinear points w.r.t that line; otherwise they are non-collinear points.



Points A, B lie on straight line 'l', these points are collinear with respect to line 'l'. Points A, B, C are not collinear w.r.t the line l.

We can prove the collinearity of more than two points with the help of distance formula.



We can prove by using distance formula that points P,Q,R are not collinear with respect to line 'm'.

$$|PQ| + |QR| = |PQ|$$

While points P,Q,R S are not collinear w.r.t m

as $\quad |PQ| + |QS| \neq |PS|$

and $\quad |PR| + |RS| \neq |PS|$

Use of distance formula to different types of a triangle.

(i) Equilateral triangle. (ii) Isosceles triangle.
(iii) Right angle triangle. (iv) Scalene triangle.

Definitions:-

(i) Equilateral Triangle:

Equilateral triangle has its sides equal in length.

(ii) Isosceles Triangle:

Two of its sides are of equal in length while its third side has a different length.

(iii) Right Angle Triangle:

It has one of its angle is of 90^0.

(iv) Scalene Triangle:

Measures of all the three sides are different.

Using distance formula to show that given four non-collinear points form a square, rectangle or a parallelogram.

(i) ***Square:***

It is a closed plane figure formed by four non-collinear points such that length of all sides are equal and measures of each angle is 90°.

(ii) ***Rectangle:***

It is a closed plane figure formed by four non-collinear points if

(i) its opposite sides are equal in length.

(ii) angle of each vertex is of 90°.

(iii) ***Parallelogram:***

It is a closed plane figure formed by four non-collinear points if

(i) lengths of opposite sides are equal.

(ii) its opposite sides are parallel.

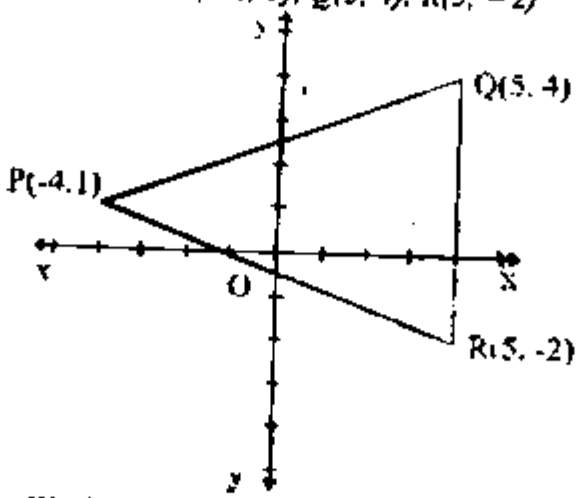
(iii) measure of none of the angle is 90°.

EXERCISE 9.2

Q.1. Show whether the points with vertices (5, − 2), (5, 4) and (− 4, 1) are vertices of an equilateral or an isosceles triangle?

Solution:-

Let $P(-4, 1), Q(5, 4), R(5, -2)$



We find $|RP|$, $|QR|$ and $|PQ|$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(Formula)

$|PQ| = \sqrt{[5-(-4)]^2 + [4-1]^2}$

$= \sqrt{(5+4)^2 + (3)^2} = \sqrt{81+9} = \sqrt{90}$ (i)

$Q(5, 4); R(5, -2)$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(Formula)

$|QR| = \sqrt{(5-5)^2 + (-2-4)^2}$

$= \sqrt{0 + (-6)^2} = \sqrt{36^2} = 6$

(ii)

$R(5, -2); P(-4, 1)$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

(Formula)

$$|RP| = \sqrt{(-4-5)^2 + [1-(-2)]^2}$$
$$= \sqrt{(-9)^2 + [1+2]^2}$$
$$= \sqrt{81 + (3)^2} = \sqrt{81+9} = \sqrt{90}$$

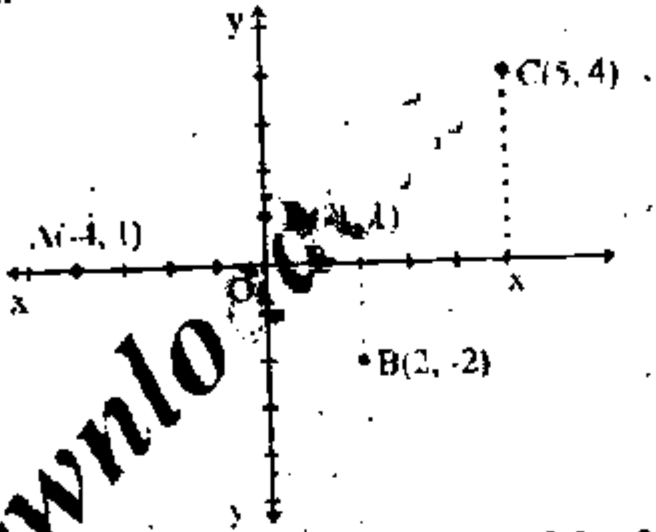
(iii)

$|PQ| = |RP|$ (From i and iii)

Thus PQR is an isosceles triangle.

Q.2. Show whether or not the points with vertices $(-1, 1)$, $(5, 4)$, $(2, -2)$ and $(-4, 1)$ form a square.

Solution:-



Let $A(-4, 1)$; $B(2, -2)$; $C(5, 4)$; $D(-1, 1)$

We find $|AB|$, $|BC|$, $|CD|$, $|DA|$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$\therefore |AB| = \sqrt{[2-(-4)]^2 + [(-2)-(1)]^2}$$
$$= \sqrt{(2+4)^2 + (-2-1)^2}$$
$$= \sqrt{(6)^2 + (-3)^2} = \sqrt{36+9} = \sqrt{45} \quad (i)$$

$$|BC| = \sqrt{(5-2)^2 + [4-(-2)]^2}$$

$$= \sqrt{(3)^2 + (6)^2} = \sqrt{9 + 36} = \sqrt{45}$$

(ii)

$$|CD| = \sqrt{[(-1) - (5)]^2 + [1 - 4]^2}$$
$$= \sqrt{(-1-5)^2 + [(-3)]^2}$$
$$= \sqrt{(-6)^2 + (-3)^2} = \sqrt{36 + 9} = \sqrt{45}$$

(iii)

$$|DA| = \sqrt{[(-4) - (-1)]^2 + [1 - 1]^2}$$
$$= \sqrt{(-4+1)^2 + 0}$$
$$= \sqrt{(-3)^2} = \sqrt{9} = 3$$

(iv)

We find that

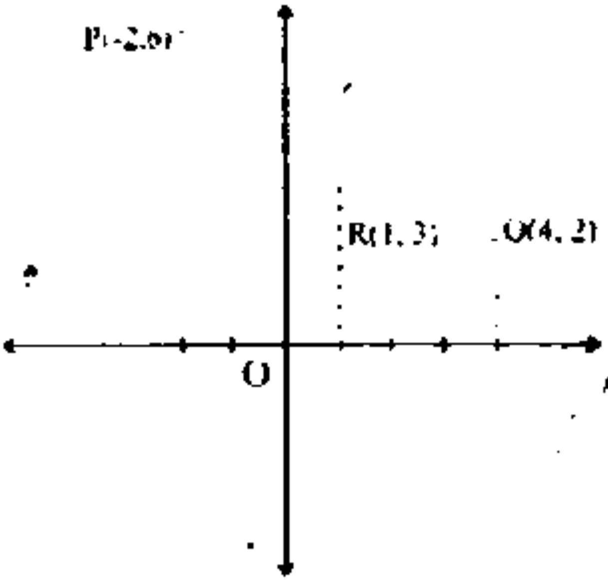
$|AB| = |BC| = |CD|$ (From i, ii, iii)

But

$|AB| = |BC| = |CD| \neq |DA|$

Therefore $(-4, 1)$ $(2, -2)$, $(5, 4)$, $(-1, 1)$ do not Form a square figure.

Q.3. ***Show whether or not the points with coordinates (1, 3), (4, 2) and (−2, 6) are vertices of a right triangle.***



Solution:-

Let $P(-2, 6), Q(4, 2), R(1, 3)$ be the points

We find $|PQ|, |QR|, |RP|$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$|PQ| = \sqrt{[4-(-2)]^2 + [2-6]^2}$$

$$= \sqrt{(4+2)^2 + (-4)^2}$$

$$= \sqrt{(6)^2 + (-4)^2} = \sqrt{36+16} = \sqrt{52}$$

$$PQ^2 = (\sqrt{52})^2 = 52 \qquad (i)$$

and

$$|QR| = \sqrt{(1-4)^2 + (3-2)^2}$$

$$= \sqrt{(-3)^2 + (1)^2}$$

$$= \sqrt{9+1} = \sqrt{10}$$

$$|QR|^2 = (\sqrt{10})^2 = 10 \qquad (ii)$$

$$|RP| = \sqrt{[(-2)-(1)]^2 + (6-3)^2}$$

$$= \sqrt{(-2-1)^2 + (3)^2}$$

$$= \sqrt{(-3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18}$$

and $|RP|^2 = (\sqrt{18})^2 = 18$ (iii)

$|PQ|^2 = 52$ From (iii), (ii), (i)

$|QR|^2 = 10$

$|RP|^2 = 18$

We see that

$$52 + 10 \neq 18$$
$$10 + 18 \neq 52$$
$$18 + 52 \neq 10$$

Therefore, points $(1, 3)$; $(4, 2)$; $(-2, 6)$ do not form a right angle triangle.

Q.4. ***Use the distance formula to prove whether or not the points (1, 1), (− 2, − 8), (4, 10) lie on a straight line?***

Solution:-

Let $P(4, 10)$; $Q(-2, -8)$, $R(1, 1)$ be the points.

We find $|RP|$, $|\overline{QR}|$ and $|PQ|$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
(Formula)

$$|PQ| = \sqrt{(-2-4)^2 + (-8-10)^2}$$
$$= \sqrt{(-6)^2 + (-18)^2}$$
$$= \sqrt{36 + 324} = \sqrt{360} = 6\sqrt{10}$$
(i)

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
(Formula)

$$|\overline{QR}| = \sqrt{[1-(-2)]^2 + [1-(-8)]^2}$$
$$= \sqrt{(1+2)^2 + (1+8)^2}$$
$$= \sqrt{(3)^2 + (9)^2}$$
$$= \sqrt{9 + 81} = \sqrt{90} = 3\sqrt{10}$$
(ii)

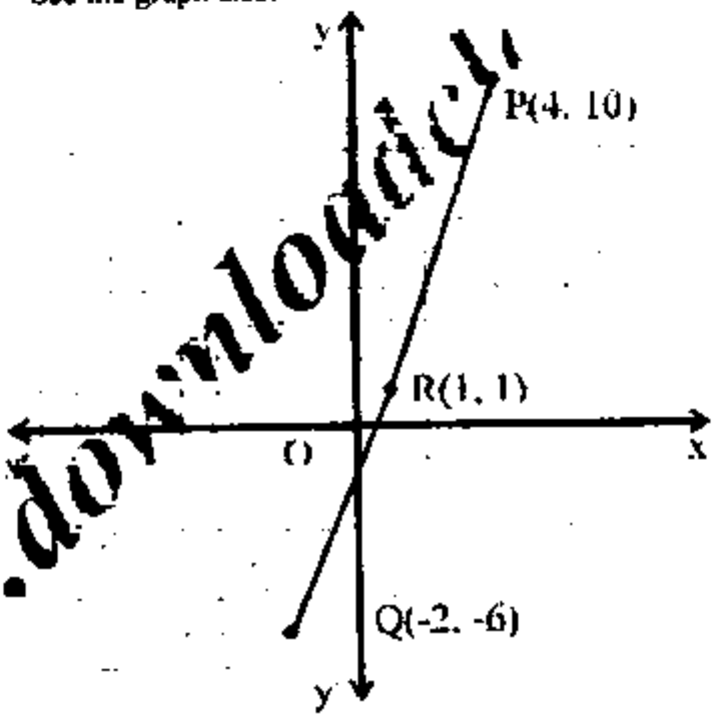
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
(Formula)

$$|RP| = \sqrt{(4-1)^2 + (10-1)^2}$$
$$= \sqrt{(3)^2 + (9)^2}$$
$$= \sqrt{9 + 81} = \sqrt{90} = 3\sqrt{10}$$

(iii)

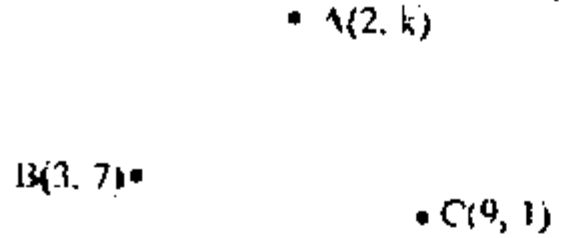
$$|QR| = 3\sqrt{10}$$
$$|RP| = 3\sqrt{10}$$
$$|QR| + |RP| = 3\sqrt{10} + 3\sqrt{10}$$
$$= \sqrt{10}(3+3)$$
$$= 6\sqrt{10}$$
$$|PQ| = 6\sqrt{10} \quad \text{From (i)}$$
$$\therefore |QR| + |RP| = |PQ|$$

Points are collinear.
See the graph also.



Q.5. Find k, given that the points (2, k) is equidistant from (3, 7) and (9, 1)

Solution:-



Let $A(2, k); B(3,7); C(9, 1)$

We find $|AB|, |AC|$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

(Formula)

$$|AB| = \sqrt{(3-2)^2 + (7-k)^2}$$
$$= \sqrt{(1)^2 + (7-k)^2}$$
$$= \sqrt{1 + (7-k)^2} \quad (i)$$

and

$$|AC| = \sqrt{(9-2)^2 + (1-k)^2}$$
$$= \sqrt{(7)^2 + (1-k)^2}$$
$$= \sqrt{49 + (1-k)^2} \quad (ii)$$

According to the given condition

$$|AB| = |AC|$$
$$\sqrt{1 + (7-k)^2} = \sqrt{49 + (1-k)^2} \quad \text{(from i, ii)}$$

Or $1 + (7-k)^2 = 49 + (1-k)^2$

$$1 + 49 + k^2 - 14k = 49 + 1 + k^2 - 2k$$
$$-12k = 0$$
$$k = 0$$

Q.6. Use distance formula to verify that the points A(0, 7), B(3, −5), C(−2, 15) are collinear.

Solution:- $A(0, 7); B(3, -5); C(-2, 15)$ are given

We find $|CA|, |BC|, |AB|$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

(Formula)

$$|AB| = \sqrt{(3-0)^2 + [(-5) - 7]^2}$$
$$= \sqrt{(3)^2 + (-5) - 7]^2}$$
$$= \sqrt{(3)^2 + (-12)^2}$$

$$= \sqrt{9 + 144} = \sqrt{153}$$
$$= \sqrt{3 \times 3 \times 17} = 3\sqrt{17}$$

(i)

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

(Formula)

$$|BC| = \sqrt{(-2-3)^2 + [15-(-5)]^2}$$
$$= \sqrt{(-5)^2 + (15+5)^2}$$
$$= \sqrt{25 + (20)^2}$$
$$= \sqrt{25 + 400} = \sqrt{425}$$
$$= \sqrt{5 \times 5 \times 17} = 5\sqrt{17}$$

(ii)

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

(Formula)

$$|CA| = \sqrt{[0-(-2)]^2 + (7-15)^2}$$
$$= \sqrt{(2)^2 + (-8)^2}$$
$$= \sqrt{4 + 64} = \sqrt{68}$$
$$= \sqrt{2 \times 2 \times 17} = 2\sqrt{17}$$

(iii)

$$|AB| + |CA| = 3\sqrt{17} + 2\sqrt{17}$$ (from i + iii)

$$= \sqrt{17}\,(3+2)$$
$$= 5\sqrt{17}$$

$$|BC| = 5\sqrt{17}$$ (from ii)

$$\therefore |AB| + |CA| = |BC|$$

Therefore $A(0, 7)$, $B(3, -5)$ and $C(-2, 15)$ are collinear.

Q.7: ***Verify whether or not the points $O(0, 0)$, $A(\sqrt{3}, 1)$, $B(\sqrt{3}, -1)$ are the vertices of an equilateral triangle.***

Solution:-

We find the lengths of the sides of the triangle

$$O(0, 0);\ A(\sqrt{3}, 1);\ B(\sqrt{3}, -1)$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ (Formula)

We find

$|AB| = \sqrt{(\sqrt{3} - \sqrt{3})^2 + [(-1) - (-1)]^2}$

$= \sqrt{0 + (-2)^2}$

$= \sqrt{4} = 2$ (i)

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ (Formula)

$|OB| = \sqrt{(\sqrt{3} - 0)^2 + (-1 - 0)^2}$

$= \sqrt{(\sqrt{3})^2 + (-1)^2}$

$= \sqrt{3 + 1} = \sqrt{4} = 2$ (ii)

$|OA| = \sqrt{(\sqrt{3} - 0)^2 + (1 - 0)^2}$

$= \sqrt{(\sqrt{3})^2 + (1)^2}$

$= \sqrt{3 + 1} = \sqrt{4} = 2$ (iii)

$|AB| = |OB| = |OA| = 2$ (From i, ii, iii)

The lengths of the sides of the triangle are equal therefore, the triangle in equilateral.

Q.8. Show that the points $A(-6, -5)$, $B(5, -5)$, $C(5, -8)$ and $D(-6, -8)$ are the vertices of rectangle. Find the lengths of the diagonals. Are they equal?

Solution:-

Points are: $D(-6, -8)$, $C(5, -8)$, $B(5, -5)$, $A(-6, -5)$

We find $|DA|$, $|CD|$, $|BC|$, $|AB|$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ (Formula)

$|AB| = \sqrt{[(5) - (-6)]^2 + [(-5) - (-5)]^2}$

$$= \sqrt{(5+6)^2 + (-5+5)^2}$$

$$= \sqrt{(11)^2 + 0} = \sqrt{11^2} = 11$$

(i)

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

(Formula)

$$|BC| = \sqrt{(5-5)^2 + [(-8) - (-5)]^2} =$$

$$= \sqrt{0 + (-8+5)^2}$$

$$= \sqrt{(-3)^2} = \sqrt{9} = 3$$

(ii)

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

(Formula)

$$|CD| = \sqrt{[(-6) - (5)]^2 + [(-8) - (-8)]^2}$$

$$= \sqrt{(-6-5)^2 + (-8+8)^2}$$

$$= \sqrt{(-11)^2} = \sqrt{11^2} = 11$$

(iii)

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

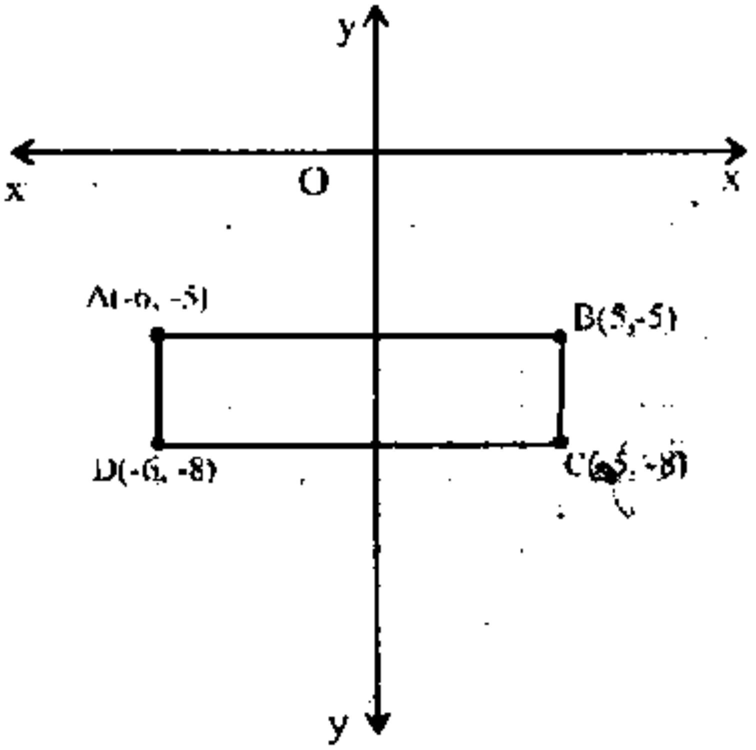
(Formula)

$$|DA| = \sqrt{[(-6) - (-6)]^2 + [(-5) - (-8)]^2}$$

$$= \sqrt{(-6+6)^2 + (-5+8)^2}$$

$$= \sqrt{0 + (3)^2} = \sqrt{3^2} = 3$$

(iv)



$$|\overline{AB}| = |CD| \quad \text{(From i, iii)}$$
$$|BC| = |DA| \quad \text{(From ii, iv)}$$

Lengths of opposite sides are equal we find the lengths of the diagonals.

$$|AC| = \sqrt{[5-(-6)]^2 + [(-8)-(-5)]^2}$$
$$= \sqrt{(5+6)^2 + (-8+5)^2}$$
$$= \sqrt{(11)^2 + (-3)^2} = \sqrt{121+9} = \sqrt{130}$$

$$|AC|^2 = 130 \quad (iv)$$
$$|AB|^2 = (11)^2 = 121 \quad (v)$$
$$|BC|^2 = (3)^2 = 9 \quad (vi)$$
$$121 + 9 = 130 \quad \text{(From iv, v and vi)}$$

$$|AB|^2 + |BC|^2 = |AC|^2 = 130$$

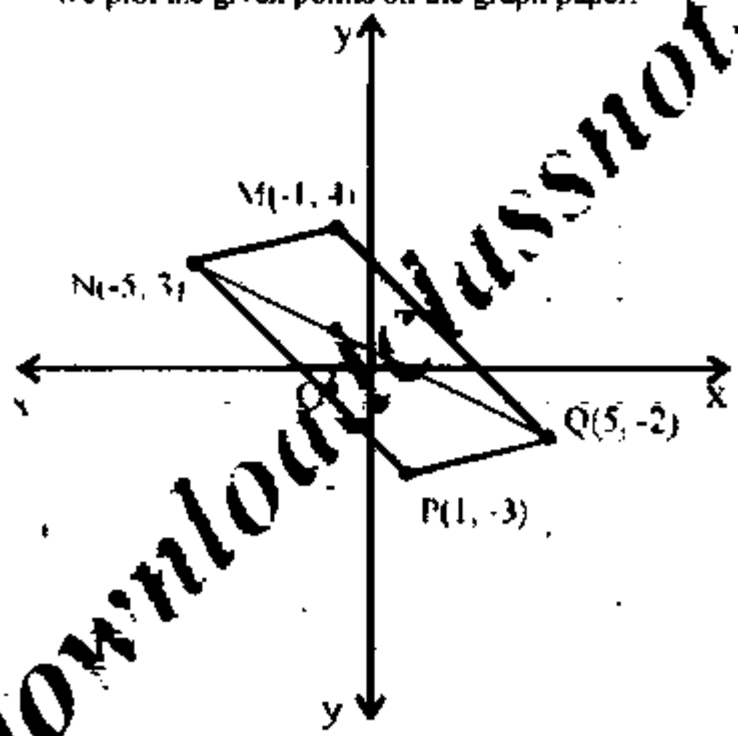
ΔABC is a right angled triangle $m\angle B = 90^O$

Similarly all the angle of ABCD are of 90^O. Therefore point A, B, C, D are the vertices of a rectangle.

Q.9. Show that the points M(− 1, 4), N(− 5, 3), p(1, − 3) and Q(5, − 2) are the vertices of a parallelogram.

Solution:-

We plot the given points on the graph paper.



We find the lengths of the sides with distance formula.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$ (Formula)

$$|NM| = \sqrt{(-1) - (-5)^2 + [(4-3)]^2}$$
$$= \sqrt{(-1+5)^2 + (1)^2}$$
$$= \sqrt{(4)^2 + 1} = \sqrt{16+1} = \sqrt{17}$$

(I)

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ (Formula)

$|PQ| = \sqrt{(5-1)^2 + [(-2)-(-3)]^2}$

$= \sqrt{(4)^2 + (-2+3)^2}$

$= \sqrt{16 + 1^2} = \sqrt{16+1} = \sqrt{17}$ (ii)

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ (Formula)

$|MQ| = \sqrt{[5-(-1)]^2 + [(-2)-(4)]^2}$

$= \sqrt{(5+1)^2 + (-2-4)^2}$

$= \sqrt{(6)^2 + (-6)^2} = \sqrt{36+36} = \sqrt{72}$ (iii)

$|NP| = \sqrt{[1-(-5)]^2 + [(-3)-(3)]^2}$

$= \sqrt{(1+5)^2 + (-3-3)^2}$

$= \sqrt{(6)^2 + (-6)^2} = \sqrt{36+36} = \sqrt{72}$ (iv)

$|NM| = |PQ|\ ;\ |MQ| = |NP|$

(From i, ii, iii and iv)

Lengths of opposite sides are equal.

We find the length of the diagonal $\overline{NQ}$,

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ (Formula)

$|NQ| = \sqrt{[(5)-(-5)]^2 + [(-2)-(3)]^2}$

$= \sqrt{(5+5)^2 + (-2-3)^2}$

$= \sqrt{(10)^2 + (-5)^2} = \sqrt{100+25} = \sqrt{125}$

Now in Δ NPQ

$|NQ|^2 = 125;\ |NP|^2 = 72;\ |PQ|^2 = 17$

and $|NP|^2 + |PQ|^2 = 72 + 17 = 89$

thus $|NQ|^2 \neq |NP|^2 + |PQ|^2$

$$M\left(0, \frac{-5}{2}\right)$$

Q.2. The end point P of a line segment PQ is (−3, 6) and its end point is (5, 8). Find the coordinate of the end point Q.

Solution:

Let Q (x, y) be the other end point.
Mid-point is R (5, 8)
One end point is P (−3, 6)

$$M(x, y) = M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \text{ (Mid-point Formula)}$$

$$\therefore \quad 5 = \frac{-3 + x}{2}$$

$$10 = -3 + x$$

$$x = 13$$

and $$8 = \frac{6 + y}{2}$$

$$16 = 6 + y$$

$$y = 10$$

thus $Q(x, y) = Q(13, 10)$

Q.3. Prove that mid-point of the hypotenuse of a right triangle is equidistant from its three vertices P (−2, 5), Q (1, 3) and R (−1, 0)

Solution: Given points are

R (−1, 0), Q (1, 3), P (−2, 5)

We find the lengths of the sides of the triangle.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{(Formula)}$$

$$|RQ| = \sqrt{[1 - (-1)]^2 + (3 - 0)^2}$$

$$= \sqrt{(1 + 1)^2 + (3)^2}$$

$$= \sqrt{(2)^2 + (3)^2}$$

$$= \sqrt{4 + 9} = \sqrt{13} \quad \text{(i)}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{(Formula)}$$

$$|QP| = \sqrt{[(-2) - (1)]^2 + [5 - 3]^2}$$

$$= \sqrt{(-2 - 1)^2 + (2)^2}$$

$= \sqrt{(-3)^2 + (2)^2}$

$= \sqrt{9+4} = \sqrt{13}$ (ii)

$|RP| = \sqrt{[(-2)-(-1)]^2 + [5-0]^2}$

$|RP| = \sqrt{(-2+1)^2 + (5)^2}$

$= \sqrt{(-1)^2 + (5)^2}$

$= \sqrt{1+25} = \sqrt{26}$ (iii)

$|QP|^2 + |RQ|^2 = 13 + 13$ (From i, ii)

$= 26$ (iv)

$|RP|^2 = 26$ (v)

$|RP|^2 = |QP|^2 + |RQ|^2$ (From iv, v)

Thus hypotenuse is $|RP|$ where R $(-1, 0)$ and $P(-2, 5)$ are its end point.

Let M (x, y) be end point of $\overline{RP}$.

$(x,y) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ (Mid-point Formula)

$M(xy) = M\left(\frac{-2-1}{2}, \frac{5+0}{2}\right)$

$= M\left(\frac{-3}{2}, \frac{5}{2}\right)$

Now, we find distance of $M\left(\frac{-3}{2}, \frac{5}{2}\right)$ from the three vertices.

We find $|MP|, |MQ|, |MR|$

$M\left(\frac{-3}{2}, \frac{5}{2}\right)$; R $(-1, 0)$, Q $(1, 3)$, P $(-2, 5)$

$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$ (Formula)

$|MP| = \sqrt{[(-2)-(\frac{-3}{2})]^2 + (5-\frac{5}{2})^2}$

$= \sqrt{(-2+\frac{3}{2})^2 + (\frac{10-5}{2})^2}$

$= \sqrt{(\frac{-4+3}{2})^2 + (\frac{10-5}{2})^2}$

$$= \sqrt{\left(\frac{-1}{2}\right)^2 + \left(\frac{5}{2}\right)^2}$$

$$= \sqrt{\frac{1}{4} + \frac{25}{4}} = \sqrt{\frac{26}{4}} = \frac{1}{2}\sqrt{26} \quad \text{(A)}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{(Formula)}$$

$$|MQ| = \sqrt{\left(1 + \frac{3}{2}\right)^2 + \left(3 - \frac{5}{2}\right)^2}$$

$$= \sqrt{\left(\frac{2+3}{2}\right)^2 + \left(\frac{6-5}{2}\right)^2}$$

$$= \sqrt{\left(\frac{5}{2}\right)^2 + \left(\frac{1}{2}\right)^2}$$

$$= \sqrt{\frac{25}{4} + \frac{1}{4}} = \sqrt{\frac{26}{4}} = \frac{1}{2}\sqrt{26} \quad \text{(B)}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{(Formula)}$$

$$|MR| = \sqrt{\left(-1 + \frac{3}{2}\right)^2 + \left(0 - \frac{5}{2}\right)^2}$$

$$= \sqrt{\left(\frac{-2+3}{2}\right)^2 + \left(\frac{-5}{2}\right)^2}$$

$$= \sqrt{\left(\frac{1}{2}\right)^2 + \frac{25}{4}}$$

$$= \sqrt{\frac{1}{4} + \frac{25}{4}} = \sqrt{\frac{26}{4}} = \frac{1}{2}\sqrt{26} \quad \text{(C)}$$

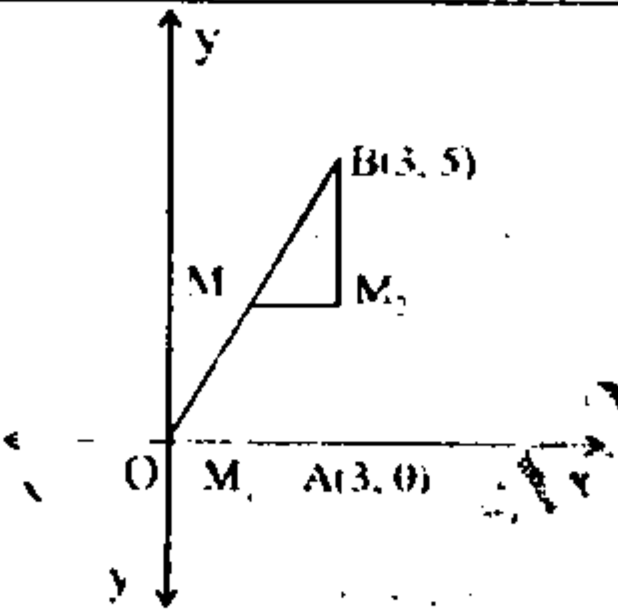
$$|MP| = |MQ| = |MR| = \frac{1}{2}\sqrt{26} \quad \text{(From A, B, C)}$$

M is equidistant from P, Q, R.

Q.4. If O(0, 0), A (3, 0) and B(3, 5) are three points in the plane, find M_1 and M_2 as

mid-points of the line segments AB and $\overline{OB}$ respectively. Find M_1, M_2:

Solution: A (3, 0), O(0, 0) and B(3, 5) are the vertices of aΔ. M1 is mid-point of $\overline{OB}$.



$$M(x, y) = M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \quad \text{(Formula)}$$

$$M_1\left(\frac{0 + 3}{2}, \frac{0 + 5}{2}\right)$$

$$M_1\left(\frac{3}{2}, \frac{5}{2}\right)$$

M_2 is mid-point of $\overline{AB}$.

Therefore: $M_2\left(\frac{3 + 3}{2}, \frac{0 + 5}{2}\right)$

$$= M_2\left(\frac{6}{2}, \frac{5}{2}\right)$$

$$= M_2\left(3, \frac{5}{2}\right)$$

Now $\left(\frac{3}{2}, \frac{5}{2}\right)$ and $\left(3, \frac{5}{2}\right)$ are mid points. We find $|M_1M_2|$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{(Formula)}$$

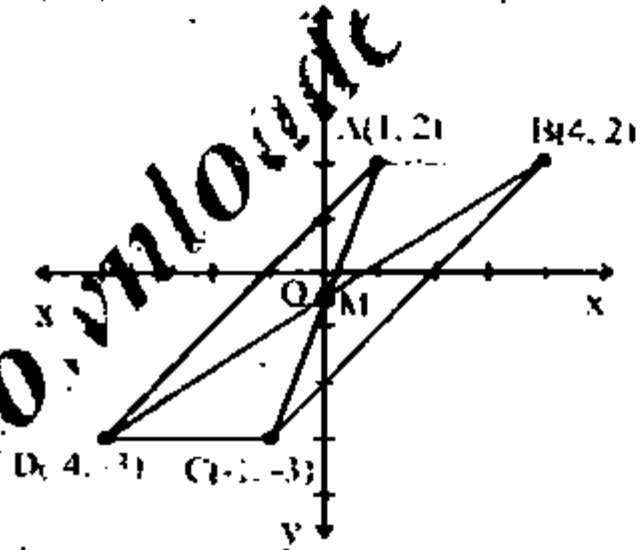
$$|M_1M_2| = \sqrt{\left(3-\frac{3}{2}\right)^2 + \left(\frac{5}{2}-\frac{5}{2}\right)^2}$$
$$= \sqrt{\left(\frac{6-3}{2}\right)^2 + 0}$$
$$= \sqrt{\left(\frac{3}{2}\right)^2} = \frac{3}{2}$$

Q.5. ***Show that the diagonals of the parallelogram having vertices A (1, 2), B (4, 2), C (– 1, – 3) and D (– 4, – 3) bisect each other.***

Solution: ABCD is a parallelogram with vertices A(1, 2), B(4, 2), C(–1, – 3) and

D (– 4, – 3). Suppose diagonals $\overline{DB}$ and $\overline{AC}$ intersect at M.

A (1, 2) and C (–1, –3) , we find mid-point of $\overline{AC}$



Mid point is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ **Formula**

Mid-point of AC $=\left(\frac{-1+1}{2}, \frac{-3+2}{2}\right)$

$$= \left(0, -\frac{1}{2}\right) \qquad \text{(i)}$$

Now B (4, 2), and D (− 4, − 3) are end points of the second diagonal $\overline{BD}$.

$$\text{Mid-point of } \overline{BD} = \left(\frac{-4+4}{2}, \frac{-3+2}{2}\right)$$

$$= \left(0, \frac{-1}{2}\right) \qquad \text{(ii)}$$

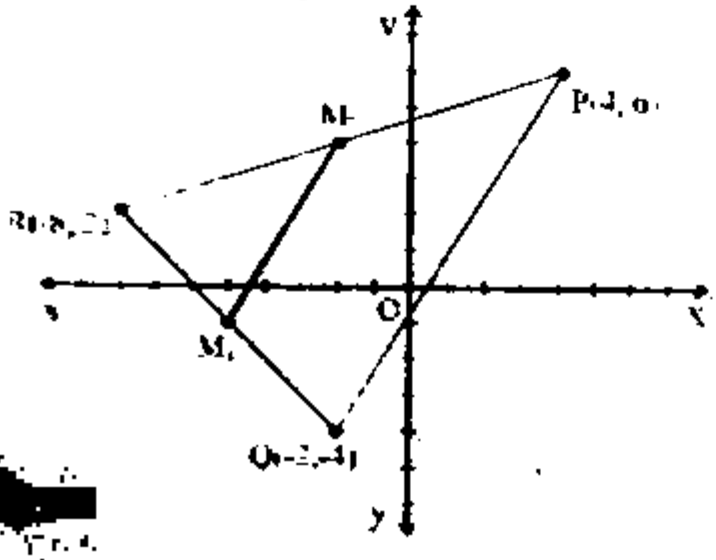
Mid-point of diagonal $\overline{AC}$ and $\overline{BD}$ is the same.

therefore, diagonals $\overline{AC}$ and $\overline{BD}$ bisect each other.

Q.6. ***The vertices of a triangle are P (4, 6) Q (−2, −4) and R (−8, 2). Show that the length of the line segment joining the mid-points of the line segment $\overline{PR}$, $\overline{QR}$ is $\frac{1}{2}$ $\overline{PQ}$.***

Solution: Vertices of ΔPQR are P(4, 6), Q (−2, −4) and R (−8, 2)

Let M_1 be mid-point of $\overline{RQ}$



Then M_1 is

$$M_1\left(\frac{-8-2}{2},\frac{2-4}{2}\right)\quad\left(\text{Formula } \frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

$$= M_1\left(\frac{-10}{2},\frac{-2}{2}\right)$$

$$= M_1\ (-5,-1)$$

M_2 is mid-point of $\overline{RP}$ then M_2 is

$$M_2\left(\frac{-8+4}{2},\frac{2+6}{2}\right)$$

$$= M_2\left(\frac{-4}{2},\frac{8}{2}\right)$$

$$= M_2\ (-2,4)$$

Now we use distance formula

$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \text{ to find } |PQ|$$

P (4, 6), Q (−2, −4)

$$|PQ| = \sqrt{(-2-4)^2+(-4-6)^2}$$

$$= \sqrt{(-6)^2+(-10)^2}$$

$$= \sqrt{36+100} = \sqrt{136}$$

$$= \sqrt{2\times2\times34} = 2\sqrt{34} \quad \textbf{(A)}$$

and $M_2(-2,4)$ & $M_1(-5,-1)$

Therefore:

$$|M_1M_2| = \sqrt{(-2+5)^2+(4+1)^2}$$

$$= \sqrt{(3)^2+(5)^2}$$

$$= \sqrt{9+25} = \sqrt{34} \quad \textbf{(B)}$$

$$\therefore |M_1M_2| = \frac{1}{2}|PQ| \quad \text{(From A and B)}$$

Review Exercise 9

Q.1. Choose the correct answer.

(i) Distance between points (0, 0) and (1, 1) is:
- (a) 0 (b) 1
- (c) 2 (d) $\sqrt{2}$

(ii) Distance between the points (1, 0) and (0, 1) is:
- (a) 0 (b) 1
- (c) $\sqrt{2}$ (d) 2

(iii) Mid-point of the points (2, 2) and (0, 0) is:
- (a) (1, 1) (b) (1, 0)
- (c) (0, 1) (d) (−1, −1)

(iv) Mid-point of the points (2, −2) and (−2, 2) is:
- (a) (2, 2) (b) (−2, −2)
- (c) (0, 0) (d) (1, 1)

(v) A triangle having all sides equal is called:
- (a) Isosceles (b) Scalene
- (c) Equilateral (d) None of these

(vi) A triangle having all sides equal is called:
- (a) Isosceles (b) Scalene
- (c) Equilateral (d) None of these

Answers:

(i)	d	(ii)	c	(iii)	a
(iv)	c	(v)	c	(vi)	b

2. Answers the following, which is true and which is false.

(i) A line has two end points.
(ii) A line segments has one end point.
(iii) A triangle is formed by three collinear points.
(iv) Each side of a triangle has two collinear vertices.
(v) The end points of each side of rectangle are collinear.
(vi) All the points that lie on the x-axis are collinear.
(vii) Origin is the only point collinear with the points of both the axes separately.

Answers:

(i)	F	(ii)	F	(iii)	F	(iv)	T
(v)	T	(vi)	T	(vii)	T		

3. Find the distance between the following pairs of points.

(i) (6, 3), (3, −3) (ii) (7, 5), (1, −1)

(iii) (0, 0), (−4, −3)

3(i) Let P(6, 3) Q (3, −3)

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$|PQ| = \sqrt{(3-6)^2 + (-3-3)^2}$$

$$= \sqrt{(-3)^2 + (-6)^2}$$

$$= \sqrt{9+36} = \sqrt{45}$$

$$= \sqrt{3\times3\times5} = 3\sqrt{5} \text{ Units}$$

3(ii) (7, 5), (1, −1)

Let A (7, 5), B (1, −1)

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$|AB| = \sqrt{(1-7)^2 + (-1-5)^2}$$

$$= \sqrt{(-6)^2 + (-6)^2}$$

$$= \sqrt{36+36} = \sqrt{72}$$

$$= \sqrt{6\times6\times2} = 6\sqrt{2} \text{ Units}$$

3(iii) (0, 0), (−4, −3)

Let O (0, 0), Q (−4, −3)

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$|OQ| = \sqrt{(-4-0)^2 + (-3-0)^2}$$

$$= \sqrt{(-4)^2 + (-3)^2}$$

$$= \sqrt{16+9} = \sqrt{25} = 5 \text{ Units}$$

4. Find the mid-point between following pairs of points.

(i) (6, 6), (4, −2) (ii) (−5, −7), (−7, −5)

(iii) (8, 0), (0, −12)

Solutions:-

4(i) Let M be mid-point of P(6, 6), (4, −2)

Mid point formula $\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}$

Thus $= M\left(\frac{6+4}{2}, \frac{6-2}{2}\right)$

$= M\left(\frac{10}{2}, \frac{4}{2}\right)$

= M (5, 2) is the mid-point.

4(ii) **Let M be mid-point of B(−7,−5), A(−5, −7)**

Mid point formula $\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}$

Thus $= M\left(\frac{-5-7}{2}, \frac{-7-5}{2}\right)$

$= M\left(\frac{-12}{2}, \frac{-12}{2}\right)$

= M (−6, −6)

4(iii) **Points P(8, 0) and Q(1, −12)**

Let M be the mid-point of P(8, 0) and Q(1, −12)

Mid point formula $\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}$

Thus mid-point M is $\left(\frac{8+0}{2}, \frac{0-12}{2}\right)$

$= \left(\frac{8}{2}, \frac{-12}{2}\right)$

$= (4, -6)$

5. **Define the following**

(i) Co-ordinate Geometry (ii) Collinear
(iii) Non-collinear (iv) Equilateral Triangle
(v) Scalene Triangle (vi) Isosceles Triangle
(vii) Right Triangle (viii) Square

Definitions:

(i) ***Co-ordinate Geometry***

Co-ordinate geometry is the study of geometrical shapes in the Cartesian plane (co-ordinate plane)

(ii) ***Collinear Points***

Two or more than two points which lie the same straight line are called collinear points w.r.t that line.

(iii) ***Non-collinear***

Three or more than three points that do not lie on a straight line are called non-collinear points.

(iv) ***Equilateral Triangle***

A triangle having lengths of its three sides the same is called an equilateral triangle.

(v) ***Scalene Triangle***

If measures of three sides of a triangle are different then the triangle is called a scalene triangle.

(vi) ***Isosceles Triangle***

If lengths of two sides of a triangle is equal and length of the third side is different then that triangle is called an isosceles triangle.

(vii) ***Right Triangle***

A triangle having one of its angle as a right angle (90^0) is called a right angled triangle.

(viii) ***Square***

A figure formed by four non-collinear points in the plane is called a square if:

(i) All its sides have equal lengths.

(ii) Measure of each angle is 90^0.

Unit 10

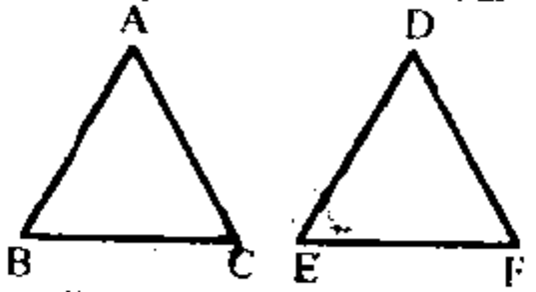
CONGRUENT TRIANGLES

Congruent Triangles:

Two triangles are said to be congruent if at least one (1 - 1) correspondence can be established between them in which the corresponding sides and angles are congruent.

For example:

If in the correspondence Δ ABC $\leftrightarrow$ Δ DEF



(i) $\angle A \cong D$ *(ii)* $\angle B \cong \angle E$

(iii) $\angle C \cong \angle F$ *(iv)* $\overline{BC} \cong \angle EF$

(v) $\overline{CA} \cong \overline{FD}$ *(vi)* $\overline{AB} \cong DE$

Then Δ ABC $\cong$ Δ DEF

In such a case we can also say that the correspondence Δ ABC $\leftrightarrow$ Δ DEF is a congruence.

Side Angle Side Postulate (S.A.S)

If in a given correspondence of two triangles, the two sides and the included angle of the one triangle are congruent to the corresponding two sides and the included angle of the other, the triangles are congruent.

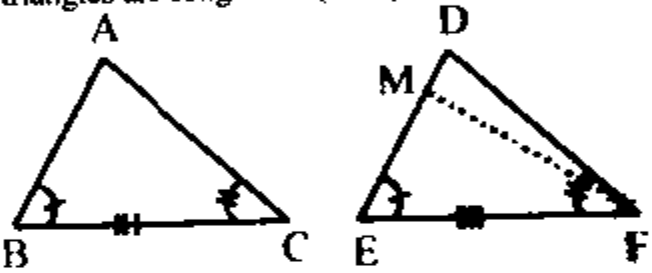
In Δ LMN $\leftrightarrow$ Δ PQR

If $\overline{LM} \cong \overline{PQ}$, $m\angle M = m\angle Q$, $\overline{MN} \cong \overline{QR}$

Then Δ LMN $\cong$ Δ PQR.

Theorem

In any correspondence of two triangles, if one side and any two angles of one triangle are congruent to the corresponding side and angles of the other, then the triangles are congruent. (A.S.A ≅ A.S.A)



Given

In $\Delta ABC \leftrightarrow \Delta DEF$

$\angle B \cong \angle E$, $\overline{BC} \cong \overline{EF}$, $\angle C \cong \angle F$

To Prove

$\Delta ABC \leftrightarrow \Delta DEF$

Construction

Suppose $\overline{AB} \ncong \overline{DE}$, take a point M on $\overline{DE}$ such that $\overline{AB} \cong \overline{ME}$. Join M to F

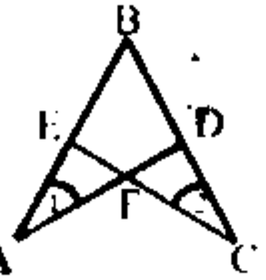
Proof:

Statements	Reasons
In $\Delta ABC \longleftrightarrow \Delta MEF$	
$\overline{AB} \cong \overline{ME}$ ......(i)	Construction
$\overline{BC} \cong \overline{EF}$ ......(ii)	Given
$\angle B \cong \angle E$......(iii)	Given
$\therefore$ $\Delta ABC \cong \Delta MEF$	S.A.S postulate
So, $\angle C \cong \angle MFE$	(Corresponding angles of congruent triangles)
But $\angle C \cong \angle DFE$	Given Both congruent to $\angle C$

∴ $\angle DFE \cong \angle MFE$ This is possible only if D and M are the same points. and $\overline{AB} \cong \overline{DE}$	$\overline{AB} \cong \overline{ME}$ (construction) and $\overline{ME} \cong \overline{DE}$ (proved)
So. $\overline{AB} \cong \overline{DE}$ .....(iv) Thus from (ii), (iii) and (iv), we have $\Delta ABC \cong \Delta DEF$	S.A.S. postulate

Exercise 10.1

Q.1. ***In the given figure $\overline{AB} \cong \overline{CD}$ $\angle 1 \cong \angle 2$ prove that $\Delta ABD \cong \Delta CBE$.***



Given:
In the given figure

$$\overline{AB} \cong \overline{CB}$$
$$\angle 1 \cong \angle 2$$

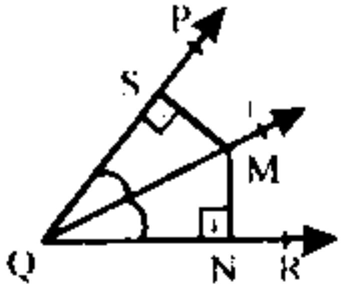
To Prove: $\Delta ABC \cong \Delta CBE$

Proof:

Statements	Reasons
In Δ^s ABC and CBE	
$\overline{AB} \cong \overline{CD}$	Given
$\angle A \cong \angle C$	Given
And $\angle B \cong \angle B$	Common
∴ $\Delta ABC \cong \Delta CBE$	S.A.S postulate

Q.2. From a point on the bisector of an angle perpendiculars are drawn to the arms of the angle. Prove that these perpendiculars are equal in measure.

Solution:



Given:

$\angle PQR$ is given $\overrightarrow{QT}$ is bisector of $\angle PQR$ i.e: $\angle 1 \cong \angle 2$

(i) M is a point on $\overrightarrow{QT}$

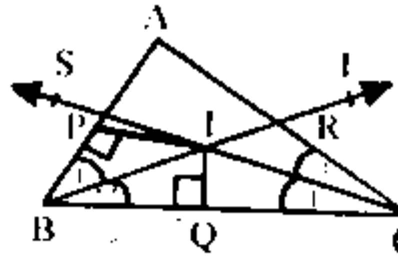
(ii) $\overline{MN} \perp \overline{QR}$, $\overline{MS} \perp \overline{QP}$
i.e: $\angle 3 \cong \angle 4$ (each 90°)

To Prove: $\overline{MN} \cong \overline{MS}$

Proof:

Statements	***Reasons***
$\Delta QMS \leftrightarrow \Delta QMN$	
$\angle 1 \cong \angle 2$	Given
$\angle 3 \cong \angle 4$	Given
$\overline{QM} \cong \overline{QM}$	Common
$\therefore \Delta QMS \cong \Delta QMN$	S.A.S postulate
$\overline{MN} \cong \overline{MS}$	Corresponding sides of congruent triangles

Q.3. ***In a triangle ABC the bisectors of $\angle B$ and $\angle C$ meet in a point I. Prove that I is equidistant from the three sides of ΔABC.***



Given:
In triangle ABC

(i) $\overrightarrow{BI}$ is bisector of $\angle B$ i.e $\angle 1 \cong \angle 2$

(ii) $\overrightarrow{CS}$ is bisector of $\angle C$ i.e $\angle 3 \cong \angle 4$

(iii) I is point of intersection of $\overrightarrow{BI}$ and $\overrightarrow{CS}$

(iv) $\overrightarrow{IP} \perp \overrightarrow{AB}$, $\overrightarrow{IQ} \perp \overrightarrow{BC}$, $\overrightarrow{IR} \perp \overrightarrow{AC}$

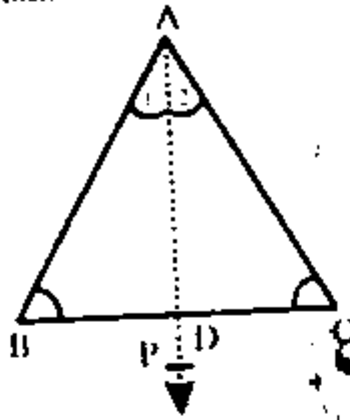
To Prove: $\overline{IP} \cong \overline{IQ} \cong \overline{IR}$

Proof:

Statements	***Reasons***
$\Delta IPB \leftrightarrow \Delta IQB$	
$\angle 1 \cong \angle 2$	Given
$\angle P \cong \angle Q$	Each 90°
$\overline{BI} \cong \overline{BI}$	Common
$\Delta IPB \cong \Delta IQB$	S.A.S postulate
(iv) ∴ $\overline{IP} \cong \overline{IQ}$	Corresponding sides of congruent triangles
From IRC, IQC	
(v) $\overline{IQ} \cong \overline{IR}$	From (iv) and (v)
$\overline{IP} \cong \overline{IQ} \cong \overline{IR}$	

Theorem

If two angles of a triangle are congruent then the sides opposite in them are also congruent.



Given: In ΔABC, $\angle B \cong \angle C$

To Prove: $\overline{AB} \cong \overline{AC}$

Construction:

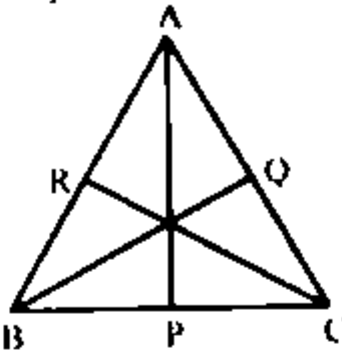
Draw $\overrightarrow{AP}$ bisector of angle A i.e. $\angle 1 \cong \angle 2$. $\overrightarrow{AP}$ cuts $\overline{BC}$ at D.

Proof:

Statements	Reasons
In $\Delta ABD \leftrightarrow \Delta ACD$	
$\angle A \cong \angle B \cong \angle C$	Given
$\angle 1 \cong \angle 2$	Construction
$\overline{AD} \cong \overline{AD}$	Common
$\therefore \Delta ABD \cong \Delta ACD$	S.A.S postulate
$\overline{AB} \cong \overline{AC}$	Corresponding sides of congruent triangles

Exercise 10.2

Q.1. Prove that any two medians of an equilateral triangle are equal in measure.



Given: In ΔABC

(i) $\overline{AB} \cong \overline{BC} \cong \overline{CA}$ points P, Q, R are respectively mid points of $\overline{BC}$, $\overline{CA}$ and $\overline{AB}$ i.e; $m\overline{AR} = m\overline{BR} = \frac{1}{2}m\overline{AB}$

(ii) $m\overline{BP} = m\overline{CP} = \frac{1}{2}m\overline{BC}$

$m\overline{CQ} = m\overline{AQ} = \frac{1}{2}m\overline{CA}$

Thus $\overline{AR} \cong \overline{BR} \cong \overline{BP} \cong \overline{CP} \cong \overline{CQ} \cong \overline{AQ}$ (From i, ii)

Medians are $\overline{CR}$, $\overline{BQ}$, $\overline{AP}$

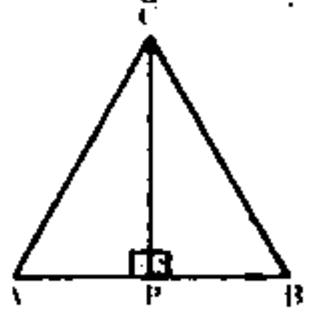
To Prove: $\overline{AP} \cong \overline{BQ} \cong \overline{CR}$

Proof:

Statements	***Reasons***
In ΔABC	Given
$\overline{AB} \cong \overline{BC} \cong \overline{CA}$	Opposite angles of equal sides
$\angle ABC \cong \angle BCA \cong \angle CAB$	
Now $\Delta BCR \leftrightarrow \Delta CBQ$	
$\overline{BC} \cong \overline{BC}$	Common

$\angle ABC \cong \angle ACB$	Proved
$\overline{BR} \cong \overline{CQ}$	Given
$\therefore \Delta BCR \cong \Delta CBQ$	S.A.S $\cong$ S.A.S
Thus $\overline{CR} \cong \overline{BQ}$	Corresponding sides of congruent triangles
Similarly $\overline{BQ} \cong \overline{AP}$	
$\overline{AP} \cong \overline{BQ} \cong \overline{CR}$	

Q.2. ***Prove that a point, which is equidistant from the end points of a line segment is on the right bisector of the line segment.***



Given:

$\overline{AB}$ is a line-segment. There is a point C that $\overline{CA} \cong \overline{CB}$

To Prove:

Point C lies on the right bisector of $\overline{AB}$

Construction:

(i) Take P as mid-point of $\overline{AB}$ i.e. $\overline{AP} \cong \overline{BP}$

(ii) Join point C to A, P, B.

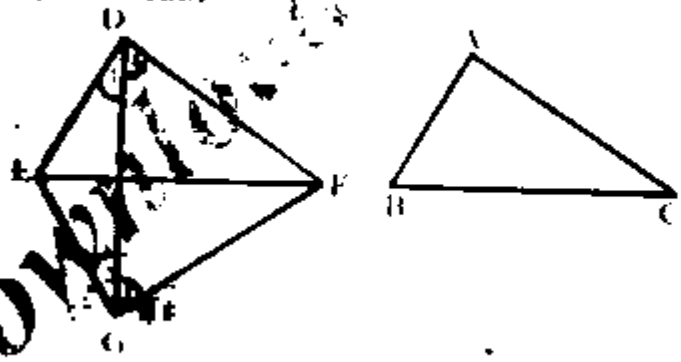
Proof:

Statements	*Reasons*
In ΔABC	
$\overline{CA} \cong \overline{CB}$	Given
$\angle A \cong \angle B$	Corresponding angles of congruent Δ's

Now $\Delta CAP \leftrightarrow \Delta CBP$		
	$\overline{CA} \cong \overline{CB}$	Given
	$\angle A \cong \angle B$	Proved
	$\overline{AP} \cong \overline{BP}$	Construction
Thus	$\Delta CAP \cong \Delta CBP$	S.A.S ≅ S.A.S
(i)	$\therefore \angle 1 \cong \angle 2$	
(ii)	$m\angle 1 + m\angle 2 = 180^\circ$	Adj. angles on one side of a line
Thus	$m\angle 1 = m\angle 2 = 90^\circ$	
Hence $\overline{CP}$ is right bisector of $\overline{AB}$ and point C lies on $\overline{CP}$		

Theorem

In a correspondence two triangles, if three sides of one triangle are congruent to the corresponding three sides of the other, then the two triangles are congruent (S.S.S ≅ SSS)



Given: In $\Delta ABC \leftrightarrow \Delta DEF$

$\overline{AB} \cong \overline{DE}$

$\overline{BC} \cong \overline{EF}$

$\overline{CA} \cong \overline{FD}$

To Prove: $\Delta ABC \cong \Delta DEF$

Construction:

Suppose the side $\overline{BC}$ is not shorter than any of the other two sides of the triangle ABC. Construct Δ GEF with G and D on opposite sides of $\overline{EF}$ such that $\overline{GE} \cong \overline{AB}$ and $\angle GEF \cong \angle B$.
Join G and D. Let the angles be named by the numbers as shown in the figure.

Proof:

Statements	*Reasons*
In $\Delta ABC \leftrightarrow \Delta GEF$	
$\overline{BC} \cong \overline{EF}$	Given
$\angle B \cong \angle GEF$	Construction
and $AB \cong GE$	Construction
Thus $\Delta ABC \cong \Delta GEF$	S.A.S ≅ S.A.S
Now in ΔEGD	
$\overline{DE} \cong \overline{GE}$	Each $\cong \overline{AB}$
$\angle 2 \cong \angle 1$ ....(i)	Opposite angles of congruent sides
Thus $\angle 3 \cong \angle 4$ ....(ii)	
$m\angle 2 + m\angle 3 = m\angle 1 + m\angle 4$	From(i)+(ii)
or $m\angle G = m\angle D$	Addition postulate of angles
i.e. $\angle D \cong \angle G$	
Now In $\Delta DEF \leftrightarrow \Delta GEF$	From (iv) and (v)
$\overline{DE} \cong \overline{GE}$	Each $\cong \overline{AB}$
$\angle D \cong \angle G$	Proved
$\overline{DF} \cong \overline{GF}$	Each $\cong \overline{CA}$
$\Delta DEF \cong \Delta DEF$	S.A.S ≅ S.A.S
But $\Delta ABC \cong \Delta GEF$	Transitive property of congruency.
$\Delta ABC \cong \Delta DEF$	

Exercise 10.3

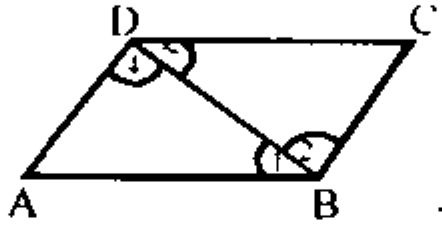
Q.1. In the figure

$$\overline{AB} \cong \overline{DC}$$

$$\overline{AD} \cong \overline{BC}$$

Prove that $\angle A \cong \angle C$

And $\angle ABC \cong \angle ADC$



Given: In the given figure

$$\overline{AB} \cong \overline{DC}$$

$$\overline{AD} \cong \overline{BC}$$

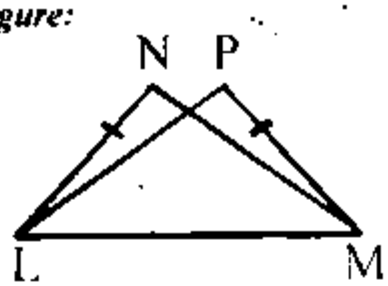
To Prove: $\angle A \cong \angle C$

$\angle ABC \cong \angle ADC$

Proof:

Statements	***Reasons***
In $\Delta ABD \leftrightarrow \Delta CDB$	
$\overline{AB} \cong \overline{DC}$	Given
$\overline{AD} \cong \overline{BC}$	Given
$\overline{BD} \cong \overline{BD}$	Common
$\therefore$ $\Delta ABD \cong \Delta CDB$	$S.S.S \cong S.S.S$
Hence. $\angle A \cong \angle C$	Corresponding angles of congruent Δ'
(i) $\angle 1 \cong \angle 3$	
$\angle 2 \cong \angle 4$	Corresponding angles of congruent Δ'
$m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	From (i)
or $m\angle ABC = m\angle ADC$	
$\angle ABC \cong \angle ADC$	

Q.2. In the figure:



$\overline{LN} \cong \overline{MN}$
$MN \cong LP$

Prove that $\angle N \cong \angle P$ (i)
and $\angle NML \cong \angle PLM$

Given: In the figure:

$\overline{LN} \cong \overline{MN}$
$MN \cong LP$

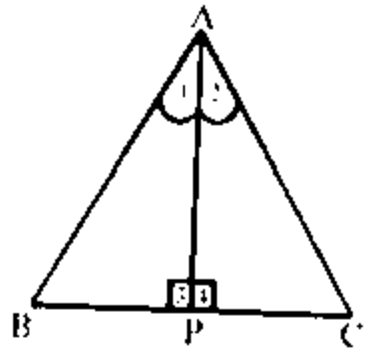
To Prove:

$\angle N \cong \angle P$ (i)
$\angle NML \cong \angle PLM$

Proof:

Statements	***Reasons***
In $\Delta LMN \leftrightarrow \Delta LMP$	
$\overline{LN} \cong \overline{MP}$	Given
$\overline{MN} \cong \overline{LP}$	Given
$\overline{LM} \cong \overline{LM}$	
$\therefore$ $\Delta LMN \cong \Delta LMP$	Common
Thus $\angle N \cong \angle P$ $\angle NML \cong \angle PLM$	S.A.S ≅ S.A.S

Q.3. ***Prove that the median bisecting the base of an isosceles triangle bisects the vertex angle and it is perpendicular to the base.***



Given: In ΔABC

(i) $\overline{AB} \cong \overline{AC}$

(ii) Point P is mid-point of $\overline{BC}$ i.e; $\overline{BP} \cong \overline{CP}$

(iii) P is joined to A. i.e. $\overline{AP}$ is median.

To Prove:

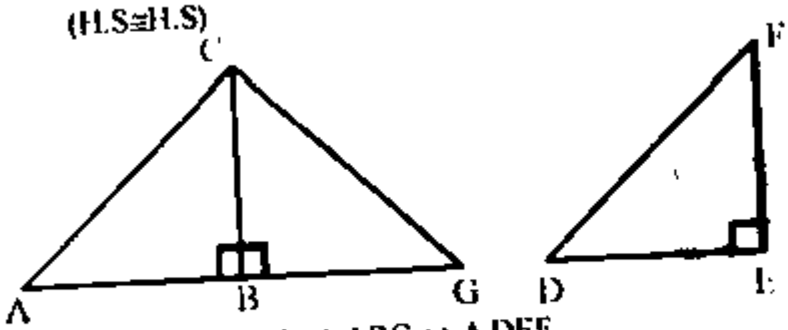
$$\angle 1 \cong \angle 2$$
$$\overline{AP} \perp \overline{BC}$$

Proof:

Statements	Reasons
In $\Delta ABP \leftrightarrow \Delta ACP$	
$\overline{AB} \cong \overline{AC}$	
$\overline{BP} \cong \overline{CP}$	Given
$\overline{AP} \cong \overline{AP}$	Given
$\therefore$ $\Delta ABP \cong \Delta ACP$	Common
Thus $\angle 1 \cong \angle 2$	S.A.S $\cong$ S.A.S
	Corresponding angles of congruent triangles
(i) $\angle 3 \cong \angle 4$	
But(ii) $m\angle 3 + m\angle 4 = 180^\circ$	Corresponding angles of congruent triangles
Thus $m\angle 3 = m\angle 4 = 90^\circ$	
therefore $\overline{AP} \perp \overline{BC}$	From (i) and (ii)

Theorem

If in the corresponding of the two right-angled triangles, the hypotenuse and one side of one triangle are congruent to the hypotenuse and the corresponding side of the other, then the triangles are congruent. (H.S≅H.S)



Given: In $\Delta\ ABC \leftrightarrow \Delta\ DEF$

$\angle ABC \cong \angle E$ (Right angle)

$\overline{AC} \cong \overline{DF}$ Hypotenuse

$\overline{BC} \cong \overline{EF}$

To Prove: $\Delta\ ABC \cong \Delta\ DEF$

Construction:

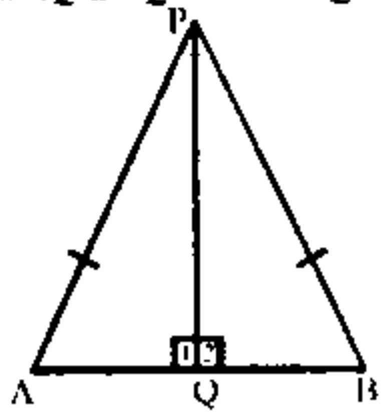
Produce $\overline{AB}$ towards B and cut off $\overline{BC} \cong \overline{DE}$

Statements	Reasons
In $\Delta\ GBC$	
$m\angle CBG = 90^\circ$	Supplementary of 90°
In $\Delta GBC \leftrightarrow \Delta DEF$	
$\overline{GB} \cong \overline{DE}$	Construction
$\angle GBE \cong \angle E$	Each 90°
$\overline{BC} \cong \overline{EF}$	Given
$\Delta\ GBC \cong \Delta\ DEF$	S.A.S ≅ S.A.S
$\overline{GC} \cong \overline{DF}$	Corresponding sides of congruent triangles
But $\overline{DF} \cong \overline{AC}$	Given
	Transitive property

Thus $\overline{GC} \cong \overline{AC}$	Corresponding angles of congruent sides
Now in Δ ACG	
$\angle A \cong \angle G$	
Now in $\Delta ABC \leftrightarrow \Delta GBC$	
	Proved
	Proved
$\overline{AC} \cong \overline{GC}$	Each 90°
$\angle A \cong \angle G$	S.A.A.≅S.A.A
$\angle ABC \cong \angle GBC$	Proved
Thus $\Delta ABC \cong \Delta GBC$	Transitive property of congruency.
But $\Delta GBC \cong \Delta DEF$	
Hence $\Delta ABC \cong \Delta DEF$	

Exercise 10.4

Q.1. In Δ PAB of figure, $\overline{PQ} \perp \overline{AB}$ and $\overline{PA} \cong \overline{PB}$, prove that $\overline{AQ} \cong \overline{BQ}$ and $\angle APQ \cong \angle BPQ$



Given: In Δ PAB

(i) $\overline{PA} \cong PB$

(ii) $\overline{PQ} \perp \overline{AB}$ i.e. $m\angle 1 = m\angle 2 = 90°$

To Prove: $\overline{AQ} \cong \overline{BQ}$

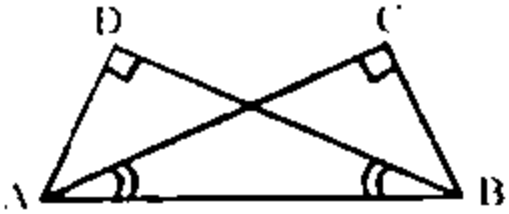
$\angle APQ \cong \angle BPQ$

Proof:

Statements	Reasons
In $\Delta APQ \leftrightarrow \Delta BPQ$	
$\angle 1 \cong \angle 2$	Given - each 90°
$\overline{PA} \cong \overline{PB}$	Given
$\overline{PQ} \cong \overline{PQ}$	Common
Thus $\Delta APQ \cong \Delta BPQ$	H.S ≅ H.S
$\therefore$ $\overline{AQ} \cong \overline{BQ}$	Corresponding sides of congruent triangles.
$\angle APQ \cong \angle BPQ$	Corresponding angles of congruent triangles.

Q.2. In the figure $m\angle C = m\angle D = 90^\circ$ and $\overline{BC} \cong \overline{AD}$.

Prove that $\overline{AC} \cong \overline{BD}$ and $\angle BAC \cong \angle ABD$.



Given: In the given figure:

$m\angle C = m\angle D = 90^\circ$ (i)

$\overline{BC} \cong \overline{AD}$

To Prove: $\overline{AC} \cong \overline{BD}$ (i)

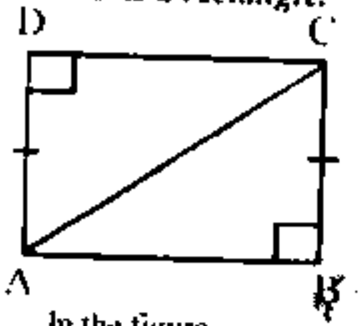
$\angle BAC \cong \angle ABD$

Proof:

Statements	Reasons
In $\Delta ABD \leftrightarrow \Delta BAC$	
$\overline{AD} \cong \overline{BC}$	Given
$\angle D \cong \angle C$	Each 90°
$\overline{AB} \cong \overline{BA}$	Common
Thus $\Delta ABD \cong \Delta BAC$	H.S ≅ H.S

$\therefore \quad \overline{AC} \cong \overline{BD}$	Corresponding sides of congruent triangles.
$\angle BAC \cong \angle ABD$	Corresponding angles of congruent triangles.

Q.3. In the figure, $m\angle B = m\angle D = 90^\circ$ and $\overline{AD} \cong \overline{BC}$ Prove that ABCD is a rectangle.



Given: In the figure

(i) $m\angle B = m\angle D = 90^\circ$

(ii) $\overline{AD} \cong \overline{BC}$

To Prove:

ABCD is a rectangle. i.e. $\overline{AB} \cong \overline{DC}$

Construction:

Join A to C.

Proof:

Statements	***Reasons***
In $\triangle ABC \leftrightarrow \triangle CDA$	
$\angle B \cong \angle D$	Given (each 90°)
$\overline{BC} \cong \overline{AD}$	Given
$\overline{AC} \cong \overline{AC}$	Common
$\therefore \triangle ABC \cong \triangle CDA$	$H.S \cong H.S$
Hence $\overline{AB} \cong \overline{DC}$	Corresponding sides of congruent triangles.

REVIEW EXERCISE 10

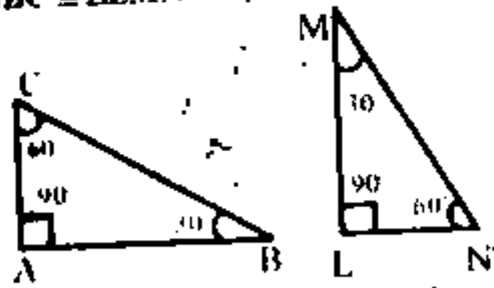
1. Which of the following are true and which are false?

(i) A ray has two end points.
(ii) In a triangle, there can be only one right angle.
(iii) Three points are said to be collinear if they lie on same line.
(iv) Two parallel lines intersect at a point.
(v) Two lines can intersect only at one point.
(vi) A triangle of congruent sides has non-congruent angles.

Answers:

(i)	False	*(ii)*	True	*(iii)*	True
(iv)	False	*(v)*	True	*(vi)*	False

Q.2. If $\Delta ABC \cong \Delta LMN$

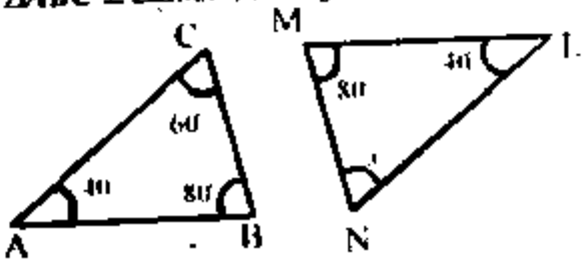


Then m $\angle M$ = ________ (i)
m $\angle N$ = ________ (ii)
m $\angle A$ = ________ (iii)

Answers:

(i) $m\angle M = 30^\circ$ *(ii)* $m\angle N = 60^\circ$ *(iii)* $m\angle A = 90^\circ$

Q.3. If $\Delta ABC \cong \Delta LMN$, then find the unknown x.

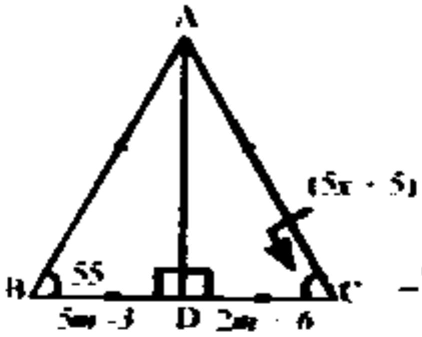


Answer:

$$m\angle N = x^\circ = 60^\circ.$$

Q.4. Find the value of unknowns for the given congruent triangles.

Solution:-



(i) $(5x + 5)^\circ = 55^\circ$

$5x + 5 = 55$

$5x = 55 - 5$

$5x = 50$

$x = 10$

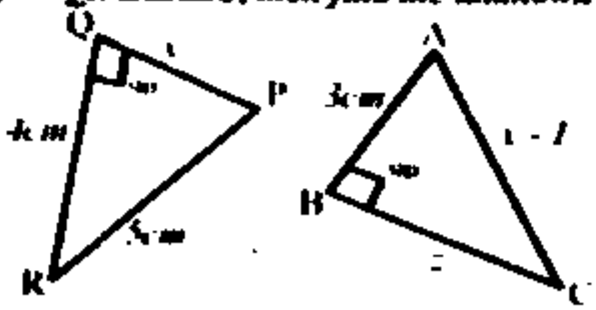
(ii) $5m - 3 = 2m + 6$

$5m - 2m = 6 + 3$

$3m = 9$

$m = 3$

Q.5. If $\Delta PQR \cong \Delta ABC$, then find the unknowns.



Solution:-

(i) $x = 3\text{cm}$

(ii) $y - 1 = 5$

$y = 5 + 1$

y = 6

(iii) z = 4cm

Remember:

- In any correspondence of two triangles, if one side and any two angles of one triangle are congruent to the corresponding side and angles of the other, the two triangles are congruent. (A.S.A ≅ A.S.A)
- If two angles of a triangle are congruent, then the sides opposite to them are also congruent.
- In a correspondence of two triangles, if three sides of one triangle are congruent to the corresponding three sides of the other, then the two triangles are congruent (S.S.S ≅ S.S.S)
- If in the correspondence of the two right-angled triangles, the hypotenuse and one side of one triangle are congruent to the hypotenuse and the corresponding side of the other, then the triangles are congruent. (H.S ≅ H.S)
- Two triangles are said to be congruent, if there exists a correspondence between them such that all the corresponding sides and angles are congruent.

Unit 11 PARALLELOGRAMS AND TRIANGLES

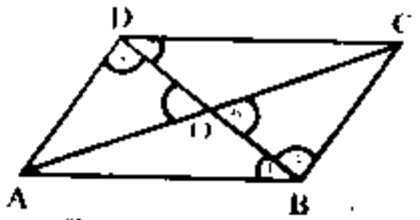
Theorem

In a parallelogram

(i) Opposite sides are congruent

(ii) Opposite angles are congruent

(iii) The diagonals bisect each other.



Given: ABCD is a ||^{m} that $\overline{AD} \parallel \overline{BC}$ and $\overline{AB} \parallel \overline{DC}$ Diagonals $\overline{AC}$ and $\overline{BD}$ cut each other at O.

To prove.... (i) $\overline{AD} \cong \overline{BC}$, $\overline{AB} \cong \overline{DC}$

(ii) $\angle ABC \cong \angle ADC$

$\angle BAD \cong \angle BCD$

(iii) $\overline{OA} \cong \overline{OC}$

$\overline{OB} \cong \overline{OD}$

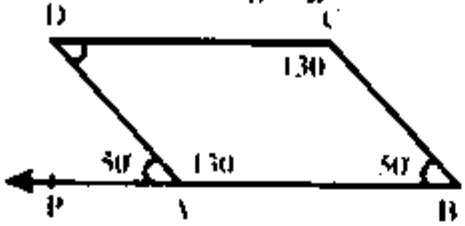
Proof

Statements	Reasons
$\Delta ABD \leftrightarrow \Delta CDB$	
$\angle 2 \cong \angle 1$	Alternate angles
$\overline{BD} \cong \overline{BD}$	Common
$\angle 4 \cong \angle 3$	Alternate angles
$\Delta ABD \cong \Delta CDB$	$A.S.A \cong A.S.A$
	Corresponding sides

$\therefore \quad \overline{AB} \cong \overline{DC}$	of congruent triangles
and $\quad \overline{AD} \cong \overline{BC}$	proved
Now $\quad \angle 2 \cong \angle 1 \; (i)$	proved
$\angle 4 \cong \angle 3 \; (ii)$	from (i) + (ii)
$m\angle 2 + m\angle 4 = m\angle 1 + m\angle 3$	
or $\quad m\angle ADC = m\angle ABC$	
or $\quad \angle ADC \cong \angle ABC$	
and $\angle BAD \cong \angle BCD$	
or $\Delta BOC \leftrightarrow \Delta DOA$	
$\overline{BC} \cong \overline{AD}$	proved
$\angle 6 \cong \angle 5$	proved
$\angle 3 \cong \angle 4$	
$\therefore \quad \Delta BOC \cong \Delta DOA$	proved
Thus $\quad \overline{OC} \cong \overline{OA}$	opp. Vertical angles
and $\quad \overline{OB} \cong \overline{OD}$	Alternate angles
	$A.A.S \cong A.A.S$

EXERCISE 11.1

Q.1 One angle of a parallelogram is 130°. Find the measure of its remaining angles.



Given: ABCD is a parallelogram in which $m\angle BAD = 130°$.

To prove: To find $m\angle B$, $m\angle D$, $m\angle C$

Construction: Produce $\overline{BA}$ towards A.

Proof

Statements	Reasons
ABCD is a parallelogram in which	
$m\angle DAB = 130°$	Given
$m\angle C = 130°$	Opposite angles of ▱
$m\angle DAP = 180° - 130°$ $= 50°$	Adj. supplementary angles
$m\angle B = 50°$	Corresponding angle of $\angle DAP$
$m\angle D = 50°$	Opposite angle of $\angle B$

Q.2. One exterior angle formed on producing one side of a parallelogram is 40°. Find the measures of its interior angles.

Given: ABCD is a parallelogram. $\overline{BA}$ is produced towards A and angle $\angle DAP = 40°$

Required: $m\angle BAD = ?$

$m\angle B = ?$

$m\angle C = ?$

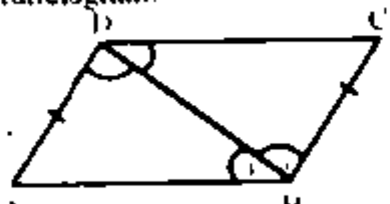
$m\angle D = ?$

Proof:

Statements	Reasons
ABCD is a parallelogram and	
$m\angle DAP = 40^0$	Given
$m\angle DAP + m\angle DAB = 180^0$	Adj. supp. angles
$40^0 + m\angle DAB = 180^0$	$m\angle DAP = 40^0$ given
$m\angle DAB = 180^0 - 40^0$	
$m\angle DAB = 140^0$ (i)	
$m\angle C = 140^0$ (ii)	Opp. Angles of $\parallel^{m}$
$m\angle B = m\angle DAP$	Corresponding angles
$m\angle B = 40^0$	
$m\angle D = 40^0$ $m\angle D = m\angle B$	Opp. Angles of $\parallel^{m}$
Thus $m\angle DAB = 140^0$	Proved
$m\angle B = 40^0$	
$m\angle C = 140^0$	
$m\angle D = 40^0$	

Theorem

If two opposite sides of a quadrilateral are congruent and parallel, it is a parallelogram.



Given: In a quadrilateral ABCD, we have

(i) $\overline{AB} \cong \overline{DC}$

(ii) $\overline{AB} \parallel \overline{DC}$

Required: ABCD is a parallelogram

i.e: $\overline{AD} \cong \overline{BC}$

$\overline{AD} \parallel \overline{BC}$

Construction: Join B to D.

Proof

Statements	Reasons
In $\Delta ABD \leftrightarrow \Delta CDB$	
$\overline{AB} \cong \overline{DC}$	Given
$\angle 2 \cong \angle 1$	Alternate angles of $\parallel$ lines
$\overline{BD} \cong \overline{BD}$	Common
$\therefore \Delta ABD \cong \Delta CDB$	$S.A.S \cong S.A.S$
Now $\angle 4 \cong \angle 3$ (*i*)	Corresponding angles of congruent triangles
$\therefore \overline{AD} \parallel \overline{BC}$ (*ii*)	from (i)
and $\overline{AD} \cong \overline{BC}$ (*iii*)	Corresponding sides of congruent triangles
also $\overline{AB} \parallel \overline{DC}$	given
$\therefore$ *ABDC is a* $\parallel^{m}$	

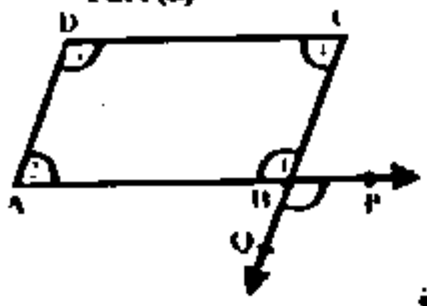
Exercise 11.12

Q.1. **Prove that a quadrilateral is a parallelogram if its**

(a) opposite angles are congruent

(b) diagonals bisects each other.

Part (a)



Given: In quadrilateral ABCD

(i) $\angle 3 \cong \angle 1$

(ii) $\angle 2 \cong \angle 4$

To prove: ABCD is a ||gm

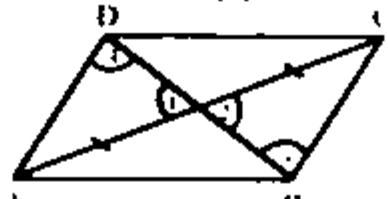
i.e: $\overline{AD} \parallel \overline{BC}$ and $\overline{AB} \parallel \overline{CD}$.

Const: Produce $\overline{AB}$ towards B and $\overline{CB}$ towards B.

Proof

Statements	Reasons
Now $\angle 3 \cong \angle 1$ (i)	Given
$\angle 5 \cong \angle 1$ (ii)	Vertical opposite angles
$\angle 3 \cong \angle 5$	From (i) and (ii)
i.e. $\overline{CQ} \parallel \overline{AD}$	
$\overline{AP} \parallel \overline{CD}$	
or $\overline{BC} \parallel \overline{AD}$	
and $\overline{AB} \parallel \overline{CD}$	
$\therefore$ ABCD is a ||gm	

Part (b)



Given: ABCD is a quadrilateral in which

$\overline{AC}$ and $\overline{BD}$ diagonals that bisect each at O. i.e.

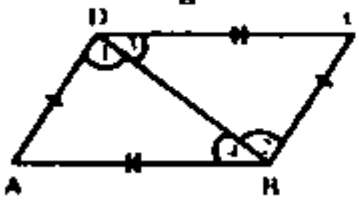
$\overline{OA} \cong \overline{OC}$ and $\overline{OB} \cong \overline{OD}$

Required: Quadrilateral ABCD is parallelogram.

Proof

Statements	Reasons
In $\Delta OBC \leftrightarrow \Delta ODA$	
$\overline{OC} \cong \overline{OA}$	Given
$\overline{OB} \cong \overline{OD}$	Given
$\angle 1 \cong \angle 2$	Vertical opp. angles
$\Delta OBC \cong \Delta ODA$	$S.A.S \cong S.A.S$
$\therefore$ $\overline{AD} \cong \overline{BC}$	Corresponding sides of congruent Δ'
$\angle 4 \cong \angle 3$	Corresponding angles of congruent Δ'
$\therefore$ $\overline{AD} \parallel \overline{BC}$	Alternate angles are congruent
and $\overline{AB} \parallel \overline{CD}$	
Thus $ABCD$ *is a* $\parallel^{m}$	

Q.2. ***Prove that a quadrilateral is a parallelogram if its opposite sides are congruent.***



Given:

In quadrilateral ABCD

(i) $\overline{AB} \cong \overline{DC}$ *(ii)* $\overline{AD} \cong \overline{BC}$

To Prove:

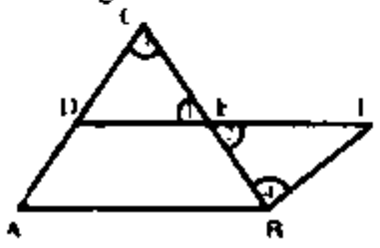
ABCD is a parallelogram i.e; $\overline{AD} \parallel \overline{BC}$; $\overline{AD} \parallel \overline{BC}$

Proof:

	Statements	Reasons
In	$\Delta ABD \leftrightarrow \Delta CDB$	
	$\overline{AD} \cong \overline{BC}$	Given
	$\overline{AB} \cong \overline{DC}$	Given
	$\overline{BD} \cong \overline{BD}$	Common
	$\Delta ABD \cong \Delta CDB$	S.S.S ≅ S.S.S
Thus	$\angle 1 \cong \angle 2$	Corresponding angles of congruent Δ'
and	$\angle 4 \cong \angle 3$	
Now	$\angle 1 \cong \angle 2$	Proved
(i)	$\overline{AD} \parallel \overline{BC}$	Alternate angles are congruent
and	$\angle 4 \cong \angle 3$	
Thus (ii)	$\overline{AB} \parallel \overline{DC}$	
∴	ABCD is a ∥gm	

Theorem

The line segment, joining the mid-points of two sides of a triangles, is parallel to the third side and is equal to one half of its length.



Given:

In ΔABC, the line segment $\overline{DE}$, joins the mid-points of $\overline{AC}$ and $\overline{BC}$.

Required:

$$\overline{DE} \parallel \overline{AB}$$

$$m\overline{DE} = \frac{1}{2}(m\overline{AB})$$

Construction:

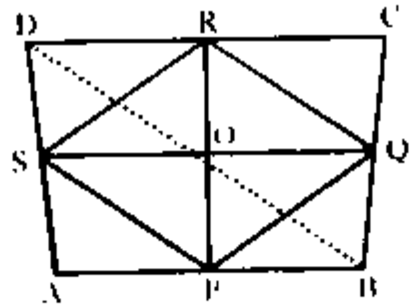
(i) Produce $\overline{DE}$ towards E and cut off $m\overline{EF} = m\overline{DE}$.
(ii) Join B to F.

Proof:

	Statements	Reasons
In	$\Delta DEC \leftrightarrow \Delta FEB$	
	$\overline{DE} \cong \overline{EF}$	Construction
	$\angle 1 \cong \angle 2$	Vertical opp. Angles
	$\overline{CE} \cong \overline{EB}$	Given
	$\Delta DEC \cong \Delta FEB$	S.A.S ≅ S.A.S
	$\angle 3 \cong \angle 4$	Corresponding angles of congruent triangles
	$\overline{CD} \cong \overline{BF}$	Corresponding sides of congruent triangles.
	$\overline{AD} \cong \overline{BF}$	As $\overline{CD} \cong \overline{AD}$
	$\overline{AC} \parallel \overline{BF}$	As Alternate angles are congruent
Thus	$\overline{AD} \parallel \overline{BF}$ ABFD is $\parallel^{m}$	As $\overline{AD} \parallel \overline{BF}$ and $\overline{AD} \cong \overline{BF}$
So	$\overline{DF} \parallel \overline{AB}$	
i.e.	$\overline{DE} \parallel \overline{AB}$	
and	$\overline{DF} \cong \overline{AB}$	
But $m\overline{DE} = \frac{1}{2}(m\overline{DF})$		
Thus $m\overline{DE} = \frac{1}{2}(m\overline{AB})$		

Exercise 11.3

Q.1. ***Prove that the line-segment joining the mid-points of the opposite sides of a quadrilateral bisect each other.***



Given:

(i) ABCD is a quadrilateral points P, Q, R, S are the mid-points of $\overline{AB}$, $\overline{BC}$, $\overline{CD}$ and $\overline{DA}$ respectively.

(ii) P is joined to R.

(iii) S is joined to Q.

$\overline{PR}$ and $\overline{SQ}$ intersect each other at O.

Required:

$$\overline{OS} \cong \overline{OQ}$$

$$\overline{OR} \cong \overline{OP}$$

Construction:

(i) Join P, Q, R, S in order.

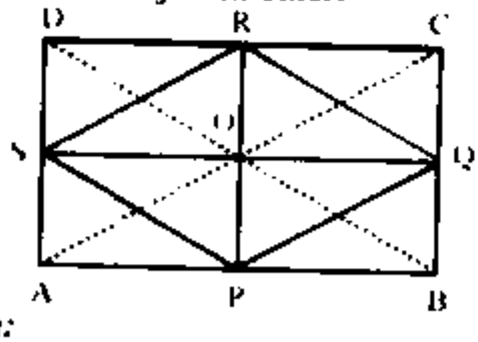
(ii) Join B to D.

Proof:

Statements	Reasons
In ΔBDC	
R is mid-point of $\overline{DC}$	Given
Q is mid-point of $\overline{BC}$	Given
Therefore(i) $\overline{RQ} \parallel \overline{DB}$	

and $m\overline{RQ} = \frac{1}{2}(m\,\overline{DB})$	
In $\rightarrow \Delta$ BAD	
S is mid-point of $\overline{AD}$ P is mid-point of AB	
(iii) $\therefore \overline{PS} \parallel \overline{DB}$	
(iv) $m\overline{PS} = \frac{1}{2}(m\,\overline{DB})$	
$\overline{PQ} \parallel \overline{PS}$	From (i). (ii) each is $\parallel$ to $\overline{DB}$
$\overline{PQ} \cong \overline{PS}$	From ii and iv each $= \frac{1}{2}(m\,\overline{DB})$
BQRS is a $\parallel^{m}$	
$\overline{PR}$. $\overline{SQ}$ are its diagonals	
Thus $\overline{OS} \cong \overline{OQ}$ $\overline{OR} \cong \overline{OP}$	Diagonals of a $\parallel$m bisect each other.

Q.2. ***Prove that the line-segments joining the mid-points of the opposite sides of a rectangle are the right-bisectors of each other.***



Given:

(i) ABCD is a rect-angle

(ii) P, Q, R, S are mid-points of $\overline{AB}$, $\overline{BC}$, $\overline{CD}$ and $\overline{DA}$ respectively.

(iii) $\overline{RP}$ and $\overline{SQ}$ cut other at O.

Required:

$\overline{OP} \cong \overline{OR}$

$\overline{OS} \cong OQ$

and $\overline{RP} \cong \overline{SQ}$

Construction:

(i) Join P, Q, R, S in order.

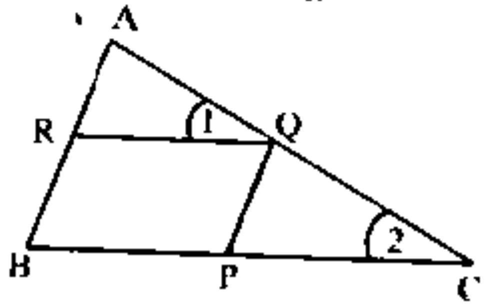
(ii) Join A to C.

Proof:

Statements	*Reasons*
In ΔABC	
P is mid-point of $\overline{AB}$	Given
Q is mid-point of $\overline{BC}$	Given
(i) $\therefore$ $\overline{PQ} \parallel \overline{AC}$	
(ii) $m\overline{PQ} = \frac{1}{2}(m\ AC)$	
Similarly, In ΔADC	
(iii) $\overline{SR}$ $\overline{AC}$	
(iv) $m\overline{SR} = \frac{1}{2} m\ \overline{AC}$	
$\overline{PQ} \parallel \overline{SR}$	From (i), (ii) each $\parallel \overline{AC}$
$m\overline{RQ} \parallel m\ \overline{SP}$	Each $= \frac{1}{2} m\ \overline{DB}$
By joining B to D, we can prove	
$\overline{RQ} \parallel \overline{SP}$	
$m\overline{RQ} \parallel m\ \overline{SP}$	Each $= \frac{1}{2} m\ \overline{AC}$

$m\overline{AC} \parallel m\overline{DB}$	Diagonals of rect-angle
∴ PQRS is a $\parallel^{m}$ All its sides are equal	∴ PQRS is $\parallel^{m}$
Now $\overline{PR}$ and $\overline{SQ}$ are diagonals of $\parallel^{m}$	
Now $\overline{OP} \cong \overline{OR}$	
$\overline{OS} \cong \overline{OQ}$	
In $\Delta OQR \leftrightarrow \Delta OQP$	
$\overline{OR} \cong \overline{OP}$	Proved
$\overline{OQ} \cong \overline{OQ}$	Common
$\overline{RQ} \cong \overline{PQ}$	Proved
∴ $\Delta OQR \cong \Delta OQP$	
Thus (M) $\angle ROQ \cong \angle POQ$	
(N) $m\angle ROQ + m\angle POQ = 180^{\circ}$	Adj. supp. angles.
Thus $m\angle ROQ = m\angle POQ = 90^{\circ}$	From M, N
Thus $\overline{RP} \perp \overline{SQ}$	

Q.3. Prove that the line-segment passing through the mid-point of one side and parallel to another side of a triangle also bisects the third side.



Given: (i) ΔABC point Q is mid-point of $\overline{AC}$

i.e., $\overline{AQ} \cong \overline{CQ}$

(ii) $\overline{QR} \parallel \overline{BC}$ which cuts $\overline{AB}$ at R.

Required: (i) $\overline{RA} \cong \overline{BR}$

Construction: (i) Take P as mid-point of $\overline{BC}$

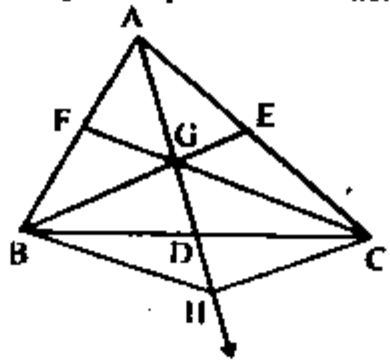
(ii) Join R to Q.

Proof:

Statements	Reasons
Now $\overline{RQ} \parallel \overline{BC}$	Given
$\therefore$ $m\overline{RQ} \; \frac{1}{2}(m\overline{BC})$	
$m\overline{RQ} = m\overline{PC}$	P is mid point of $\overline{BC}$
In $\Delta AQR \leftrightarrow \Delta QCP$	
$\overline{AQ} \cong \overline{QC}$	given
$\angle 1 \cong \angle 2$	Corresponding angles
$\overline{RQ} = \overline{PC}$	Proved
$\therefore$ $\Delta AQR \cong \Delta QCP$	S.A.S $\cong$ S.A.S
$\overline{AR} \cong \overline{QP}$	..... construction
(M) $\overline{BP} \cong \overline{PC}$	..... proved
(N) But $\overline{RQ} = \overline{PC}$	From M, N
$\overline{RQ} \cong \overline{BP}$	Given
$\overline{RQ} \parallel \overline{BP}$	
$\therefore$ BPQR $\parallel^{gm}$	
(I) $\overline{RB} = \overline{QP}$	Proved
(II) $\overline{AR} \cong \overline{QP}$	
Thus $\overline{AR} = \overline{RB}$	from (i) and (ii)
or $\overline{RA} \cong \overline{BR}$	

Theorem

The medians of a triangle are concurrent and their point of concurrency is the point of trisection of each median.



Given:

In ΔABC $\overline{BE}$ and $\overline{CF}$ are medians that intersect at point G.

(i) All the three medians of triangle ABC are congruent at point G.

(ii) Point G is the point of trisection of each medians.

Required:

(i) Join A to G and produce ahead. It cuts $\overline{BC}$ at D.

Take $\overline{GH} \cong \overline{AG}$

(ii) Join H to B and C.

Proof:

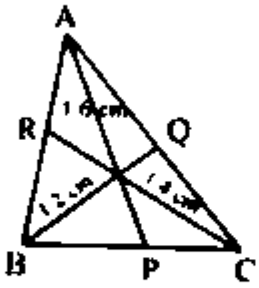
Statements	*Reasons*
In ΔABH (i) F is mid-point of $\overline{AB}$	Given

(ii) G is mid-point of $\overline{AH}$	Construction
$\therefore$ $\overline{FG} \parallel \overline{EH}$	
or $\overline{GC} \parallel \overline{EH}$	
Similarly $\overline{GB} \parallel \overline{HC}$	
Thus BHCG is a $\overline{BC}$ and $\overline{GH}$ are its diagonals.	
Hence	
(M) Thus $\overline{BD} \cong \overline{CD}$	
(N) and $\overline{GD} \cong \overline{HD}$	
Point D is mid-point of $\overline{BC}$	
We can say $\overline{AD}$ is third median	Construction
It passes through G	
$\overline{AD}$ & $\overline{GF}$	As $\overline{GD} = \overline{DH}$
Are concurrent	
Now $m\overline{AG} = m\overline{GH}$	
$m\overline{AG} = 2m\overline{GD}$	
$\dfrac{m\overline{AG}}{m\overline{GD}} = \dfrac{2}{1}$	
Or $m\overline{AG} : m\overline{GD} = 2 : 1$	
Point G is point of trisection	
Similarly it can be proved that G is point of trisection of $\overline{CF}$ and $\overline{BE}$ is called centroid of the triangle ABC.	

Exercise 11.14

Q.1. The distances of the point of concurrency of the medians of a triangle from its vertices are respectively 1.2 cm, 1.4 cm and 1.6 cm. Find the lengths of the medians.

Solution:-



Let $\overline{AP}$, $\overline{BQ}$, $\overline{CR}$ be medians of ΔABC

Length of $\overline{AP}$ $= 1.6 + \frac{1.6}{2}$

$= 1.6 + .8$

$= 2.4$ cm

Length of $\overline{BQ}$ $= 1.2 + \frac{1.2}{2}$

$= 1.2 + .6$

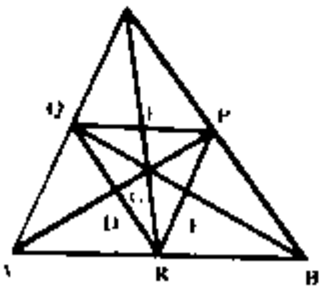
$= 1.8$ cm

Length of $\overline{CR}$ $= 1.4 + \frac{1.4}{2}$

$= 1.4 + .7$

$= 2.1$ cm

Q.2. Prove that the point of concurrency of the medians of a triangle and the triangle which is made by joining the mid-points of the sides is the same.



Given:

ABC is a triangle $\overline{AP}$, $\overline{BQ}$, $\overline{CR}$ are its medians that are concurrent at point G; $\overline{AR} \cong \overline{BR}$; $\overline{BP} \cong \overline{CP}$; $\overline{CQ} \cong \overline{AQ}$ points P, Q and R are joined together. PQR is a triangle formed by joining the mid-point of the sides of ΔABC.

Required:

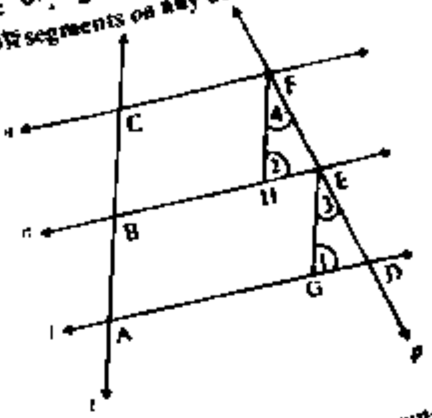
$\overline{RF}$, $\overline{PD}$, $\overline{QE}$ are the medians of triangle PQR and these pass through G also.

Proof:

Statements	Reasons
In ΔABC $\overline{QP} \parallel \overline{AB}$ or $\overline{QF} \parallel \overline{AB}$ or $\overline{QF} \parallel \overline{AR}$ Thus F is mid-point of $\overline{QP}$ Similarly E is mid-point of $\overline{PR}$ and D is mid-point of $\overline{QR}$ Therefore, RF, PD and $\overline{QE}$ are medians of triangle PQR and these pass through G. Hence the result	As P, Q are mid-point of $\overline{AB}$, $\overline{AC}$.

Theorem

If three or more parallel lines make congruent segments on a transversal they also intercept congruent segments on any other line that cuts them.



Given:

(iii) $l \parallel m \parallel n$ and transversal t cuts these parallel lines at A, B, C respectively that $\overline{AB} \cong \overline{BC}$

(iv) Another transversal p cuts these lines at D, E, F respectively.

Gi

Required: $\overline{DE} \cong \overline{EF}$

Construction:
Passing through F and E take line segment FH, EG parallel to t.

Require

Solution:

Proof:

Statements	Reasons
AGEB is a $\parallel^{gm}$	$\overline{AB} \parallel \overline{GE}$ and $\overline{AG} \parallel \overline{BE}$

And

$\overline{AB}$

Transversal l

L, M,

as $\overline{AB} \cong \overline{B}$

$\overline{LM} \cong \overline{M}$

when as

Therefore mL

$= m\overline{PQ} = 1cm$

$\overline{GE} \cong \overline{AB}$

Similarly $\overline{HF} \cong \overline{BC}$ opp. Si...

Thus $m\overline{LN} = m\overline{LM} + m\overline{MN}$

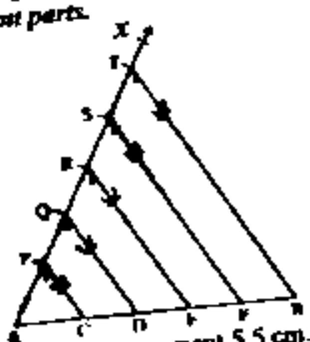
$= 1 + 1 = 2c$

$m\overline{LQ} = m\overline{LM} + m\overline{MN} + m\overline{NP} + m\overline{PQ}$

$= 1 + 1 + 1 + 1$

$= 4$ cm

Q.2 ***Take a line segment of length 5.5 cm and divide it into five congruent parts.***



Construction: (i) Take a line segment 5.5 cm.

(ii) Draw an acute angle $\angle BAX$

(iii) On AX take $\overline{AP} \cong \overline{PQ} \cong \overline{QR} \cong \overline{RS} \cong \overline{ST}$

(iv) Join T to B

(v) Draw line parallel to TB from the points P, Q, R and S.

Result: Line segment $\overline{AB}$ is divided into congruent line segments $\overline{AC} \cong \overline{CD} \cong \overline{DE} \cong \overline{EF} \cong \overline{FB}$

Proof: $\overline{AP} \cong \overline{PQ} \cong \overline{QR} \cong \overline{RS} \cong \overline{ST}$

and $\overline{TB} \parallel \overline{SF} \parallel \overline{RE} \parallel \overline{QD} \parallel \overline{PC}$

Therefore: $\overline{AC} \cong \overline{CD} \cong \overline{DE} \cong \overline{EF} \cong \overline{FB}$

REVIEW EXERCISE 11

1. Fill in the blanks.

(i) In a parallelogram opposite sides are ....

(ii) In a parallelogram opposite angles are ....

(iii) Diagonals of a parallelogram ....each other at a point

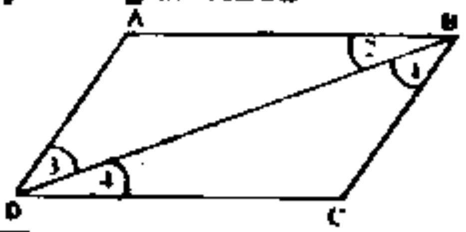
(iv) Medians of a triangle are ....

(v) Diagonal of a parallelogram divides the parallelogram into two .... triangles.

Answers

(i)	congruent	(ii)	congruent
(iii)	bisect	(iv)	concurrent
(v)	congruent		

Q.2. In the parallelogram ABCD



(i) $m\overline{AB}$..............$m\overline{DC}$ (i)

(ii) $m\overline{BC}$..............$m\overline{AD}$ (ii)

(iii) $m\angle 1 \cong$ _______ (iii)

(iv) $m\angle 2 \cong$ _______ (iv)

Answers

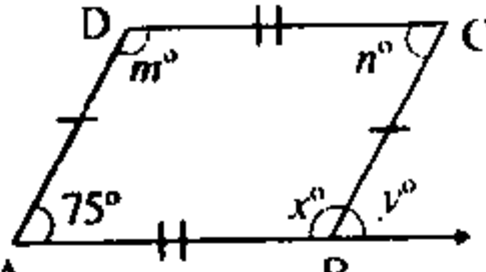
(i) $m\overline{AB} = m\overline{DC}$

(ii) $m\overline{BC} \cong m\overline{AD}$

(iii) $m\angle 1 = m\angle 3$

(iv) $m\angle 2 = m\angle 4$

Q.3 Find the unknowns in the given figure.



Solution: $m\angle C = m\angle A$

$n^\circ = 75^\circ$ (i)

$y^\circ = n^\circ$ (Alternate angle)

$y^\circ = 75^\circ$ (ii)

$x^\circ + y^\circ = 180^\circ$ Adj.Supp.angle.

$x^\circ + 75^\circ = 180^\circ$

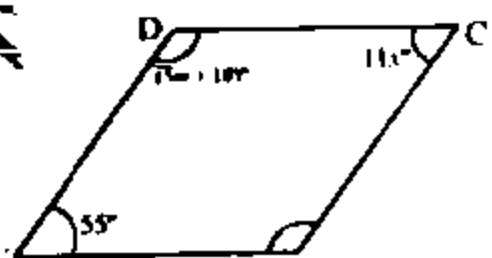
$x^\circ = 180^\circ - 75^\circ$

$x^\circ = 105^\circ$ (iii)

$m^\circ = x^\circ$

$m^\circ = 105^\circ$ (iv)

Q.4 If the given figure ABCD is a parallelogram, then find x, m



Solution: $11x^\circ = 55^\circ$ opp.angles of $\|^{m}$

$$x = \frac{55}{11} = 5 \quad (i)$$

Now $(5m+10)+55=180$

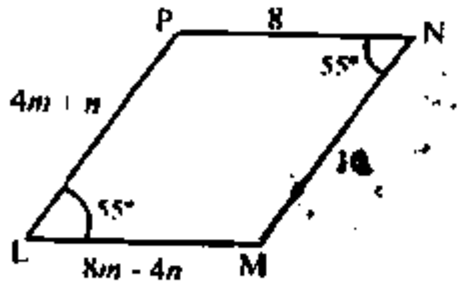
$$5m+10+55=180$$

$$5m=180-10-55$$

$$5m=115$$

$$m=\frac{115}{5}=23 \qquad (ii)$$

Q.5. The given figure LMNP is a parallelogram. Find the value of *m*, *n*.



Solution: $4m+n=10$ *opp sides of* $\|^{gm}$ (i)

$$8m-4n=8 \qquad (ii)$$

$$16m+4n=40$$

$$8m-4n=8 \quad \textit{multiplying by 4}$$

$$24m=48 \quad (\textit{Adding})$$

$$m=\frac{48}{24}=2$$

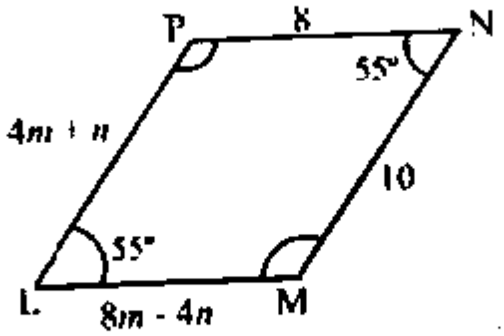
Putting $m=2$

$$4(2)+n=10$$

$$8+n=10$$

$$n=10-8=2$$

Q.6. In the question 5, sum of the opposite angles of the parallelogram is 110°, find the remaining angles.



Solution: $m\angle P + m\angle L = 180°$

$$m\angle P + 55 = 180°$$

$$m\angle P = 180 - 5$$

$$m\angle P = 125°$$

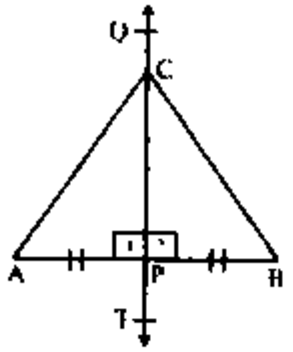
$$m\angle M = m\angle P$$

$$m\angle M = 125°$$

Unit 12 LINE BISECTORS AND ANGLE BISECTORS

Theorem

Any point on the right bisector of a line segment is equidistant from its end points.



Given: (i) $\overline{AB}$ is a line segment

(ii) $\overleftrightarrow{QT}$ is a right bisector of $\overline{AB}$, i.e. $\overline{AP} \cong \overline{BP}$

$\overleftrightarrow{QT} \perp \overline{AB}$

$m\angle 1 = m\angle 2 = 90^\circ$

(iii) Point C lies on $\overleftrightarrow{QT}$

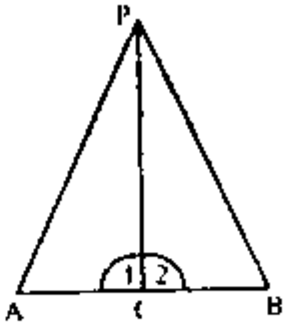
(iv) C is joined to A and B

Required: $\overline{CA} \cong \overline{CB}$

Proof: **Statements**	***Reasons***
In $\Delta CAP \longleftrightarrow \Delta CBP$	
$\angle 1 \cong \angle 2$	Given
$\overline{CP} \cong \overline{CP}$	Common
$\overline{AP} \cong \overline{BP}$	Given
So $\Delta CAP \cong \Delta CBP$	S.A.S ≅ S.A.S
$\overline{CA} \cong \overline{CB}$	Corresponding sides of congruent Δ^s.

Converse of Theorem

Any point equidistant from the end points of a line segment is on the right bisector of it.



Given: $\overline{AB}$ is a line segment
Point P is such that $\overline{PA} \cong \overline{PB}$

Required: Point P is on the right bisector of $\overline{AB}$

Construction: Take C mid point of $\overline{AB}$ i.e.
$\overline{AC} \cong \overline{BC}$ and point P to C

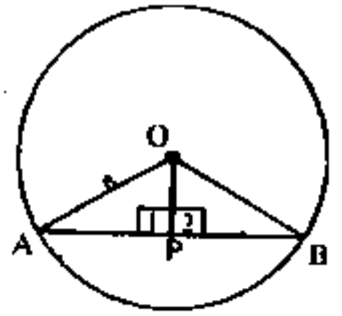
Proof:

	Statements	***Reasons***
	In $\Delta ACP \longleftrightarrow \Delta BCP$	
	$\overline{CA} \cong \overline{BC}$	Construction
	$\overline{PA} \cong \overline{PB}$	Given
	$\overline{PC} \cong \overline{PC}$	Common
So	$\Delta ACP \cong \Delta BCP$ (i)	S.S.S ≅ S.S.S
∴	$m\angle 1 = m\angle 2$ (ii)	Corresponding angle of congruent Δ^s.
But	$m\angle 1 + m\angle 2 = 180^\circ$	Adj. Supp. Angles
	$m\angle 1 + m\angle 2 = 180^\circ$	$m\angle 2 = m\angle 1$
	$2m\angle 1 = 180$	
	$m\angle 1 = 90^\circ$	
	$\overline{PC} \perp \overline{AB}$	

Thus $PC \perp \overline{AB}$ and point C on it.

EXERCISE 12.1

Q.1 ***Prove that the centre of a circle is on the right bisector of each of its chords.***



Given: (i) O is the centre of a circle

(ii) $\overline{AB}$ is chord of the circle.

Required: Centre O lies on the right bisector of $\overline{AB}$

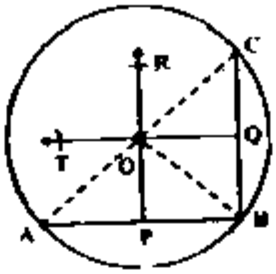
Construction: (i) Take mid-point of $\overline{AB}$ as P.

(ii) Join P to O, O to A and B.

Proof:

Statements	*Reasons*
In $\Delta OPA \longleftrightarrow \Delta OPB$	
$\overline{PA} \cong \overline{PB}$	Construction
$\overline{OP} \cong \overline{OP}$	Common
$\overline{OA} \cong \overline{OB}$	Radii of a circle
Therefore $\Delta OPA \cong \Delta OPB$	S.S.S $\cong$ S.S.S
$\therefore$ $m\angle 1 = m\angle 2$	Corresponding angles of congruent Δ^s.
(i) $m\angle 1 = m\angle 2$	
(ii) $m\angle 1 + m\angle 2 = 180^\circ$	Adj. Supp. Angles
Thus $m\angle 1 = m\angle 2 = 90^\circ$	From (i) and (ii)
$\overline{OP}$ is right bisector of $\overline{AB}$ and O lies on it.	

Q.2 ***Where will be the centre of a circle passing through three non-collinear points? And why?***



Given: A, B, C are three non-collinear points and a circle is passing through these points

Required: To find the centre of the circle and to prove that it is the centre.

Construction: (i) Join B to A and C.

(ii) Take $\overleftrightarrow{PR}$ right bisector of $\overline{AB}$.

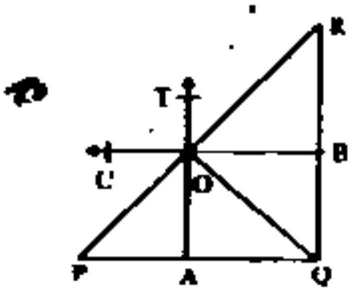
(iii) Take $\overleftrightarrow{QT}$ right bisector of $\overline{BC}$

$\overleftrightarrow{PR}$ and $\overleftrightarrow{QT}$ intersect at point O

Proof:

Statements	Reasons
$\overline{PO}$ is right bisector of $\overline{AB}$	
$\therefore \quad \overline{OA} \cong \overline{OB}$ (i)	
$\overline{QO}$ is right bisector $\overline{BC}$	
$\therefore \quad \overline{OB} \cong \overline{OC}$ (ii)	
$\therefore \quad \overline{OA} \cong \overline{OB} \cong \overline{OC}$	From (i) and (ii)
Hence O is centre of the circle	

Q.3 ***Three villages P, Q and R are not on the same line. The people of these villages want to make a Children Park at such place which is equidistant from these three villages. After fixing the place of Children Park, prove that the park is equidistant from the three villages.***



Given: P, Q, R three villages are non-collinear

Required: To locate a point for a park equidistant from these villages.

Construction: (i) Join Q to P and R.

(ii) Take $\overleftrightarrow{AT}$ right bisector of $\overline{PQ}$.

(iii) Take $\overleftrightarrow{BU}$ right bisector of $\overline{QR}$

$\overleftrightarrow{AT}$ and $\overleftrightarrow{BU}$ intersect at point O.

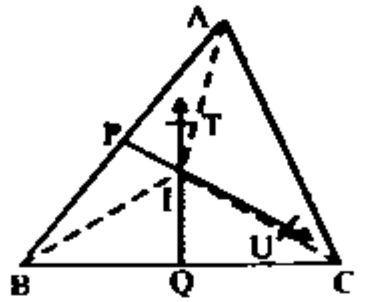
Result: O is the required place.

Proof:

Statements	***Reasons***
$\overline{OP} \cong \overline{OQ}$ (i)	Point O is on the right bisector of $\overline{AQ}$
$\overline{OQ} \cong \overline{OR}$ (ii)	Point O is on the right bisector of $\overline{QR}$
$\therefore$ $\overline{OP} \cong \overline{OQ} \cong \overline{OR}$	From (i) + (ii)
Hence O is equidistant from P, Q and R.	

Theorem

The right bisectors of the sides of a triangle are concurrent.



Given: In ΔABC

$\overleftrightarrow{PU}$, $\overleftrightarrow{QT}$ are right bisectors of $\overline{AB}$ and $\overline{BC}$.

$\overleftrightarrow{PU}$ and $\overleftrightarrow{QT}$ intersect at *I*.

Required: Point *I* lies on the right bisector of $\overline{AC}$.

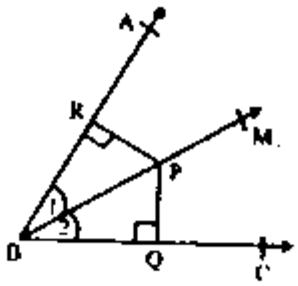
i.e. $\overline{IC} \cong \overline{IA}$

Proof:

Statements	*Reasons*
$\overline{BI} \cong \overline{CI}$ (i)	Point *I* is on the right bisector of $\overline{BC}$
$\overline{BI} \cong \overline{AI}$ (ii)	Point I is on the right bisector of $\overline{AB}$
$\therefore$ $\overline{CI} \cong \overline{IA}$	From (i) and (ii)
Hence Point *I* lies on the right bisector of $\overline{AC}$	
Therefore three right bisectors of sides of a triangle are concurrent.	

Theorem

Any point on the bisector of an angle is equidistant from its arms.



Given: (i) $\angle ABC$ is an angle.

(ii) $\overrightarrow{BM}$ is bisector of $\angle ABC$

(iii) $\overline{PQ} \perp \overrightarrow{BC}$

(iv) $\overline{PR} \perp \overrightarrow{BA}$

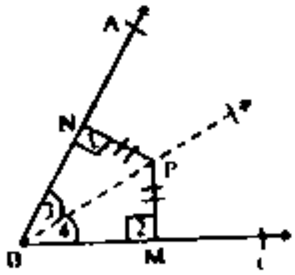
To prove: $\overline{PQ} \cong \overline{PR}$

Proof:

Statements	Reasons
In $\Delta PBR \longleftrightarrow \Delta PBQ$	
$\overline{BP} \cong \overline{BP}$	Common
$\angle 1 \cong \angle 2$	Given
$\angle PRB \cong \angle PQB$	Each 90°
$\Delta PBR \cong \Delta PQB$	S.A.A. ≅ S.A.A.
$\overline{PR} \cong \overline{PQ}$	Corresponding sides of congruent triangles.

Converse Theorem

Any point inside an angle, equidistant from the arms, is on the bisector of it.



Given: (i) A point P lies inside of angle $\angle ABC$.

(ii) $\overline{PM} \perp \overline{BC}$, $\overline{PN} \perp \overline{BA}$

(iii) $\overline{PM} \cong \overline{PN}$

Required: Point P is on the bisector of $\angle ABC$

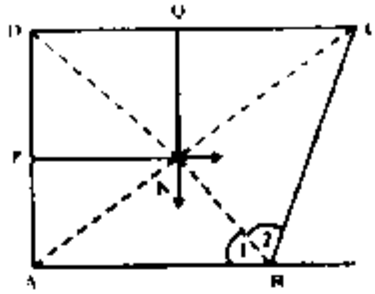
Construction: Join B to P and produce.

Proof:

Statements	***Reasons***
In $\Delta PMB \longleftrightarrow \Delta PNB$	
$\overline{BP} \cong \overline{BP}$	Common
$\angle 1 \cong \angle 2$	Each 90° (given)
$\overline{PM} \cong \overline{PN}$	Given
$\Delta PMB \cong \Delta PNB$	H.S. ≅ H.S.
Thus $\angle 3 \cong \angle 4$	Corresponding sides of congruent triangles.
Hence $\overline{BP}$ is bisector of $\angle ABC$ and point P lies on it.	

EXERCISE 12.2

Q.1 ***In a quadrilateral ABCD, $\overline{AB} \cong \overline{BC}$ and the right bisector of $\overline{AD}$, CD meet each other at point N. Prove that $\overline{BN}$ is bisector of $\angle ABC$.***



Given: (i) ABCD is a quadrilateral.

(ii) $\overline{AB} \cong \overline{BC}$

(iii) $\overline{PN}$ is right bisector of $\overline{AD}$

(iv) $\overline{QN}$ is right bisector of $\overline{CD}$

$\overline{PN}$ and $\overline{QN}$ intersect each other at N.

(v) N is joined to N.

Required: $\overline{BN}$ is bisector of angle $\angle ABC$

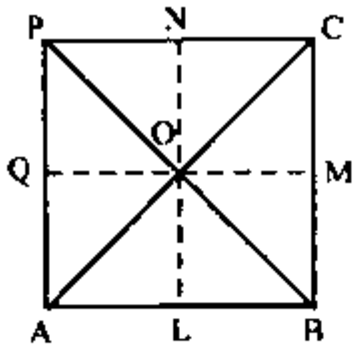
i.e. $\angle 1 \cong \angle 2$

Const: Join N to A, D, C.

Proof:

Statements	**Reasons**
$\overline{NA} \cong \overline{ND}$ (i)	N lies on right bisector of $\overline{AD}$
$\overline{ND} \cong \overline{NC}$ (ii)	N lies on right bisector of $\overline{CD}$
$\overline{NA} \cong \overline{NC}$	From (i) and (ii)
In $\Delta NAB \longleftrightarrow \Delta NCB$	
$\overline{NA} \cong \overline{NC}$	Proved
$\overline{AB} \cong \overline{AC}$	Given
$\overline{BN} \cong \overline{BN}$	Common
$\therefore$ $\Delta NAB \cong \Delta NCB$	S.S.S $\cong$ S.S.S
So $\angle 1 \cong \angle 2$	
Hence $\overline{BN}$ is bisector of $\angle ABC$	

Q.2 The bisector of ∠A, ∠B and ∠C of a quadrilateral ABCP meet each other at point O. Prove that the bisector of ∠P unit also pass through the point O.



Given: (i) ABCP is a quadrilateral.

(ii) Bisectors of ∠A, ∠B and ∠C meet at point O.

Required: Point O lies on the bisector of ∠P

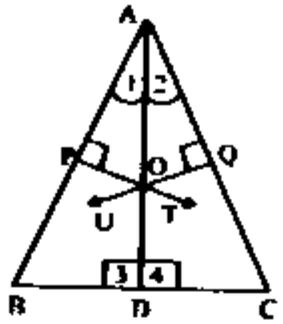
Construction: (i) Join O to P.

(ii) Draw $\overline{OL} \perp AB$, $\overline{OM} \perp \overline{BC}$ and $\overline{ON} \perp \overline{PC}$ also $\overline{OQ} \perp \overline{AP}$

Proof:

Statements	***Reasons***
$\perp \overline{OL} \cong \perp \overline{OQ}$ (i)	Point O is on the bisector of ∠A
$\perp \overline{OL} \cong \overline{OM}$ (ii)	Point O is on the bisector of ∠B
$\perp \overline{OM} \cong \overline{ON}$ (iii)	Point O is on the bisector of ∠C
∴ $\perp \overline{OQ} \cong \perp \overline{ON}$	From (i), (ii) and (iii)
Thus Point O lies on the **bisector** of ∠P	
Therefore $\overline{OP}$ is bisector of ∠P	

Q.3 Prove that the right bisector of congruent sides of an isosceles triangle and its altitude are congruent.



Given: In ΔABC

(i) $\overline{AB} \cong \overline{AC}$

(ii) $\overline{PT}$ is right bisector of $\overline{AB}$

(iii) $\overline{QU}$ is right bisector of $\overline{AC}$

$\overline{PT}$ and $\overline{QU}$ intersect each other at O.

Required: Point O lies on the altitude of ΔABC

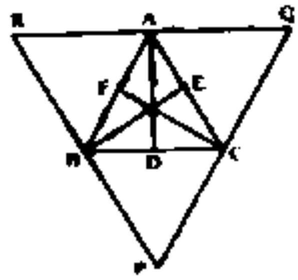
Construction: Join A to O and produce it. It cuts $\overline{BC}$ at D.

Proof:

Statements	***Reasons***
$m\overline{AB} = m\overline{AC}$	Given
$\frac{1}{2}m\overline{AB} \cong \frac{1}{2}m\overline{AC}$	Dividing by 2
$\therefore \overline{AP} \cong \overline{AQ}$	
In $\Delta APO \longleftrightarrow AQO$	
$\overline{AP} \cong AQ$	proved
$\overline{AO} \cong \overline{AO}$	common
$\angle APO \cong \angle AQO$	each 90°
$\Delta APO \cong \Delta AQO$	H.S. ≅ H.S.

Thus $\angle 1 \cong \angle 2$	
Now in $\Delta BAD \longleftrightarrow \Delta CAD$	
$\overline{AB} \cong \overline{AC}$	Given
$\angle 1 \cong \angle 2$	Proved
$\overline{AD} \cong \overline{AD}$	Common
$\Delta BAD \cong \Delta CAD$	S.A.S. ≅ S.A.S.
(i) $\angle 3 \cong \angle 4$	Corresponding angles of Congruent Δ^s.
(ii) $m\angle 3 + m\angle 4 = 180°$	Adj. supp. angles
Thus $m\angle 3 = m\angle 4 = 90°$	
Therefore $\overline{AD} \perp \overline{AB}$	
Point O lies on altitude $\overline{AD}$	

Q.4 Prove that the altitudes of a triangle are concurrent.



Given: In ΔABC

$\overline{AD}$, $\overline{BE}$, $\overline{CF}$ are its altitudes.

i.e. $\overline{AD} \perp \overline{BC}$, $\overline{BE} \perp \overline{AC}$, $\overline{CF} \perp \overline{AB}$

Required: $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ are concurrent.

Construction: Passing through A, B, C take $\overline{RQ} \parallel \overline{BC}$,

$\overline{RP} \parallel \overline{AC}$ and $\overline{QP} \parallel \overline{AB}$ respectively.

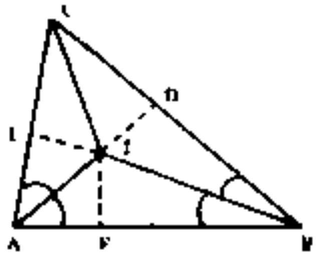
Forming a triangle PQR.

Proof:

Statements	*Reasons*
$\overline{BC} \parallel \overline{PQ}$	Construction
$\overline{AB} \parallel \overline{QC}$	Construction
$\therefore$ ABCQ is a $\parallel^{m}$	
Hence $\overline{AQ} \cong \overline{BC}$	
Similarly $\overline{RA} \cong \overline{BC}$	
Hence point A is mid-point of $\overline{RQ}$	
And $\overline{AD} \perp \overline{BC}$	
$\overline{BC} \parallel \overline{RQ}$	
$\therefore$ $\overline{AD} \perp \overline{RQ}$	
Thus $\overline{AD}$ is right bisector of $\overline{RQ}$	
Similarly $\overline{BE}$ is right bisector of $\overline{RP}$ and $\overline{CF}$ is right bisector of $\overline{PQ}$.	Right bisector of sides of Δ are con-current.
Therefore $\perp$'s $\overline{AD}$, $\overline{BE}$, $\overline{CF}$ are right bisector of sides of ΔPQR	
$\therefore$ $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ are concurrent	

Theorem

The bisectors of the angle of a triangle are concurrent.



Given:

I ΔABC bisectors of $\angle B$, $\angle C$ meet at I

Required:

Bisector of $\angle A$ passes through I.

Construction:

(i) Join A to I

(ii) Take $IF \perp \overline{AB}$

(iii) $ID \perp \overline{BC}$ and ,

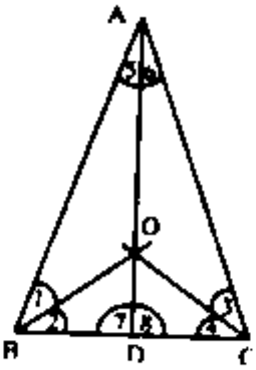
(iv) $IE \perp \overline{AC}$

Proof:

Statements	Reasons
(i) $\perp \overline{IF} \cong \perp ID$	I lies on bisector of angle B.
(ii) $\perp IE \cong \perp \overline{ID}$	
$\therefore$ $\perp IE \cong \perp IF$	I lies on bisector of angle C.
Hence I lies on bisector of $\angle A$.	From (i) and (ii)
Therefore $\overline{AI}$ is bisector of $\angle A$. Thus the bisector of angles of a triangle are concurrent.	

EXERCISE 12.3

Q.1 ***Prove that the bisectors of the angle of base of an isosceles triangle intersect each other on its altitude.***



Given: In ΔABC

(i) $\overline{AB} \cong \overline{AC}$

Bisectors of $\angle B$ and $\angle C$ intersect at O

(ii) A is joined to O and produce that cuts $\overline{BC}$ at D.

Required: $\overline{AD}$ is altitude of ΔABC

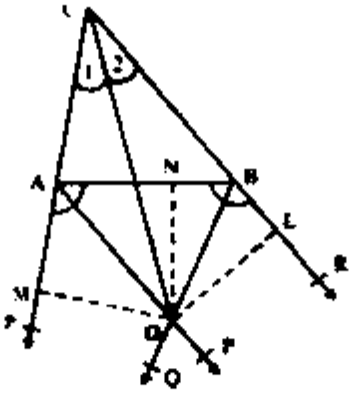
i.e. $\overline{AD} \perp \overline{BC}$

Proof:

Statements	Reasons
In ΔABC	
$\overline{AB} \cong \overline{AC}$	Given
$\angle ABC \cong \angle ACB$	Angles opposite to congruent sides.
$\therefore \frac{1}{2} m\angle ABC = \frac{1}{2} m\angle ACB$	
Thus $\angle 1 \cong \angle 3$	

Now in ΔOBC	Proved
$\angle 1 \cong \angle 3$	Opp. Sides of congruent angles
$\therefore$ $\overline{BO} \cong \overline{CO}$	
Now $\Delta AOC \longleftrightarrow \Delta AOB$	Common
$\overline{AO} \cong \overline{AO}$	Given
$\overline{AC} \cong \overline{AB}$	Proved
$\overline{BO} \cong \overline{CO}$	S.S.S. $\cong$ S.S.S
$\therefore$ $\Delta AOC \cong \Delta AOB$	
Thus $\angle 5 \cong \angle 6$	Common
Now in $\Delta ABD \longleftrightarrow \Delta ACD$	Proved
$\overline{AD} \cong \overline{AD}$	Given
$\angle 5 \cong \angle 6$	S.A.S. $\cong$ S.A.S.
$\overline{AB} \cong \overline{AC}$	Corresponding angles of congruent triangles
$\therefore$ $\Delta ABD \cong \Delta ACD$	Adj. supp. angles.
(i) $\therefore$ $m\angle 7 \cong 8$	From (i) and (ii)
(ii) point $m\angle 7 + m\angle 8 = 180^\circ$	
Thus $m\angle 7 = m\angle 8 = 90^\circ$	
Hence $\overline{AD} \perp \overline{BC}$	
And $\overline{AD}$ passes through point O	

Q.2 Prove that the bisectors of two exterior and third interior angle of triangle are concurrent.



Given: In ΔABC

$\angle ABR$ and $\angle BAP$ are two exterior angles.

Their $\overrightarrow{AP}$ and $\overrightarrow{BQ}$ bisectors intersect each other at O. C is joined to O.

Required: CO is bisector of angle $\angle C$. i.e. $\angle 1 \cong \angle 2$

Construction: $\overline{OL} \perp \overline{CR}$, $\overline{ON} \perp \overline{AB}$ and $\overline{OM} \perp \overline{CP}$

Proof:

Statements	Reasons
$\perp \overline{OL} \cong \perp \overline{ON}$ (i)	$\overline{BO}$ is bisector of $\angle ABR$
$\perp \overline{OM} \cong \perp \overline{ON}$ (ii)	$\overline{AO}$ is bisector of $\angle BAP$
$\therefore \perp \overline{OL} \cong \perp \overline{ON}$	From (i) and (ii)
Hence CO is bisector of $\angle C$	
i.e. $\angle 1 \cong \angle 2$	

Review Exercise 12

Q.1 Which of the following are true and which are false?

(i) Bisection means to divide into two equal parts.

(ii) Right bisection of line segment means to draw perpendicular which passes through the mid-point.

(iii) Any point on the right bisector of a line segment is not equidistant from its end points.

(iv) Any point equidistant from the end points of a line segment is on the right bisector of it.

(v) The right bisectors of the sides of a triangle are not concurrent.

(vi) The bisectors of the angles of a triangle are concurrent.

(vii) Any point on the bisector of an angle is not equidistant from its arms.

(viii) Any point inside an angle, equidistant from its arms, is on the bisector.

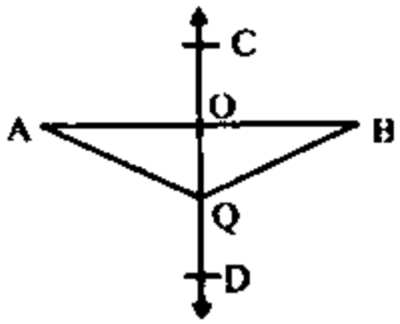
Answers:

(i)	T	*(ii)*	T	*(iii)*	F	*(iv)*	T
(v)	F	*(vi)*	T	*(vii)*	F	*(viii)*	T

Q.2 ***If $\overline{CD}$ is right bisector of line segment $\overline{AB}$, then***

(i) $m\overline{OA}$ = ________

(ii) $m\overline{AQ}$ = ________



Answers:

$m\overline{OA} = m\overline{OB}$

$m\overline{AQ} = m\overline{BQ}$

Q.3 ***Define the following:***

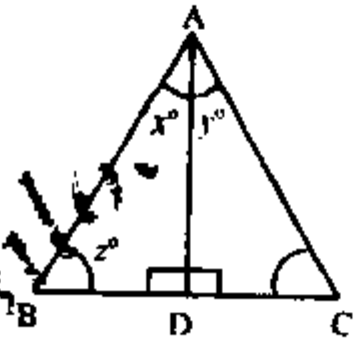
(i) Bisector of a line segment

(ii) Bisector of an angle

Answers: (i) A line is called the bisector of a line segment if it passes through its mid-point.

(ii) If a ray divides an angle into two congruent angles then that ray is called bisector of that angle.

Q.4 ***The given triangle ABC is equilateral triangle and $\overline{AD}$ is bisector of angle A, then find the values of unknowns $x°, y°$ and $z°$.***



Given: In ΔABC

$\overline{AB} \cong \overline{BC} \cong \overline{CA}$

i.e. $\angle B \cong \angle C \cong \angle BAC$ each $= 60°$

$\overline{AD}$ is bisector of $\angle A$ i.e. $x° = y°$

Required: To find $x°$, $y°$ and $z°$

Solution Proof:

$z° = 60$ given

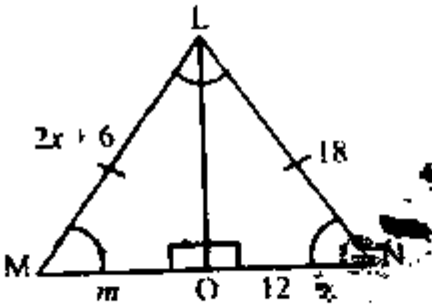
$\overline{AD}$ is bisector of $\angle A$

Now $m\angle A = 60°$

$$\frac{1}{2} m\angle A = x° = y° = \frac{60}{2}$$

$$x° = y° = 30°$$

Q.5 ***In the given congruent triangles LMO and LNO, find the unknowns x and m.***



Solution: In $\Delta LMO \longleftrightarrow \Delta LNO$

$$\overline{MO} \cong \overline{NO}$$

$$m = 12 \quad \text{(i)}$$

$$2x + 6 = 18$$

$$2x = 18 - 6$$

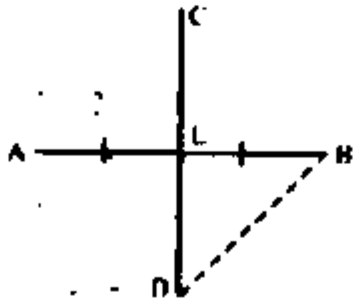
$$2x = 12$$

$$x = \frac{12}{2} = 6$$

Q.6 ***$\overline{CD}$ is right bisector of the line segment AB.***

(i) ***If $m\overline{AB} = 6$ cm, then find the $m\overline{AL}$ and $m\overline{LB}$.***

(ii) ***if $m\overline{BD} = 4$ cm, then find $m\overline{AD}$.***



In the given figure $\overline{CD}$ is right bisector of $\overline{AB}$

i.e. $\overline{AL} \cong \overline{BL}$

$m\overline{AB} = 6$ cm

$m\overline{BD} = 4$ cm

$m\overline{AL} = ?$

$m\overline{LB} = ?$

$m\overline{AD} = ?$

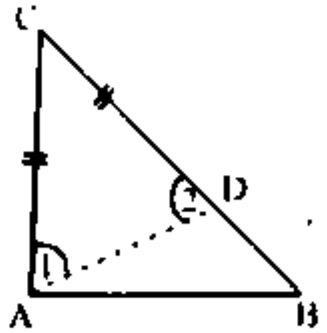
Proof:

Statements	*Reasons*
$\overline{AL} \cong \overline{LB}$ (i)	$\overline{CD}$ is right bisector of $\overline{AB}$
and $m\overline{AB} = 6$ cm	
Therefore	
$m\overline{AL} = m\overline{LB} = \frac{1}{2} m\overline{AB}$	
$= \frac{1}{2} \times 6 = 3$cm	
$m\overline{BD} = 4$cm (ii)	
$m\overline{AD} = m\overline{BD}$	Every point on right bisector is equidistant from end points
Hence $m\overline{AD} = 4$ cm	

Unit 13 SIDES AND ANGLES OF A TRIANGLE

Theorem

If two sides of a triangle are unequal in length, the longer side has an angle of greater measure opposite to it.



Given: In ΔABC
$m\overline{BC} > m\overline{AC}$

To prove: $m\angle A > m\angle B$

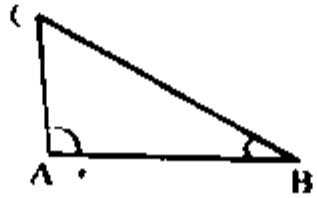
Construction: Form $\overline{CB}$ cut $\overline{CD} \cong \overline{CA}$
Join D to A.

Proof:

Statements	Reasons
In ΔACD	
$m\angle 1 = m\angle 2$ (i)	Angles opposite to congruent sides
In ΔABD	
$m\angle 2 > m\angle B$ (ii)	$\angle 2$ in exterior and $\angle B$ is interior
$\therefore$ $m\angle 1 > m\angle B$ (iii)	By (i) and (ii)
Now $m\angle A = m\angle 1 + m\angle BAD$	Addition of angles (postulate)
$m\angle A > m\angle 1$ (iv)	By (iii) and (iv)
$\therefore$ $m\angle A > m\angle 1 > m\angle B$	Transitive property of inequality of numbers.
or $m\angle A > m\angle B$	

Converse of Theorem

If two angles of a triangle are unequal in measure, the side opposite to the greater angle is longer than the side opposite to the smaller angle.



Given: In ΔABC

$m\angle A \not> m\angle B$

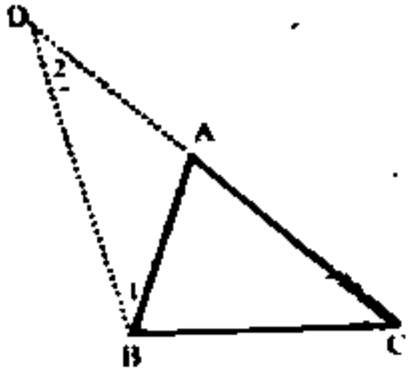
Required: $m\overline{BC} > m\overline{AC}$

Proof:

Statements	***Reasons***
If $m\overline{BC} > m\overline{AC}$	
Then (i) $m\overline{BC} = m\overline{AC}$	Trichotomy property of numbers.
or (ii) $m\overline{BC} < m\overline{AC}$	
For first condition	
If $m\overline{BC} = m\overline{AC}$	
Then $m\angle A = m\angle B$	Opposite to given
Which is not possible	
For recondition condition	
If $m\overline{BC} < m\overline{AC}$	
Then $m\angle A < m\angle B$	Contrary to given
This is also not possible	
Now $m\overline{BC} \neq m\overline{AC}$	Trichotomy property of numbers.
And $m\overline{BC} \not< m\overline{AC}$	
Hence $m\overline{BC} > m\overline{AC}$	

Theorem

The sum of the lengths of any two sides of a triangle is greater than the length of the third side



Given: ΔABC

To Prove:
(i) $m\overline{AB} + m\overline{AC} > m\overline{BC}$
(ii) $m\overline{AB} + m\overline{BC} > m\overline{AC}$
(iii) $m\overline{BC} + m\overline{CA} > m\overline{AB}$

Construction:

Take a point D on $\overrightarrow{CA}$ such that $\overline{AD} \cong \overline{AB}$. Join B to D and name the angles. $\angle 1$, $\angle 2$ as shown in the given figure.

Proof:

Statements	Reasons
In ΔABD	
$\angle 1 \cong \angle 2$ (i)	$\overline{AD} \cong \overline{AB}$ construction
$m\angle DBC > m\angle 1$ (ii)	$m\angle DBC = m\angle 1 + m\angle ABC$
$\therefore m\angle DBC > m\angle 2$ (iii)	From (i) and (ii)
In $\Delta ABD \longleftrightarrow \Delta ECD$	Given
$\overline{BD} \cong \overline{CD}$	Vertical angles
$\angle 1 \cong \angle 2$	Construction
$\overline{AD} \cong \overline{ED}$	S.A.S. Postulate
$\Delta ABD \cong \Delta ECD$	Corresponding sides of $\cong \Delta^s$
$\overline{AB} \cong \overline{EC}$ (I)	ACE is a triangle
$m\overline{AC} + m\overline{EC} > m\overline{AE}$ (II)	Form I and II
$m\overline{AC} + m\overline{AB} > m\overline{AE}$	$m\overline{AE} = 2m\overline{AD}$ (construction)
Hence $m\overline{AC} + m\overline{AB} > 2m\overline{AD}$	

EXERCISE 13.1

Q.1 ***Two sides of a triangle measure 10cm and 15cm. Which of the following measure is possible for the third side?***

(a) *5 cm* *(b)* *20 cm* *(c)* *25 cm* *(d)* *30cm*

Solution:

Length of one side = 10 cm

Length of second side = 15 cm

Sum of length of two sides = 10 + 15 = 25 cm

(a) $5 < 25$

(b) $20 < 25$

(c) $25 = 25$

(d) $30 > 25$

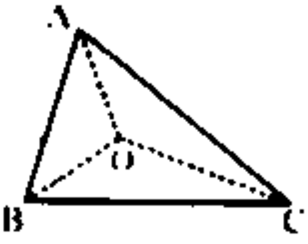
Sum of lengths of two sides is always greater than the third side of a triangle.

Therefore c, d are not possible.

a and b are possible.

Q.2 ***O is an interior point of the ΔABC. Show that***

$$m\overline{OA} + m\overline{OB} + m\overline{OC} > \frac{1}{2}(m\overline{AB} + m\overline{BC} + m\overline{CA})$$



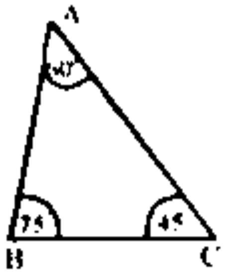
Given: Point O is in the interior of ΔABC

Required: $m\overline{OA} + m\overline{OB} + m\overline{OC} > \frac{1}{2}(m\overline{AB} + m\overline{BC} + m\overline{CA})$

Proof:

Statements	*Reasons*
In ΔOAC	
(i) $m\overline{OA} + m\overline{OC} > m\overline{AC}$	Sum of lengths of two sides is in greater than the length of the third side.
In ΔOAB	
(ii) $m\overline{OA} + m\overline{OB} > m\overline{AB}$	Sum of lengths of two sides is in greater than the length of the third side.
In $BOC\Delta$	
(iii) $m\overline{OB} + m\overline{OC} > m\overline{BC}$	Sum of lengths of two sides is in greater than the length of the third side.
$2m\overline{OA} + 2m\overline{OB} + 2m\overline{OC} > m\overline{AB} + m\overline{BC} + m\overline{CA}$	Adding (i), (ii), (iii)
$2(m\overline{OA} + m\overline{OB} + m\overline{OC}) > m\overline{AB} + m\overline{BC} + m\overline{CA}$	
$m\overline{OA} + m\overline{OB} + m\overline{OC} > \frac{1}{2}(m\overline{AB} + m\overline{BC} + m\overline{CA})$	Dividing by 2

Q.3 In the ΔABC, $m\angle B = 70°$ and $m\angle C = 45°$ which of the sides of the triangle is longest and which is the shortest?



Solution:

Given: In ΔABC

$m\angle B = 70°$, $m\angle C = 45°$

Required: (i) Which is longest from among $\overline{AB}, \overline{BC}, \overline{CA}$

(ii) Which is shortest from among $\overline{AB}, \overline{BC}$, CA

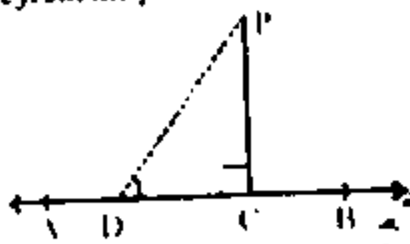
Proof:

Statements	*Reasons*
In ΔABC	
$m\angle A + m\angle B + m\angle C = 180°$	Sum of angles of Δ
$m\angle A + 75° + 45° = 180°$	$m\angle B = 75°$, $m\angle C = 45°$ given
$m\angle A + 120° = 180°$	
$m\angle A = 180° - 120°$	
Hence $m\angle A = 60°$	
Now $m\angle A = 60°$	
$m\angle B = 75°$	
$m\angle C = 45°$	
$\overline{AC}$ is the longest	Opposite to the greatest angle
$\overline{AB}$ is the shortest	Opposite to the smallest angle

$\overline{BD} > \overline{DC}$	

Theorem

From a point outside a line, the perpendicular is the shortest distance from the point to the line.



Given: (i) P is a point out side $\overleftrightarrow{AB}$
(ii) $\overline{PC} \perp \overleftrightarrow{AB}$
(iii) D is any point on $\overleftrightarrow{AB}$
(iv) P is joined to D.

Required: $m\overline{PC} < m\overline{PD}$

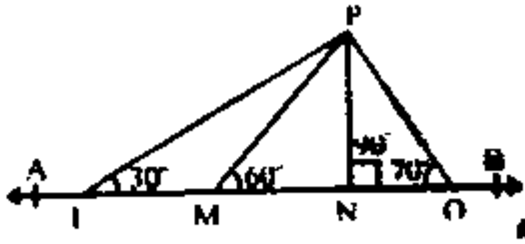
Proof:

Statements	*Reasons*
In ΔPDC	
$m\angle C = 90°$	Given
Now	
$m\angle D + m\angle P + m\angle C = 180°$	Sum of angles of ΔPDC
$m\angle D + m\angle P + 90° = 180°$	$m\angle C = 90°$ (given)
$m\angle D + m\angle P = 180° - 90°$	
$m\angle D + m\angle P = 90°$ (i)	
Therefore $m\angle D < 90°$	from (i)
And $m\angle P < 90°$	from (i)
Thus $m\angle D < m\angle C$	
And $m\overline{PC} < m\overline{PD}$	Side opposite to smaller

Hence all distance from P to $\overline{AB}$ will greater than $\overline{PC}$	angle

EXERCISE 13.2

Q.1 ***In the figure. P is any point and AB is a line. Which of te following is the shortest distance between the point P and the line $\overline{AB}$.***



(a) $m\overline{PL}$ *(b)* $m\overline{PM}$

(c) $m\overline{PN}$ *(d)* $m\overline{PO}$

Solution: $m\overline{PN}$ is the shortest distance from P to $\overline{AB}$

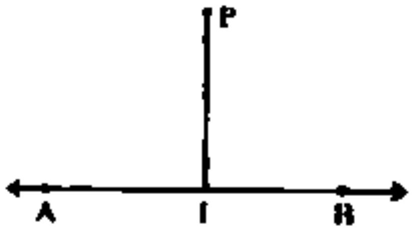
Proof:

Statements	*Reasons*
In ΔPNO $m\overline{PN} < m\overline{PO}$ (i)	$m\angle N = 90°$ $m\angle O = 70°$ smaller angle has small side opposite to
In ΔPMO $m\overline{PM} > m\overline{PO}$ (ii)	$m\angle O = 70°$, $m\angle M = 60°$
In ΔPLO $m\overline{PL} > m\overline{PO}$ (iii)	$m\angle L = 30°$ $m\angle O = 70°$ small angle has smaller side opposite to it.
Hence $mPN < m\overline{PO}$ $m\overline{PM} > m\overline{PO}$ $m\overline{PL} > mPO$ Therefore $m\overline{PN}$ is the smallest of all $\overline{PO}, \overline{PM}, \overline{PL}$	

Q.2 ***In the figure. P is any point lying away from the line*** $\overline{AB}$. ***Then*** $m\overline{PL}$ ***will be the shortest distance if***

(a) $m\angle PLA = 80^\circ$ (b) $m\angle PLB = 100^\circ$

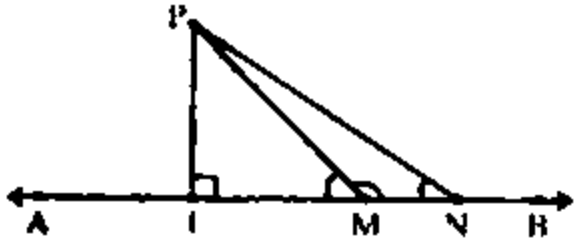
(c) $m\angle PLA = 90^\circ$



Answer: If $m\angle PLA = 90^\circ$

Reason: The perpendicular distance from a point out-side the line is the shortest distance from that point to the given line.

Q.3 ***In the figure.*** $\overline{PL}$ ***is perpendicular to the line*** $\overline{AB}$ ***and*** $m\overline{LN} > m\overline{LM}$. ***Prove that*** $m\overline{PN} > m\overline{PM}$.



Given: In the given figure

(i) $\overline{PL} \perp \overline{AB}$

(ii) $m\overline{LN} > m\overline{LM}$

Required: $m\overline{PN} > m\overline{PM}$

Proof:

Statements	***Reasons***
In ΔPLM	
$m\angle PLM = 90^\circ$	Given
$m\angle PLM < 90^\circ$ (i)	In a right angle triangle other two angles an active.
$m\angle PLM + m\angle PMN = 180^\circ$ (ii)	Adj. Supp. Angles
(iii) $m\angle PMN > 90^\circ$	
In ΔPLN	Given
$m\angle L = 90^\circ$	
(iv) $m\angle PNL \angle 90^\circ$	
$m\angle PMN > m\angle PNM$	
In ΔPMN	Proved
$m\angle PMN > m\angle PNM$	
$\therefore$ $m\overline{PN} > m\overline{PM}$	

Review Exercise 13

Q.1 Which of the following are true and which are false?

(i) The angle opposite to the longer side is greater.

(ii) In a right-angled triangle greater angle is of 60°.

(iii) In an isosceles right-angled triangle, angles other than right angle are each of 45°.

(iv) A triangle having two congruent sides is called equilateral triangle.

(v) A perpendicular from a point to line is shortest distance.

(vi) Perpendicular to line form an angle of 90°

(vii) A point out side the line is collinear.

(viii) Sum of two sides of triangle is greater than the third.

(ix) The distance between a line and a point on it is zero.

(x) Triangle can be formed of lengths 2 cm, 3 cm and 5 cm.

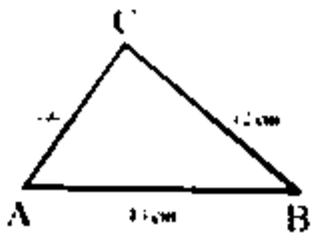
Answers:

(i)	T	*(ii)*	F	*(iii)*	T	*(iv)*	F	*(v)*	T
(vi)	T	*(vii)*	F	*(viii)*	T	*(ix)*	T	*(x)*	F

Q.2 What will be angle for shortest distance from an outside point to the line?

Answers: 90°

Q.3 If 13 cm, 12 cm and 5 cm are the length of a triangle, then verify that difference of measures of any two sides of a triangle is less than the measure of the third side.



Let $m\overline{AB}$ 13 cm

$m\overline{BC} = 12$ cm

$m\overline{CA} = 5$ cm

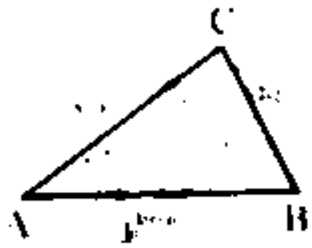
Now $m\overline{AB} - m\overline{BC} = 13 - 12 = 1$ cm $< \overline{CA}$ ($m\overline{CA} = 5$cm)

$m\overline{BC} - m\overline{CA} = 12 - 5 = 7$cm $< \overline{AB}$ ($m\overline{AB} = 13$cm)

$m\overline{AB} - m\overline{CA} = 13 - 5 = 8$cm $< \overline{BC}$ ($m\overline{BC} = 12$cm)

It show that difference of measure of any two sides of a triangle is less than the measure of the third side.

Q.4 ***If 10 cm, 6 cm and 8 cm are the lengths of a triangle, then verify that sum of measures of two sides of a triangle is greater than the third side..***



Let $m\overline{AB} = 10$ cm

$m\overline{BC} = 6$ cm

$m\overline{CA} = 8$ cm

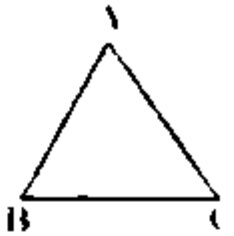
Now $m\overline{AB} + m\overline{BC} = 10 + 6 = 16$cm $> m\overline{CA}$ ($m\overline{CA} = 8$cm)

$m\overline{BC} + m\overline{CA} = 6 + 8 = 14$cm $\overline{AB}$ ($m\overline{AB} = 10$cm)

$m\overline{CA} + m\overline{AB} = 8 + 10 = 18 > m< \overline{BC}$ ($m\overline{BC} = 6$cm)

This verifies that sum of length of any two sides of a triangle is greater than the length of the third side.

Q.5 3 cm, 4 cm and 7 cm are not the lengths of the triangle. Give the reason.



Let $m\overline{AB} = 7$ cm

$m\overline{BC} = 4$ cm

$m\overline{CA} = 3$ cm

Now (i) $m\overline{AB} + m\overline{BC} = 7 + 4 = 11 > m\overline{CA}\ (m\overline{CA} = 3\text{cm})$

(ii) $m\overline{BC} + m\overline{CA} = 4 + 3 = 7 = m\overline{AB}\ (m\overline{AB} = 7\text{cm})$

(iii) $m\overline{CA} + m\overline{AB} = 3 + 7 = 10 > \overline{BC}\ (m\overline{BC} = 4\text{cm})$

Part (ii) is not possible

Therefore 7 cm, 4 cm, 3 cm are not the sides of a triangle.

Q.6. If 3 cm and 4 cm are lengths of two sides of a right triangle then what should be the third length of the triangle.

Solution:-

Let length of hypotenuse $= x$ cm

$$(x)^2 = (3)^2 + (4)^2$$
$$= 9 + 16$$
$$x^2 = 25$$
$$x = \sqrt{25} \quad \text{(taking sq. root)}$$
$$x = 5 \text{ cm}$$

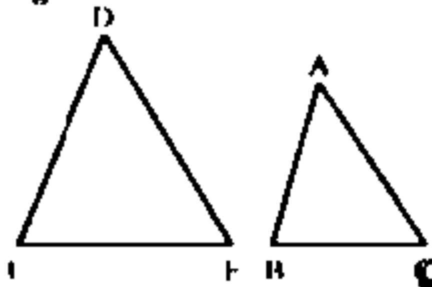
$\therefore$ Length of hypotenuse $= 5$ cm

Unit 14

RATIO AND PROPORTIONAL

Similar Triangles:

Two triangles ΔABC and ΔDEF are similar if



In $\Delta ABC \leftrightarrow \Delta DEF$

(i) $\angle A \cong \angle D$

$\angle B \cong \angle E$

$\angle C \cong \angle F$

and ***(ii)*** $\dfrac{m\overline{AB}}{m\overline{DE}} = \dfrac{m\overline{BC}}{m\overline{EF}} = \dfrac{m\overline{CA}}{m\overline{FD}}$

Symbolically we write it as

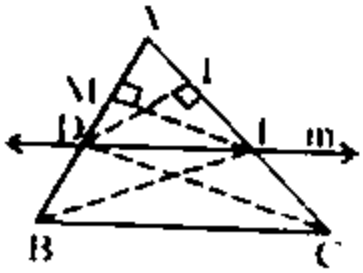
$\Delta ABC \sim \Delta DEF$

Remember:

Two congruent triangles are similar. Two similar triangles may not be congruent.

Theorem

A line parallel to one side of a triangle and intersecting the other two sides divides them proportionally.



Given:

(i) In ΔABC, line m cuts $\overline{AB}$ and $\overline{AC}$ at D and E respectively.

(ii) $\overline{DE} \parallel \overline{BC}$

Required:

$$\frac{m\overline{AD}}{m\overline{DB}} = \frac{m\overline{AE}}{m\overline{EC}}$$

Construction:

(i) Join C to D

(ii) Join E to B

(iii) Take $\overline{DL} \perp \overline{AC}$ and $\overline{EM} \perp \overline{AB}$

Statements	***Reasons***
In Δ^s ADE and CDE	
$\overline{DL}$ is common Altitude	
$\Delta CDE = \frac{1}{2} \times \overline{EC} \times \overline{DL}$ (i)	Area$\Delta = \frac{1}{2}\times$base$\times$Altitude
$\Delta ADE = \frac{1}{2} \times \overline{AE} \times \overline{DL}$ (ii)	
(iii) $\frac{\Delta CDE}{\Delta ADE} = \frac{m\overline{EC}}{m\overline{AE}}$	Dividing (i) by (ii)
Similarly	

(iv) $\frac{\Delta BED}{\Delta AED} = \frac{m\overline{DB}}{m\overline{AD}}$	Base $\overline{BC}$ is common and lie between ∥ lines
But $\Delta CDE \cong \Delta BED$	
So $\frac{m\overline{EC}}{m\overline{AE}} = \frac{m\overline{DB}}{m\overline{AD}}$	Invert do
or $\frac{m\overline{AE}}{m\overline{EC}} = \frac{m\overline{AD}}{m\overline{DB}}$	Invertendo
Thus $\frac{m\overline{AD}}{m\overline{DB}} = \frac{m\overline{AE}}{m\overline{EC}}$	

Theorem

If a line segment intersects the two sides of a triangle in the same ratio then it is parallel to the third side.

Given:

In ΔABC, a line segment cuts $\overline{AB}$ at D and $\overline{AC}$ at E

that $\frac{m\overline{AD}}{m\overline{DB}} = \frac{m\overline{AE}}{m\overline{EC}}$

Required:

$\overline{DE} \parallel \overline{BC}$

Construction:

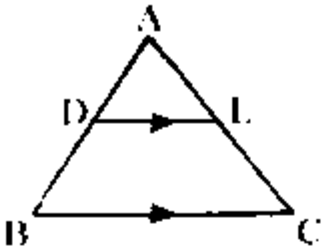
Suppose $\overline{DE} \nparallel \overline{BC}$, then take $\overline{CP} \parallel \overline{DE}$ that meets $\overline{AB}$ produced at P.

Proof:

Statements	*Reasons*
In ΔAPC	
$\overline{DE} \parallel \overline{CP}$	Construction
Therefore	
$\dfrac{m\overline{AD}}{m\overline{DP}} = \dfrac{m\overline{AE}}{m\overline{EC}}$ (i)	
But $\dfrac{m\overline{AD}}{m\overline{DP}} = \dfrac{m\overline{AE}}{m\overline{EC}}$ (ii)	Given
$\therefore$ $\dfrac{m\overline{AD}}{m\overline{DB}} = \dfrac{m\overline{AD}}{m\overline{DP}}$	From (i) and (ii)
or $m\overline{DB} = m\overline{DP}$	Property of real numbers.
Therefore P lies on P	
Hence our supposition is wrong	
Hence $\overline{DE} \parallel \overline{BC}$	

Exercise 14.1

Q.1. ***In ΔABC, $\overline{DE} \parallel BC$***



In ΔABC, $\overline{DE} \parallel \overline{BC}$.

(i) If $\overline{AD} = 1.5$ cm, $\overline{BD} = 3$ cm, $\overline{AE} = 1.3$ cm, then find $\overline{CE}$.

(ii) If $\overline{AD} = 2.4$ cm, $\overline{AE} = 3.2$ cm, $\overline{EC} = 4.8$ cm, find $\overline{AB}$.

(iii) If $\dfrac{\overline{AD}}{\overline{DB}} = \dfrac{3}{5}$ cm, $\overline{AC} = 4.8$, find $\overline{AE}$.

(iv) If $\overline{AD} = 2.4$ cm, $\overline{AE} = 3.2$ cm, $\overline{DE} = 2$ cm, $\overline{BC} = 5$ cm, find $\overline{AB}$, $\overline{DB}$, $\overline{AC}$, $\overline{CE}$.

(v) If $AD = 4x - 3$, $AE = 8x - 7$, $BD = 3x - 1$, and $CE = 5x - 3$, find the value of x.

Solutions:

In ΔABC, $\overline{DE} \parallel \overline{BC}$

(i)

$\overline{AD} = 1.5$ cm

$\overline{BD} = 3$ cm

$\overline{AE} = 1.3$ cm

$\overline{CE} = ?$

In ΔABC $\quad \overline{DE} \parallel \overline{BC}$

Let $m\overline{CE} = x$ cm

$\therefore \quad \dfrac{\overline{AD}}{\overline{DB}} = \dfrac{\overline{AE}}{\overline{EC}}$

Putting values of $\overline{AD}$, $\overline{BD}$ and $\overline{AE}$

$$\frac{1.5}{3} = \frac{1.3}{x}$$

$$x = \frac{1.3 \times 3}{1.5}$$

or $$x = \frac{3.9}{1.5} = \frac{39}{15} = 2.6 \text{ cm}$$

(ii) $\overline{AD} = 2.4$ cm

$\overline{AE} = 3.2$ cm

$\overline{EC} = 4.8$ cm

$AB = ?$

In ΔABC $\quad \overline{DE} \parallel \overline{BC}$

Now $m\overline{AB} = m\overline{AD} + m\overline{DB}$

We find $m\overline{DB}$

Let $m\overline{DB} = x$ cm

Now $\dfrac{\overline{AD}}{\overline{DB}} = \dfrac{\overline{AE}}{\overline{EC}}$

Putting values of $\overline{AD}$, $\overline{AE}$, $\overline{EC}$

$$\frac{2.4}{x} = \frac{3.2}{4.8}$$

$$x = \frac{4.8 \times 2.4}{3.2} = \frac{48 \times 24}{10 \times 10} \times \frac{10}{32}$$

$= \frac{48}{10} \cdot \frac{24}{?} \times \frac{1}{32} = 3.6\ cm$

(iii)

$\frac{m\overline{AD}}{m\overline{DB}} = \frac{3}{5}$

$\overline{AB} = m\overline{AD} + m\overline{DB} = 2.4 + 3.6 = 6\ cm$

$m\overline{AC} = 4.8\ cm$

In $\triangle ABC$

$\because \overline{DE} \parallel \overline{BC}$

$\therefore \frac{\overline{AD}}{\overline{DB}} = \frac{\overline{AE}}{\overline{EC}}$

and $\frac{\overline{AD}}{\overline{DB}} = \frac{\overline{AC} - \overline{CE}}{\overline{CE}}$

Putting values of AD, DB, AC

$\frac{3}{5} = \frac{4.8 - \overline{CE}}{\overline{CE}}$

or $3\overline{CE} = 5(4.8 - \overline{CE})$

$3\overline{CE} = 24.0 - 5\overline{CE}$

$3\overline{CE} + 5\overline{CE} = 24$

$8\overline{CE} = 24$

$\overline{CE} = \frac{24}{8} = 3\ cm$

Now $m\overline{AE} = \overline{AC} - \overline{CE}$

$\therefore m\overline{AE} = 4.8 - 3 = 1.8\ cm$

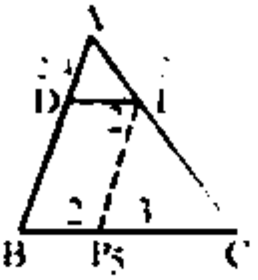
(iv) $\overline{AD} = 2.4$ cm *(i)*

$\overline{AE} = 3.2$ cm *(ii)*

$\overline{DE} = 2$ cm *(iii)*

$\overline{BC} = 5$ cm

$\overline{AB} = ?$, $\overline{DB} = ?$, $\overline{AC} = ?$, $\overline{CE} = ?$



Construction:

Take $\overline{EP} \parallel \overline{AB}$

Now DEPB is $\|^{m}$

$\therefore \quad m\overline{PB} = m\overline{DE} = 2$ cm

Thus $m\overline{CP} = 5 - 2 = 3$ cm

Now in ΔABC $\overline{EP} \parallel \overline{AB}$

$$\frac{\overline{CE}}{\overline{EA}} = \frac{\overline{CP}}{\overline{PB}}$$

Putting values of EA, CP, PB

$$\frac{\overline{CE}}{3.2} = \frac{3}{2}$$

$$\overline{CE} = \frac{3 \times 3.2}{2}$$

$\overline{CE} = 3 \times 1.6 = 4.8$ cm *(iv)*

Now in ΔABC $\overline{DE} \parallel \overline{BC}$

$$\therefore \quad \frac{\overline{BD}}{\overline{AD}} = \frac{\overline{CE}}{\overline{AE}}$$

$$\frac{\overline{BD}}{2.4} = \frac{4.8}{3.2}$$

(Putting the values of $\overline{AD}$, $\overline{CE}$, $\overline{AE}$)

$$BD = \frac{2.4 \times 4.8}{3.2} \qquad (v)$$

$= 3.6$ cm

Now $\overline{AB} = \overline{AD} + \overline{DB}$

$\therefore \quad = 2.4 + 3.6 \qquad$ *(i), (v)*

$= 6.0$ cm

Hence $\overline{DB} = 3.8$ cm $\qquad$ *(v)*

$\overline{AC} = \overline{AE} + \overline{EC}$

$= 3.2 + 4.8 \qquad$ From *(ii), (iv)*

$= 8.0$ cm.

$\overline{CE} = 4.8$ cm $\qquad$ From *(iv)*

I(v)

$AD = 4x - 3$
$AE = 8x - 7$
$\overline{BD} = 3x - 1$
$CE = 5x - 3$
$x = ?$

Now in ΔABC $\overline{DE} \parallel \overline{BC}$

$$\therefore \quad \frac{AD}{BD} = \frac{AE}{CE}$$

Putting values

$$\frac{4x-3}{3x-1} = \frac{8x-7}{5x-3}$$

Now $(4x-3)(5x-3) = (8x-7)(3x-1)$

$20x^2 - 27x + 9 = 24x^2 - 29x + 7$

or $20x^2 - 24x^2 - 27x + 29x + 9 - 7 = 0$

$-4x^2 + 2x + 2 = 0$

Dividing by 2

or $2x^2 - x - 1 = 0$

$2x^2 - 2x + x - 1 = 0$

$2x(x-1) + 1(x-1) = 0$

$(x-1)(2x+1) = 0$

or $x - 1 = 0$

$x = 1$

or $2x + 1 = 0$

$2x = -1$

$x = -\frac{1}{2}$

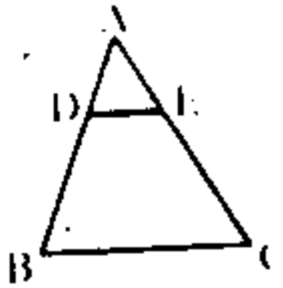
$x = -\frac{1}{2}$ (impossible)

$\therefore$ $x = 1$

Q.2. ***If ΔABC is an isosceles triangle, $\angle A$ is vertex angle and $\overline{DE}$ intersects the sides $\overline{AB}$ and $\overline{AC}$ as shown in the figure so that***

$m\overline{AD} : m\overline{DB} = m\overline{AE} : m\overline{EC}$.

prove that ΔADE is also an isosceles triangle.



Given:

In $\triangle ABC$, $\angle A$ is its vertex.

$$\overline{AB} = \overline{AC}$$

$$\frac{m\overline{AD}}{m\overline{DB}} = \frac{m\overline{AE}}{m\overline{EC}}$$

Required:

$$\overline{AD} \cong \overline{AE}$$

Proof:

Now $\dfrac{m\overline{DB}}{m\overline{AD}} = \dfrac{m\overline{EC}}{m\overline{AE}}$ given

or $\dfrac{m\overline{DB} + m\overline{AD}}{m\overline{AD}} = \dfrac{m\overline{EC} + m\overline{AE}}{m\overline{AE}}$

or $\dfrac{m\overline{AB}}{m\overline{AD}} = \dfrac{m\overline{AC}}{m\overline{AE}}$

$\therefore$ $m\overline{AD} = m\overline{AE}$

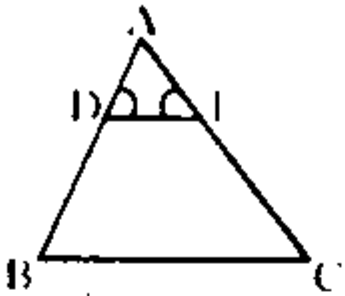
$\therefore$ $m\overline{AB} = m\overline{AC}$ (given)

$\therefore \triangle ADE$ is also an isosceles triangle.

Q.3. ***In an equilateral triangle ABC shown in the figure.***

$$m\overline{AE} : m\overline{AC} = m\overline{AD} : m\overline{AB}$$

Find all the three angles of ΔADE and name it also.



Given:

ABC is an equilateral triangle i.e. $\overline{AB} \cong \overline{BC} \cong \overline{AC}$ also

$m\angle A = m\angle B = m\angle c = 60^\circ$ and $\dfrac{m\overline{AE}}{m\overline{AC}} = \dfrac{m\overline{AD}}{m\overline{AB}}$

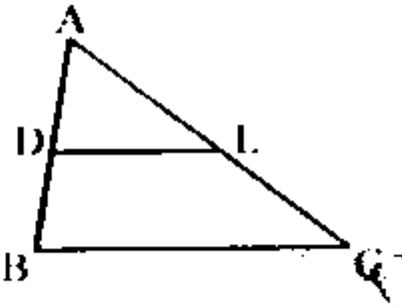
Required:

To find measures of angles of ΔADE.

Solution:

Statements	*Reasons*
$\dfrac{m\overline{AE}}{m\overline{AC}} = \dfrac{m\overline{AD}}{m\overline{AB}}$	Given
$\therefore \overline{DE} \parallel \overline{BC}$	
$m\angle A = m\angle B = m\angle C = 60^\circ$	Given
Now $\overline{DE} \parallel \overline{BC}$	Corresponding angles
$\therefore m\angle 1 = m\angle B$ $= 60^\circ$	Corresponding angles
and $m\angle 2 = m\angle C$ $= 60^\circ$	Corresponding angles
$m\angle A = 60$	
ΔADE is an isosceles triangle	Given

Q.4. ***Prove that the line segment drawn through the mid-point of one side of a triangle and parallel to another side bisects the third side.***



Given:

In ΔABC

$m\overline{AD} = m\overline{DB}$ ...... (i)

$\overline{DE} \parallel \overline{BC}$

Required:

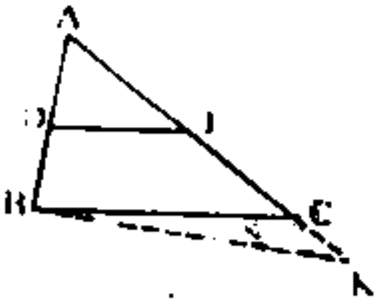
$m\overline{AE} = m\overline{EC}$

Proof:

Statements	***Reasons***
In ΔABC	
$\because$ $\overline{DE} \parallel \overline{BC}$	Given
(i) $\dfrac{m\overline{AD}}{m\overline{DB}} = \dfrac{m\overline{AE}}{m\overline{EC}}$	Theorem
Now $m\overline{AD} = m\overline{DB}$	Given
$\dfrac{m\overline{DB}}{m\overline{DB}} = \dfrac{m\overline{AE}}{m\overline{EC}}$	Putting $m\overline{AD} = m\overline{DB}$ in (i)

$1=\dfrac{m\overline{AE}}{m\overline{EC}}$	
or $m\overline{AE}=m\overline{EC}$	
Hence, $\overline{DE}$ bisects $\overline{AC}$	

Q.5. ***Prove that the line segment joining the mid-points of any two sides of a triangle is parallel to the third side.***



Given:

In ΔABC

$$m\overline{AD}=m\overline{DB}$$

$$m\overline{AE}=m\overline{EC}$$

i.e; $\dfrac{m\overline{AD}}{m\overline{DB}}=\dfrac{m\overline{AE}}{m\overline{EC}}$

Required:

$\overline{DE}\parallel\overline{BC}$

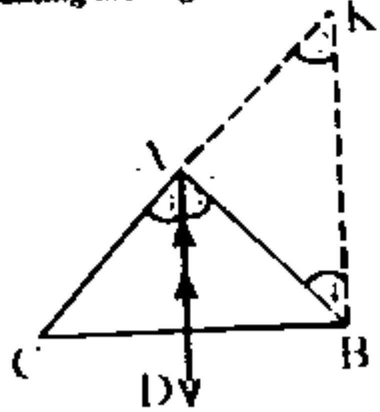
Proof:

Statements	Reasons
If $\overline{DE}\nparallel\overline{BC}$ then take $\overline{BK}\parallel\overline{DE}$ which cuts produced $\overline{AC}$ at K.	

Now	
(i) $\frac{m\overline{AD}}{m\overline{DB}} = \frac{m\overline{AE}}{m\overline{EK}}$	
(ii) Now $\frac{m\overline{AD}}{m\overline{DB}} = \frac{m\overline{AE}}{m\overline{EC}}$	Given
$\frac{m\overline{AE}}{m\overline{EK}} = \frac{m\overline{AE}}{m\overline{EC}}$	(From i. and ii)
i.e: $m\overline{EK} = m\overline{EC}$ This is possible only when K lies on C.	
Hence, $\overline{DE} \parallel \overline{BC}$	

Theorem

The internal bisector of an angle of a triangle divides the side opposite to it in the ratio of the length of the sides containing the angle.



Given:

In $\triangle ABC$ internal bisector of $\angle A$ meets $\overline{CB}$ at D.

To prove:

$$\frac{m\overline{AC}}{m\overline{AB}} = \frac{m\overline{CD}}{m\overline{BD}}$$

Construction:

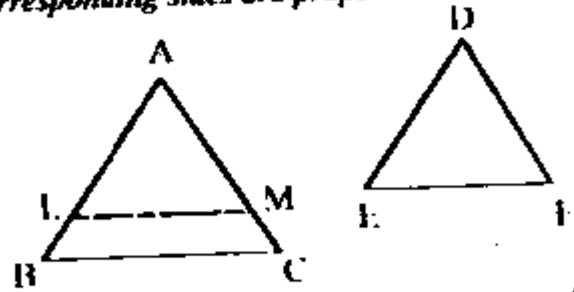
Draw $\overline{BK} \parallel \overline{DA}$ which meets $\overline{CA}$ produced at K.

Proof:

Statements	Reasons
$\overline{AD} \parallel \overline{BK}$ and $\overline{CK}$ cuts them $\therefore\ m\angle 1 = m\angle 2$	Corresponding angles
$\overline{AD} \parallel \overline{KB}$ and $\overline{AB}$ cuts them $\therefore\ m\angle 3 = m\angle 4$	Alternate angles
But $m\angle 1 = m\angle 3$	Given
$\therefore\ m\angle 2 = m\angle 4$	
and $\overline{AB} \cong \overline{AK}$ or $\overline{AK} \cong \overline{AB}$	
Now $\overline{AD} \parallel \overline{KB}$	Construction
$\therefore\ \frac{m\overline{BD}}{m\overline{DC}} = \frac{m\overline{KA}}{m\overline{AC}}$	Theorem
or $\frac{m\overline{BD}}{m\overline{DC}} = \frac{m\overline{AB}}{m\overline{AC}}$	
or $\frac{m\overline{AC}}{m\overline{AB}} = \frac{m\overline{DC}}{m\overline{BD}}$	Proved.

Theorem

If two triangles are similar, then the measures of their corresponding sides are proportional.



Given:

$\Delta ABC \sim \Delta DEF$

i.e: $\angle A \cong \angle D$, $\angle B \cong \angle E$ and $\angle C \cong \angle F$

To Prove:

$$\frac{m\overline{AB}}{m\overline{DE}} = \frac{m\overline{AC}}{m\overline{DF}} = \frac{m\overline{BC}}{m\overline{EF}}$$

Construction:

(i) Suppose that $m\overline{AB} > m\overline{DE}$

(ii) $m\overline{AB} \leq m\overline{DE}$

On $\overline{AB}$ take a point L such that $m\overline{AL} = m\overline{DE}$.

On $\overline{AC}$ take a point M such that $m\overline{AM} = m\overline{DF}$. Join L and M by the line segment LM.

Proof:

Statements	Reasons
(I) In $\Delta ALM \longleftrightarrow \Delta DEF$	
$\angle A \cong \angle D$	Given
$\overline{AL} \cong \overline{DE}$	Construction
$\overline{AM} \cong \overline{DF}$	Construction
Thus $\Delta ALM \cong \Delta DEF$	S.A.S. postulate
$\angle L \cong \angle E$	Corresponding angles

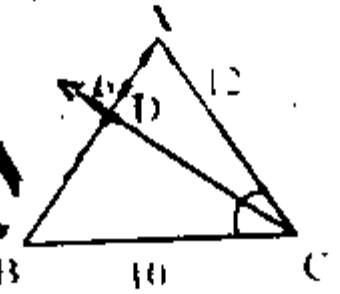
Statements	Reasons
$\angle M \cong \angle F$	of congruent Δs
and $\angle L \cong \angle E$, $\angle M \cong \angle F$	(Corresponding angles of congruent triangles)
Now $\angle E \cong \angle B$ and $\angle F \cong \angle C$	Given
$\therefore \angle L \cong \angle B$, $\angle M \cong \angle C$	Transitivity of congruence
Thus $\overline{LM} \parallel \overline{BC}$	Corresponding angles are equal.
Hence, $\dfrac{m\overline{AL}}{m\overline{AB}} = \dfrac{m\overline{DF}}{m\overline{AC}}$	by Theorem 14.1.1
or $\dfrac{m\overline{DE}}{m\overline{AB}} = \dfrac{m\overline{DF}}{m\overline{AC}}$ (i)	$m\overline{AL} = m\overline{DE}$ and $m\overline{AM} = m\overline{DF}$ (Construction)
Similarly by intercepting segments on $\overline{BA}$ and $\overline{BC}$, we can prove that $\dfrac{m\overline{DE}}{m\overline{AB}} = \dfrac{m\overline{EF}}{m\overline{BC}}$ (ii)	
Thus $\dfrac{m\overline{DE}}{m\overline{AB}} = \dfrac{m\overline{DF}}{m\overline{AC}} = \dfrac{m\overline{EF}}{m\overline{BC}}$	by (i) and (ii)
or $\dfrac{m\overline{AB}}{m\overline{DE}} = \dfrac{m\overline{AC}}{m\overline{DF}} = \dfrac{m\overline{BC}}{m\overline{EF}}$	by taking reciprocals
(II) If $m\overline{AB} < m\overline{DE}$, it can similarly be proved by taking intercepts on the sides of ΔDEF.	
If $m\overline{AB} = m\overline{DE}$, then	

In $\Delta ABC \leftrightarrow \Delta DEF$	Given
$\angle A \cong \angle D$	Given
$\angle B \cong \angle E$	
and $\overline{AB} \cong \overline{DE}$	A.S.A ≅ A.S.A
So $\Delta ABC \cong \Delta DEF$	
Thus	
$\dfrac{m\overline{AB}}{m\overline{DE}} = \dfrac{m\overline{AC}}{m\overline{DF}} = \dfrac{m\overline{BC}}{m\overline{EF}} = 1$	$\overline{AC} \cong \overline{DF}$, $\overline{BC} \cong \overline{EF}$
Hence the result is true for all the cases	

Exercise 14.2

Q.1. *In ΔABC as shown in the figure, $\overrightarrow{CD}$ bisects $\angle C$ and meets $\overline{AB}$ at D. $m\overline{BD}$ is equal to*

(a) 5 (b) 16 (c) 10 (d) 18



Solution:

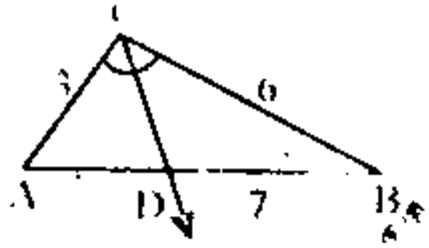
$$\frac{m\overline{BD}}{m\overline{DA}} = \frac{m\overline{BC}}{m\overline{CA}}$$

$$\frac{m\overline{BD}}{6} = \frac{10}{12}$$

$$m\overline{BD} = \frac{10}{12} \times 6$$

$$m\overline{BD} = 5$$

Q.2. *ΔABC shown in the figure. $\overrightarrow{CD}$ bisects $\angle C$. If $m\overline{AC} = 3$, $m\overline{CB} = 6$ and $m\overline{AB} = 7$, then find $m\overline{AD}$ and $m\overline{DB}$.*



Solution:

Let $m\overline{AD} = x$

$m\overline{BD} = 7 - x$

$$\frac{m\overline{AD}}{m\overline{DB}} = \frac{m\overline{AC}}{m\overline{CB}}$$

Putting values

$$\frac{x}{7-x} = \frac{3}{6}$$

or $\frac{x}{7-x} = \frac{1}{2}$

or $2(x) = 1(7 - x)$

$2x = 7 - x$

$2x + x = 7$

$3x = 7$

or $x = \frac{7}{3}$

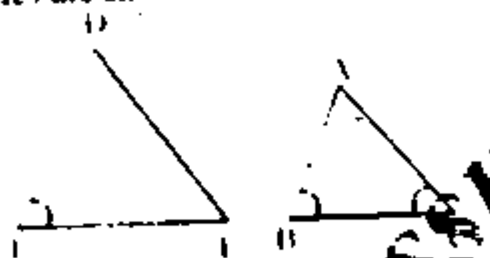
or $m\overline{AD} = \frac{7}{3}$

and $m\overline{DB} = 7 - x$

$= 7 - \frac{7}{3}$

$$m\overline{DB} = \frac{21-7}{3} = \frac{14}{3}$$

Q.3. ***Show that in any correspondence of two triangles, if two angles of one triangle are congruent to the corresponding angles of the other, then the triangles are similar.***



Given:

ΔABC and ΔDEF

$m\angle B = m\angle E$

$m\angle C = m\angle F$

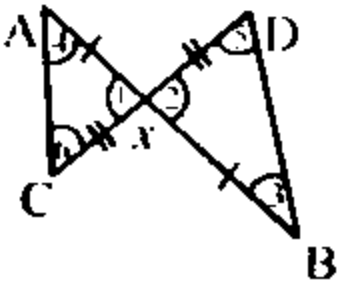
Required:

$\Delta ABC \sim \Delta DEF$

Proof:

Statements	Reasons
$m\angle B + m\angle C + m\angle A = 180^\circ$	Angles of ΔABC
$m\angle E + m\angle F + m\angle D = 180^\circ$	Angles of ΔDEF
$m\angle B + m\angle C + m\angle D = 180^\circ$	As $m\angle E = m\angle B$, $m\angle F = m\angle C$
$m\angle A - m\angle D = 0$	Subtracting (ii) from (i)
or $m\angle A = m\angle D$	
Now in Δ ABC, DEF	
$m\angle A = m\angle D$	
$m\angle B = m\angle E$	
$m\angle C = m\angle F$	
Hence $\Delta ABC \sim \Delta DEF$	

Q.4. If line segments AB and CD are intersecting at point X and $\frac{m\overline{AX}}{m\overline{XB}} = \frac{m\overline{CX}}{m\overline{XD}}$. Then show that ΔAXC and ΔBXD are similar.



Given:

$\overline{AB}$ and $\overline{CD}$ intersect at X that $\frac{m\overline{AX}}{m\overline{XB}} = \frac{m\overline{CX}}{m\overline{XD}}$

Required:

$\Delta AXC \sim \Delta BXD$

Proof:

Statements	Reasons
In BXD and AXC Δ.	
$\angle 1 \cong \angle 2$	Vertical opp. Angles
$\frac{m\overline{AX}}{m\overline{XB}} = \frac{m\overline{CX}}{m\overline{XD}}$	Given
$\overline{AC} \parallel \overline{BD}$	
$\angle 4 \cong \angle 3$	Corresponding angles
$\angle 6 \cong \angle 5$	Corresponding angles.
Thus	
$\frac{m\overline{AX}}{m\overline{XB}} = \frac{m\overline{CX}}{m\overline{XD}} = \frac{m\overline{AC}}{m\overline{DB}}$	
Hence $\Delta A \times C \sim \Delta B \times D$	

Review Exercise 14

Q.1. *Which of the following are true and which are false?*

(i) Congruent triangles are of same size and shape.
(ii) Similar triangles are of same shape but different sizes
(iii) Symbol used for congruent is '$\sim$'
(iv) Symbol used for similarity is '$\cong$'
(v) Congruent triangles are similar.
(vi) Similar triangles are congruent.
(vii) A line segment has only one mid point.
(viii) One and only one line can be draw through two points
(ix) Proportion is non-equality of two ratios.
(x) Ratio has no unit.

Answers:

(i)	T	*(ii)*	T	*(iii)*	F	*(iv)*	F	*(v)*	T
(vi)	F	*(vii)*	T	*(viii)*	T	*(ix)*	F	*(x)*	T

Q.2. *Define the following:*

(i) Ratio *(ii)* Proportion
(iii) Congruent Triangles *(iv)* Similar Triangles

Answers:

(i) ***Ratio:***

We define ratio $a : b = \frac{a}{b}$ as the comparison of two alike quantities 'a' and 'b' called the elements of a ratio 'a' is the first element and b is called the second element of the ratio.

(ii) ***Proportion:***

The relation of equality of two ratios is called proportion. If $a : b = c : d$, then a, b, c and d are said to be in proportion.

(iii) ***Congruent Triangles:***

If three sides of a triangle are equal in length to the corresponding three sides of another triangles then the triangles are called congruent triangles.

(iv) ***Similar Triangles***

If three angles of a triangle are congruent to three corresponding angles of another triangle and their corresponding sides are proportional then these triangles are called similar triangles.

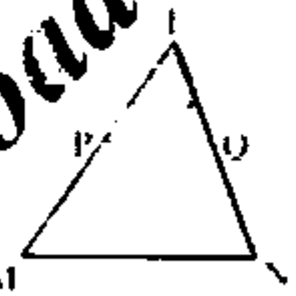
Remember:

Congruent triangles are also similar triangles but all similar triangles may or may not be congruent triangles.

Q.3. ***In ΔLMN shown in the figure, $\overline{MN} \parallel \overline{PQ}$***

(i) If $m\overline{LM} = 5cm$, $m\overline{LP} = 2.5cm$, $m\overline{LQ} = 2.3$ cm, then find $m\overline{LN}$

(ii) If $m\overline{LM} = 6cm$, $m\overline{LQ} = 2.5cm$, $m\overline{QN} = 5$ cm, then find $m\overline{LP}$



3(i) ***Given:***

In ΔLMN

$$\overline{MN} \parallel \overline{PQ}$$

$$m\overline{LM} = 5cm,\ m\overline{LP} = 2.5\text{ cm}$$

and $m\overline{LQ} = 2.3$ cm

Required:

$$m\overline{LN} = ?$$

Calculations:

Now $\frac{m\overline{LN}}{m\overline{LQ}} = \frac{m\overline{LM}}{m\overline{LP}}$ as $\overline{PQ} \parallel \overline{MN}$

$\frac{m\overline{LN}}{2.3} = \frac{5}{2.5}$ putting the values

$\overline{LN} = \frac{5 \times 2.3}{2.5}$

$= \frac{5 \times 23}{25} = 4.6$ cm

3(ii) ***Given:***

In $\triangle LMN$

$\overline{MN} \parallel \overline{PQ}$

$m\overline{QN} = 5$cm. and $m\overline{LQ} = 2.5$ cm, $m\overline{LM} = 6$cm

Required:

$m\overline{LP}$

Calculations:

$\frac{m\overline{LP}}{m\overline{LM}} = \frac{m\overline{LQ}}{m\overline{LN}}$ as $\overline{PQ} \parallel \overline{MN}$

$\frac{m\overline{LP}}{m\overline{LM}} = \frac{m\overline{LQ}}{m\overline{LQ} + m\overline{QN}}$

Putting values

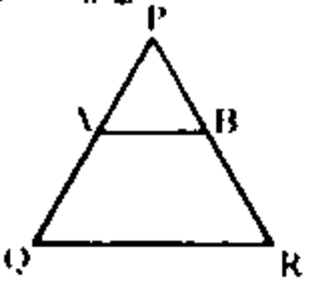
$\frac{m\overline{LP}}{6} = \frac{2.5}{2.5 + 5}$

$m\overline{LP} = \frac{2.5}{7.5} \times 6$

$= \frac{1}{3} \times 6$

$\overline{LP} = 2$ cm

Q.4. In the shown figure, let $m\overline{PA} = 8x - 7$, $m\overline{PB} = 4x - 3$, $m\overline{AQ} = 5x - 3$, $m\overline{BR} = 3x - 1$. Find the value of x if $\overline{AB} \parallel \overline{QR}$



Given:

In $\triangle PQR$: $\overline{AB} \parallel \overline{QR}$

and $m\overline{PA} = 8x - 7$

$m\overline{PB} = 4x - 3$

$m\overline{AQ} = 5x - 3$

$m\overline{BR} = 3x - 1$

$\overline{AB} \parallel \overline{QR}$

Required:

$x = ?$

Calculations:

Statements	***Reasons***
In $\triangle PQR$	
$\dfrac{m\overline{PA}}{m\overline{AQ}} = \dfrac{m\overline{PB}}{m\overline{BR}}$	$\overline{AB} \parallel \overline{QR}$
Putting values	
$\dfrac{8x-7}{5x-3} = \dfrac{4x-3}{3x-1}$	
$(8x-7)(3x-1) = (4x-3)(5x-3)$	
$24x^2 - 29x + 7 = 20x^2 - 27x + 9$	

$24x^2 - 20x - 29x + 27x + 7 - 9 = 0$

$4x^2 - 2x - 2 = 0$

$2x^2 - x - 1 = 0$

$2x^2 - 2x + x - 1 = 0$

$2x(x - 1) + 1(x - 1) = 0$

$(x - 1)(2x + 1) = 0$

$2x + 1 = 0$

$2x = -1$

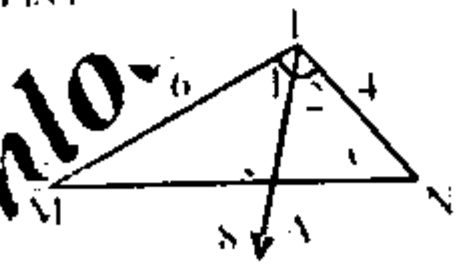
$x = -\frac{1}{2}$

or $x - 1 = 0$

$x = 1$

$x = -\frac{1}{2}$ is not possible

Q.5. In ΔLMN shown in the figure, $\overrightarrow{LA}$ bisects $\angle L$. If $m\overline{LN} = 4$, $m\overline{LM} = 6$, $m\overline{MN} = 8$, then find $m\overline{MA}$ and $m\overline{AN}$.



Given:

In ΔLMN, $\overrightarrow{LA}$ bisector of $\angle L$.
$m\overline{LM} = 6cm$, $m\overline{LN} = 4cm$, $m\overline{MN} = 8cm$.

Required:

$m\overline{MA} = ?$ and

$m\overline{AN} = ?$

Calculations:

Let $m\overline{AN} = x$ cm

$m\overline{MA} = 8 - x$ cm

Now $\dfrac{m\overline{MA}}{m\overline{AN}} = \dfrac{m\overline{LM}}{m\overline{LN}}$ ($\overrightarrow{LA}$ is bisector of $\angle L$)

Putting the values

$\dfrac{8-x}{x} = \dfrac{6}{4}$

$4(8 - x) = 6x$

$32 - 4x = 6x$

$32 = 6x + 4x$

$10x = 32$

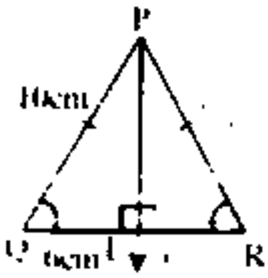
$x = 3.2$

$\therefore$ $m\overline{AN} = 3.2$ cm

and $m\overline{MA} = 8 - x$

$= 8 - 3.2 = 4.8$ cm

Q.6. ***In isosceles ΔPQR shown in the figure, find the value of x and y.***



Given:

$$\overline{PQ} \parallel \overline{PR}$$

In ΔPQR

$$\overline{PL} \perp \overline{QR}$$

Required:

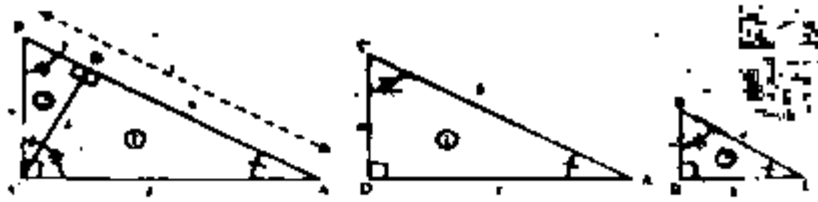
$x = ?;\ y = ?$

Proof:

Statements		Reasons
In ΔPRL and ΔPQL		
$m\overline{PQ} = m\overline{PR}$	(i)	
$\angle PLQ = \angle PLR$		Each 90°
$m\overline{PL} = m\overline{PL}$		Common
$\Delta PQL \cong \Delta PRL$		$H.S \cong H.S$
$m\overline{QL} = m\overline{LR}$		Corresponding sides of congruent triangles.
$6 = y$		
$y = 6$ cm		
$x = 10$ cm		

Unit 15 PYTHAGORAS' THEOREM

In a right angled triangle, the square of the length of hypotenuse is equal to the sum of the squares of the lengths of the other two sides.



(i) (ii) (iii)

Given

ΔACB is right angled triangle in which $m\angle C = 90^\circ$ and $m\overline{BC} = a$, $m\overline{AC} = b$ and $m\overline{AB} = c$.

To prove

$c^2 = a^2 + b^2$

Construction

Draw $\overline{CD}$ perpendicular from C on $\overline{AB}$.

Let $m\overline{CD} = h$, $m\overline{AD} = x$ and $m\overline{BD} = y$. Line segment CD splits ΔABC into two Δs ADC and BDC which are separately shown in the figures (ii) -a and (ii) -b respectively.

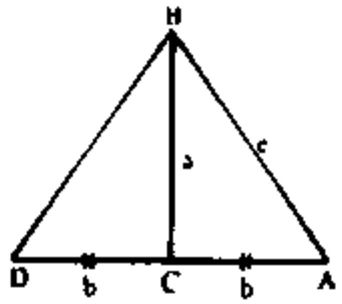
Proof (Using similar Δs)

Statements	Reasons
In $\Delta ADC \longleftrightarrow \Delta ACB$	Refer to figure (ii) a and (i)
$\angle A \cong \angle A$	common - self congruent
$\angle ADC \cong \angle ACB$	Construction – given, each angle = 90°
$\angle C \cong \angle B$	$\angle C$ and $\angle B$, complements of $\angle A$.
$\therefore$ $\Delta ADC \sim \Delta ACB$	Congruency of three angles

$\therefore \quad \frac{x}{b} = \frac{b}{c}$	(Measures of corresponding sides of similar triangles are proportional)
or $\quad x = \frac{b^2}{c}$ ....(I)	
Again in $\Delta BDC \leftrightarrow \Delta BCA$	Refer to figure (ii)-b and (i)
$\angle B \cong \angle B$	Common-self congruent
$\angle BDC \cong \angle BCA$	Construction – given, each angle = 90°
$\angle C \cong \angle A$	$\angle C$ and $\angle A$, complements of $\angle B$
$\therefore \quad \Delta BDC \sim \Delta BCA$	Congruency of three angles.
$\therefore \quad \frac{y}{a} = \frac{a}{c}$	(Corresponding sides of similar triangles are proportional).
or $\quad y = \frac{a^2}{c}$ ....(II)	
But $\quad y + x = c$	Supposition
$\therefore \quad \frac{a^2}{c} + \frac{b^2}{c} = c$	Eq (I) and (II)
or $\quad a^2 + b^2 = c^2$	Multiplying both sides by c.
i.e., $\quad c^2 = a^2 + b^2$	

Converse of Pythagoras' Theorem

If the square of length one side of a triangle is equal to the sum of the squares of the lengths of the other two sides then the triangle is a right angled triangle.



Given: In a ΔABC, m $\overline{AB} = c$, m $\overline{BC} = a$ and m $\overline{AC} = b$ such that:

$$a^2 + b^2 = c^2$$

To prove: Δ ABC is a right angled triangle.

Construction: (i) Draw $\overline{CD}$ perpendicular to $\overline{BC}$ such that $\overline{CD} \cong \overline{CA}$

(ii) Join B to D.

Proof

Statements	Reasons
ΔDCB is a right angled triangle.	Construction
$\therefore \ (m\overline{BD})^2 = a^2 + b^2$ (i)	Pythagoras theorem
But $a^2 + b^2 = c^2$ (ii)	Given
$\because \ (m\overline{BD})^2 = c^2$	From i, ii
or $(m\overline{BD}) = c$	
Now in $\Delta DCB \leftrightarrow \Delta ACB$	
$\overline{CD} \cong \overline{CA}$	Construction
$\overline{BC} \cong \overline{BC}$	Common
$\overline{DB} \cong \overline{AB}$	each = c
$\because \ \Delta DCB \cong \Delta ACB$	$s.s.s \cong s.s.s$
$\therefore \ \angle DCB \cong \angle ACB$	Corresponding angle of congruent triangles.
But $m\angle DCB = 90^\circ$	Construction
$\therefore \ m\angle ACB = 90^\circ$	
Hence ΔACB is a right angled triangle.	

EXERCISE 15

Q.1. Verify that the Δs having the following measures of sides are right angled.

(i) a = 5cm, b = 12cm, c = 13cm
(ii) a = 1.5cm, b = 2cm, c = 2.5cm
(iii) a = 9cm, b = 12cm, c = 15cm
(iv) a = 16cm, b = 30cm, c = 34cm

Solutions: 1(i)

$$\left.\begin{aligned} a &= 5cm \\ b &= 12cm \\ c &= 13cm \end{aligned}\right\} \quad \left.\begin{aligned} a^2 &= (5)^2 = 25 \\ b^2 &= (12)^2 = 144 \\ c^2 &= (13)^2 = 169 \end{aligned}\right\}$$

Now we see $c^2 = a^2 + b^2$

$169 = 25 + 144 = 169$

The given lengths are the sides of a right angled triangle.

1 (ii)

$$\left.\begin{aligned} a &= 1.5cm \\ b &= 2cm \\ c &= 2.5cm \end{aligned}\right\} \text{ and } \left.\begin{aligned} a^2 &= (1.5)^2 = 2.25 \\ b^2 &= (2)^2 = 4 \\ c^2 &= (2.5)^2 = 6.25 \end{aligned}\right\}$$

Now $c^2 = a^2 + b^2$

$6.25 = 2.25 + 4 = 6.25$

The given lengths are the lengths of the sides of a right angled triangle.

1 (iii)

$$\left.\begin{aligned} a &= 9cm \\ b &= 12cm \\ c &= 15cm \end{aligned}\right\} \text{ and } \left.\begin{aligned} a^2 &= (9)^2 = 81 \\ b^2 &= (12)^2 = 144 \\ c^2 &= (15)^2 = 225 \end{aligned}\right\}$$

Now we see $c^2 = a^2 + b^2$

$225 = 81 + 144 = 225$

The given lengths are the lengths of the sides of a right angled triangle.

1 (iv)

$$\left.\begin{array}{l} a = 16cm \\ b = 30cm \\ c = 34cm \end{array}\right\} and \quad \left.\begin{array}{l} a^2 = (16)^2 = 256 \\ b^2 = (30)^2 = 900 \\ c^2 = (34)^2 = 1156 \end{array}\right\}$$

$$c^2 = a^2 + b^2$$

Now we see $1156 = 256 + 900 = 1156$

The given lengths are the lengths of the sides of a right angled triangle.

Q.2. **Verify that $a^2 + b^2, a^2 - b^2$ and 2ab are the measures of the sides of a right angled triangle where a and b are any two real numbers (a > b).**

Solution:

$$\left.\begin{array}{l} a^2 + b^2 \\ a^2 - b^2 \\ 2ab \end{array}\right\} \left\{\begin{array}{ll} (a^2 + b^2)^2 = a^4 + b^4 + 2a^2b^2 & (i) \\ (a^2 - b^2)^2 = a^4 + b^4 - 2a^2b^2 & (ii) \\ (2ab)^2 \quad = 4a^2b^2 & (iii) \end{array}\right.$$

Now we see if

$$(a^2 + b^2)^2 = (a^2 - b^2)^2 + (2ab)^2$$

From i, ii, iii

$$a^4 + b^4 + 2a^2b^2 = (a^4 + b^4 - 2a^2b^2) + 4a^2b^2$$

$$= a^4 + b^4 - 2a^2b^2 + 4a^2b^2$$

$$a^4 + b^4 + 2a^2b^2 = a^4 + b^4 + 2a^2b^2$$

The given lengths are the lengths of the side of a right angled triangle.

Q.3. **The three sides of a triangle are of measure 8, x and 17 respectively. For what value of x will it become base of a right angled triangle?**

Solution: Lengths of the sides of a triangle.

a = 8

$b = x$

$c = 17$

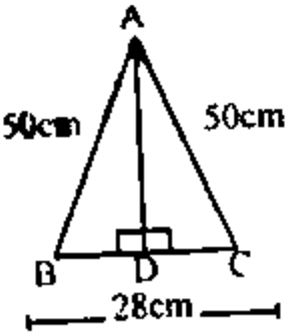
In case of right angled triangle.

$$c^2 = a^2 + b^2$$
$$(17)^2 = (8)^2 + (x)^2$$
$$289 = 64 + x^2$$
$$x^2 + 64 = 289$$
$$x^2 = 289 - 64$$
$$x^2 = 225$$

Thus $x = 15$ (Taking sq. root)

Q.4. In an isosceles Δ, the base $\overline{BC} = 28$ cm, and $\overline{AB} = \overline{AC} = 50$cm. If $\overline{AD} \perp \overline{BC}$, then find

(i) Length of $\overline{AD}$ (ii) area of ΔABC



Given in Δ ABC

$m\overline{AC} = m\overline{AB} = 50cm$

$m\overline{BC} = 28cm$

$\overline{AD} \perp \overline{BC}$

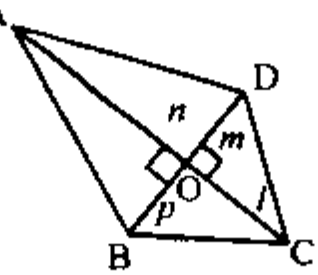
Required: $m\overline{AD} = ?$

Area of Δ ABC

Statements	Reasons
In Δ ACD $\leftrightarrow$ Δ ABD	

$m\angle ADC = m\angle ADB$	each 90°
$m\overline{AC} = m\overline{AB}$	given (Hyp)
$m\overline{AD} = m\overline{AD}$	common
$\therefore \quad \Delta ABD \cong \Delta ACD$	$Hyp.side \cong Hyp.side$
$m\overline{BD} = m\overline{CD} = \frac{28}{2}$ $= 14$ cm	
Now in right angled ΔADC	
$m\overline{CD} = 14cm$	
$\overline{AC} = 50$	given (Hyp)
$Now (mAD)^2 = (mAC)^2 - (mCD)^2$ $(50)^2 - (14)^2$ $= 2500 - 196$ $= 2304$	
$Hence \quad m\overline{AD} = 18$	Taking sq. root.
Part II	
$Area\ \Delta ABC = \frac{Base \times Alt}{2}$ $= \frac{28 \times 48}{2}$ $= 14 \times 48 = 672\, Sq.cm$	

Q.5. In a quadrilateral ABCD, the diagonals $\overline{AC}$ and $\overline{BD}$ are perpendicular to each other.
Prove that $m\overline{AB}^2 + m\overline{CD}^2 = m\overline{AD}^2 + m\overline{BC}^2$.



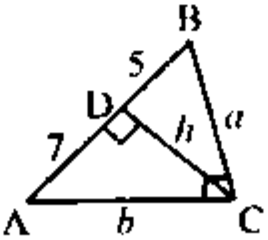
Given: ABCD is a quadrilateral in which its diagonals $\overline{AC}$ *and* $\overline{BD}$ are perpendicular to each other.

Required: $(mAB)^2+(mCD)^2=(mAD)^2+(mBC)^2$

Proof

Statements	Reasons
Let $m\overline{OC}=l$	
$m\overline{OD}=m$	
$m\overline{OA}=n$	
$m\overline{OB}=p$	
In right angled ΔOCD	
$(mCD)^2=l^2+m^2$ (i)	
In right angled ΔABO $(mAB)^2=n^2+p^2$	
Now $(mAB)^2+(mCD)^2=l^2+m^2+n^2+p^2$	from (i) + (ii) (L)
In right angled Δ AOD	
$(mAD)^2=n^2+m^2$ (iii)	
In right angled Δ BOC	
$(mBC)^2=p^2+l^2$ (iv)	
Now $(mAD)^2+(mBC)^2=l^2+m^2+n^2+p^2$	From iii + iv M
Thus $(mAB)^2+(mCD)^2=(mAD)^2+(mBC)^2$	from L and M

Q.6. (i) In the Δ ABC as shown in the figure,
$m\angle ACB=90°$ and $\overline{CD}\perp\overline{AB}$. Find the lengths a, h and b if $mBD=5$ units and $mAD=7$ units.



Given: In ΔABC

(i) $m\angle ACB=90°$

(ii) $\overline{CD} \perp \overline{AB}$

$m\overline{BD} = 5, m\overline{AD} = 7$

Required: To find measures of a, b and h

Calculations: In right angled triangle ABC

$(m\overline{AB})^2 = a^2 + b^2$

$(7+5)^2 = a^2 + b^2$

$(12)^2 = a^2 + b^2$

$a^2 + b^2 = 144$ ...(i)

In right angled triangle CDB

$a^2 = (h)^2 + (5)^2$

or $a^2 - h^2 = 25$ ...(ii)

In right-angled triangle AGD

$b^2 = h^2 + (7)^2$

or $b^2 - h^2 = 49$ ...(iii)

$a^2 - b^2 = -24$ ...(iv) From (ii)−(iii)

or $2a^2 = 144 - 24$ From (i)+(iv)

$2a^2 = 120$

$a^2 = 60$

$a = \sqrt{60}$

$= 2\sqrt{15}$ ...(v)

Now $a^2 + b^2 = 144$

$b^2 = 144 -$

$= 14$

$b^2 =$

Now

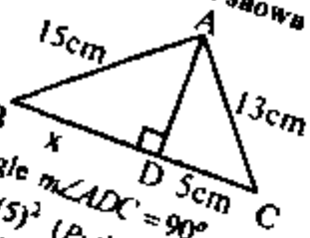
or

$= 60 - 25$ as $a^2 = 60$

or $h^2 = 35$

$h = \sqrt{35}$

6 (ii) Find the value of x in the shown figure.



Given: In ΔADC angle $m\angle ADC = 90^\circ$.

$(m\overline{AD})^2 = (13)^2 - (5)^2$ (Pythagoras theorem)

$= 169 - 25$

$= 144$

$m\overline{AD} = \sqrt{144}$ ... (i)

$= 12cm$

Now in right angle triangle ABD

angle ADB is 90°)

$x^2 = (15)^2 - (m\overline{AD})^2$

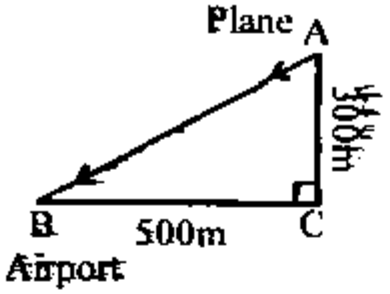
$x^2 = (15)^2 - 144$

$= 225 - 144$

81

$\sqrt{81}$

a height of 300 m and is 500 m away
port as shown in the figure. How much
travel to land at the airport?



Solution: ABC is a right-angled triangle at C.

$$\therefore \quad (m\overline{AB})^2 = (m\overline{AC})^2 + (m\overline{BC})^2 \quad (\textit{Pythagoras Theorem})$$

$$= (300)^2 + (500)^2$$

$$= 90000 + 250000$$

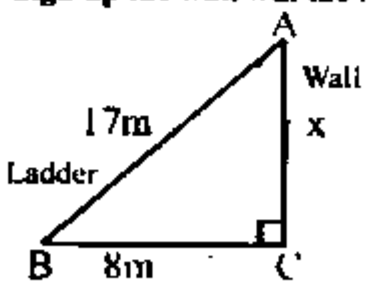
$$(m\overline{AB})^2 = 340000$$

$$m\overline{AB} = \sqrt{340000}$$

$$= \sqrt{34 \times 100 \times 100}$$

$$m\overline{AB} = 100\sqrt{34}\, m$$

8. A ladder 17m long rests against a vertical wall. The foot of the ladder is 8 m away from the base of the wall. How high up the wall will the ladder reach?



Solution: Let $\overline{AB}$ be the ladder.

Now ABC is a right-angled triangle at C.

$\therefore \quad (m\overline{AC})^2 = (m\overline{AB})^2 - (m\overline{BC})^2$ (Pythagoras Theorem)

$Now\,(m\overline{AC})^2 = (17)^2 - (8)^2$

$= 289 - 64$

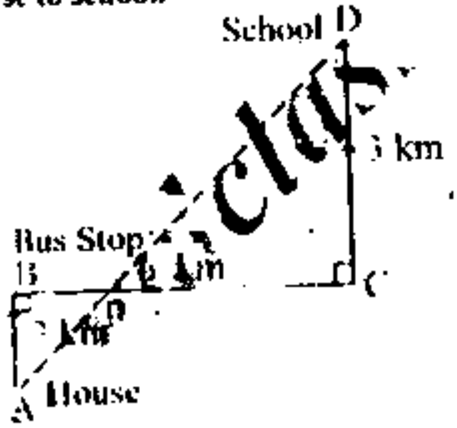
$= 225$

$m\overline{AC} = \sqrt{225}$ Taking sq. root

$m\overline{AC} = 15m$

The ladder reaches upto 15m high.

9. A student travels to his school by the route as shown in the figure. Find $m\overline{AD}$, the direct distance from his house to school.



Solution:

Let $m\overline{BP} = x\,km$

$m\overline{PC} = (6-x)km$

In $\Delta PCD \sim \Delta PBA$

$$\frac{x}{6-x} = \frac{2}{3}$$

$3x = 12 - 2x$

$5x = 12$

$$x = \frac{12}{5} = m\overline{BP}$$

$$m\overline{PC} = 6 - \frac{12}{5} = \frac{18}{5}$$

$$In\, PCD\Delta \;\; mP^2D = mP^2D + mc^2D \; in$$

$$= \left(\frac{18}{5}\right)^2 + (3)^2$$

$$= \frac{324}{25} + 9 = \frac{324 + 225}{25}$$

$$= \frac{549}{25}$$

$$m\overline{PD} = \sqrt{\frac{549}{25}} = \sqrt{\frac{3 \times 3 \times 61}{5 \times 5}}$$

$$= \frac{3}{5}\sqrt{61}\, km$$

REVIEW EXERCISE 15

Which of the following are true and which are false?

(i) In a right angled triangle greater angle is of 90^0.

(ii) In a right angled triangle right angle is of 60^0.

(iii) In a right triangle hypotenuse is a side opposite to right angle.

(iv) If a, b, c are sides of right angled triangle with c as longer side then $c^2 = a^2 + b^2$.

(v) If 3 cm and 4 cm are two sides of a right angled triangle, then hypotenuse is 5 cm.

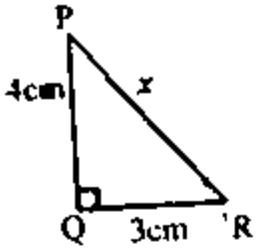
(vi) If hypotenuse of an isosceles right triangle is $\sqrt{2}$ cm then each of the other side is of length 2 cm.

Answers:

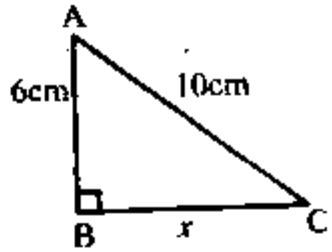
(i)	T	(ii)	F	(iii)	T
(iv)	T	(v)	T	(vi)	F

2. Find the unknown value in each of the following figures.

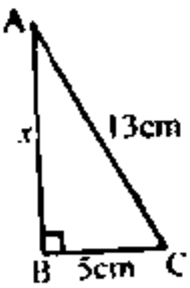
(i)



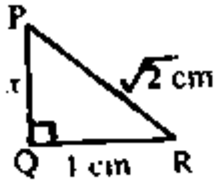
(ii)



(iii)



(iv)



Solutions: In high angled ΔPQR

2(i) $mPR^2 = mPQ^2 + mQ^2R$ (*Pythagoras Theorem*)

$x^2 = (4)^2 + (3)^2$ *Putting values*

$= 16 + 9$

$x^2 = 25$

$x = \sqrt{25} = 5cm$

2(ii) **In right angled triangle ABC (Pythagoras Theory)**

$mA^2C = mA^2B + mB^2C$ (*Pythagoras Theorem*)

$(10)^2 = (6)^2 + x^2$ *Putting values*

$100 = 36 + x^2$

$x^2 = 100 - 36 = 64$

$x = \sqrt{64} = 8cm$

2(iii) **In right angled Δ ABC**

$$(mAC)^2 = (mAB)^2 + (mBC)^2 \quad (Pythagoras\ Theorem)$$

$$(13)^2 = x^2 + (5)^2 \quad Putting\ values$$

$$169 = x^2 + 25$$

$$x^2 = 169 - 25 = 144$$

$$x = \sqrt{144} = 12cm \quad (Taking\ sq.root)$$

2(iv) **In right angled Δ PQR**

$$(mPR)^2 = (mPQ)^2 + (mQR)^2 \quad (Pythagoras\ Theorem)$$

$$(\sqrt{2})^2 = x^2 + (1)^2 \quad Putting\ values$$

$$2 = x^2 + 1$$

$$x^2 = 2 - 1 = 1$$

$$x = \sqrt{1}\,cm \quad (Taking\ sq.root)$$

$$x = 1cm$$

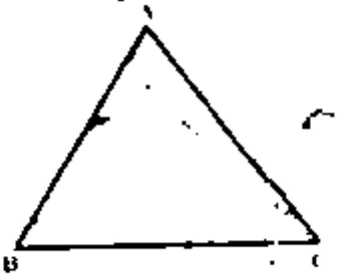
Unit 16 THEOREMS RELATED WITH AREA

Area of a Figure

The region enclosed by the bounding lines of a closed figure is called the Area of the figure.

Triangular Region

The interior of a triangle is the part of the plane enclosed by the triangle.

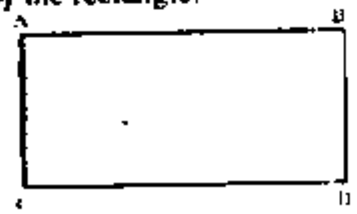


A triangular region is union of a triangle and its interior.

Area of a triangle means the area its triangular region.

Rectangular Region

The interior of a rectangular is the part of the plane enclosed by the rectangle.



A rectangular region is the union of a rectangle and its interior.

Area of a rectangular region is the product of the length and width of the region.

Altitude of a parallelogram

If one side of a parallelogram is taken as its base, the perpendicular distance between that side and the side parallel to it is called the Altitude or Height of the parallelogram.

Altitude of a Triangle

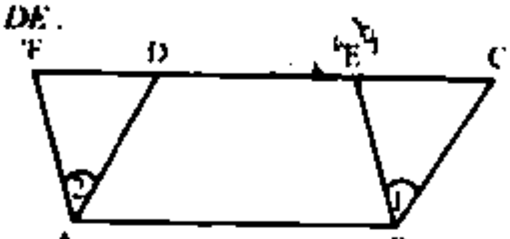
If one side of a triangle is taken as the base, the perpendicular to that side from the opposite vertex is called the Altitude or Height of the triangle.

Theorem

Parallelograms on the same base and between the same parallel lines (or of the same altitude) are equal in area.

Given

Two parallelograms ABCD and ABEF having the same base $\overline{AB}$ and between the same parallel lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{DE}$.



To prove

area of parallelogram ABCD = area of parallelogram ABEF

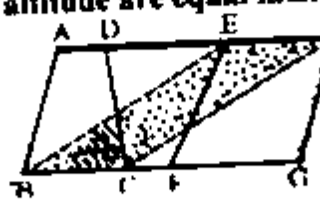
Proof

Statement	*Reason*
In $\Delta EBC \leftrightarrow \Delta FAD$	
$\overline{CB} \cong \overline{DA}$	opp. Sides of ‖^{gm} ABCD
$\overline{BE} \cong \overline{AF}$	opp. Sides of ‖^{gm} ABEF
$\angle 1 \cong \angle 2$	$\overline{BC} \parallel \overline{AD}$, $\overline{BE} \parallel \overline{AF}$
$\therefore \Delta\ EBC \cong \Delta\ FAD$	
$\therefore$ Area of $\Delta\ EBC$ = Area of $\Delta\ FAD$	

Also are of quad ABED = Area of quad ABED	(same quad) (ii)
Adding (i) and (ii)	
ΔEBC + area quad ABED = ΔFAD + area quad ABED	
Hence $\parallel^{m}$ ABCD (area) = $\parallel^{m}$ ABEF	
Q.E.D	

Theorem

Parallelograms on equal bases and having the same (or equal) altitude are equal in area.



Given

Parallelograms ABCD, EFGH are on the equal bases $\overline{BC}, \overline{FG}$, having equal altitudes.

To Prove

area of (parallelogram ABCD) = area of (parallelogram EFGH)

Construction

Place the parallelograms ABCD and EFGH so that their equal bases $\overline{BC}, \overline{FG}$ are in the straight line BCFG. Join $\overline{BC}$ and $\overline{CH}$.

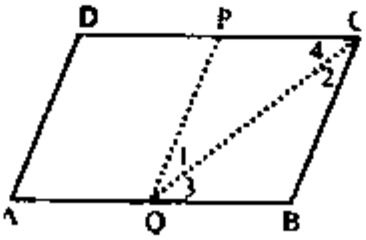
Proof

Statements	Reasons
The given $\parallel^{gms}$ ABCD and EFGH are between the same parallels	Their altitudes are equal (given)
Hence ADEH is a straight line $\parallel$ to $\overline{BC}$	
$\therefore \quad m\overline{BC} = m\overline{FG}$	Given
$= m\overline{EH}$	EFGH is a parallelogram

Now $m\overline{BC} = m\overline{EH}$ and they are ‖	
∴ $\overline{BE}$ and $\overline{CH}$ are both equal and ‖	
Hence EBCH is a parallelogram	A quadrilateral with two opposite sides congruent and parallel is a parallelogram
Now ‖gm ABCD = ‖gm EBCH ...... (i)	Being on the same base $\overline{BC}$ and between the same parallels
But ‖gm EBCH = ‖gm EFGH ...... (ii)	Being on the same base $\overline{EH}$ and between the same parallels
Hence area (‖gm ABCD) = area(‖gm EFGH)	From (i) and (ii)

EXERCISE 16.1

Q.1 ***Show that the line segment joining the mid-points of opposite sides of a parallelogram. Divides it into two equal parallelograms.***



Given:

(i) ABCD is a ‖gm

(ii) P is mid point of $\overline{DC}$ i.e. $\overline{DP} \cong \overline{PC}$

(iii) Q is mid point of $\overline{AB}$ i.e. $\overline{AQ} \cong \overline{QB}$

(iv) P is joined to Q.

Required:

‖gm AQPD ≅ ‖gm QBCP.

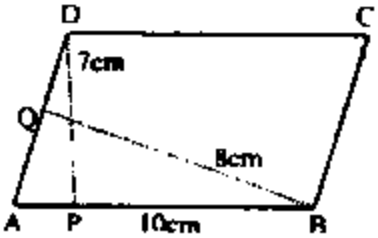
Construction:

Join Q to C.

Proof:

Statements	Reasons
$m\overline{AB} = m\overline{DC}$	Opp. sides of ||^{m} ABCD
$\frac{1}{2}m\overline{AB} = \frac{1}{2}m\overline{DC}$	Dividing by 2
or $m\overline{QB} = m\overline{PC}$	
Now in ΔPQC ΔQBC	
$\overline{QC} \cong \overline{QC}$	Common
$\overline{QB} \cong \overline{PC}$	Proved
$\angle 3 \cong \angle 4$	Alternate angles $\overleftrightarrow{AB} \parallel \overleftrightarrow{DC}$
$\Delta QBC \cong \Delta PQC$	S.A.S ≅ S.A.S.
(i) $\overline{PQ} \cong \overline{CB}$	Corresponding sides of cong. triangles
But (ii) $\overline{AD} \cong \overline{CB}$	Opp. side of ||^{m}
Thus $\overline{PQ} \cong \overline{AD} \cong \overline{BC}$	
And $\angle 1 \cong \angle 2$	Corresponding angles of cong. Triangles
or $m\angle 1 + m\angle 3 = m\angle 2 + m\angle 4$	
or $\angle PQB \cong m\angle PCB$	
and $\angle A \cong m\angle PCB$	Given
or $\angle A \cong m\angle PQB$	Proved
Now in QBCP = AQPD ||^{m}	Proved
$\overline{AQ} \cong \overline{QB}$	
$\overline{AD} \cong \overline{PQ}$	
$\angle A \cong \angle PQB$	
Therefore ||^{m} AQPD ≅ ||^{m} QBCP	

Q.2 ***In a parallelogram ABCD,*** $m\overline{AB}$ = ***10cm. The altitudes corresponding to sides AB and AD are respectively 7 cm and 8 cm. Find*** $\overline{AD}$



Given:

ABCD is a $\|^{m}$ in which.

(i) $m\overline{AB} = 10$ cm.

(ii) Alt. $\overline{DP}$

Required:

To find $m\overline{AD}$

Solution:

In $\|^{m}$ ABCD

$$\text{Base} = 10 \text{ cm}$$
$$\text{Alt} = 7 \text{ cm}$$
$$\text{Area of } \|^{m} ABCD = 10 \times 7$$
$$= 70 \text{ sq. cm} \quad \text{(i)}$$

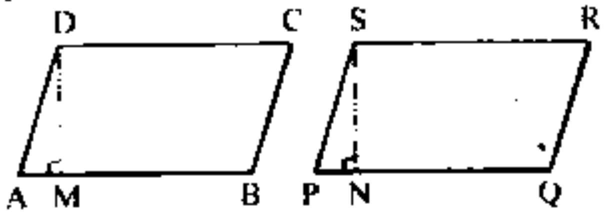
In second case:

$$\text{Base} = \overline{AD}$$
$$\text{Alt } \overline{BQ} = 8 \text{ cm.}$$
$$\text{Area of } \|^{m} ABCD = (8) \times (m\overline{AD})$$

Now $70 = 8 \times (m\overline{AD})$ from (i)

$\therefore$ $m\overline{AD} = \dfrac{70}{8} = \dfrac{35}{4}$ cm

Q.3 ***If two parallelograms of equal areas have the same or equal bases, their altitudes are equal.***



Given:

In $\|^{m}$ ABCD and PQRS

Area $\|^{m}$ ABCD = Area $\|^{m}$ PQRS

And Base $\overline{AB}$ = Base $\overline{PQ}$

Alt. of ABCD is $\overline{DM}$

Alt. of PQRS is SN.

Required:

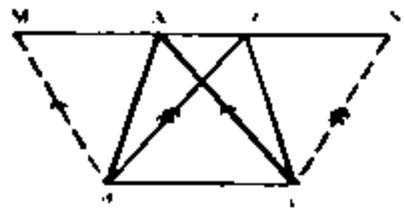
$m\overline{DM} = m\overline{SN}$

Proof:

Statements	*Reasons*
Area ABCD $\|^{m}$ = Area PQRS $\|^{m}$	
Now (mAB)(mDM) = (mPQ)(mSN)	Area of $\|^{m}$ = Base × Alt.
(mAB)(mDM) = (mAB)(mSN)	$m\overline{AB} = m\overline{PQ}$ (given)
∴ $m\overline{DM} = m\overline{SN}$	

Theorem

Triangles on the same base and of the same (i.e. equal) altitudes are equal in area.



Given: Δs ABC, DBC on the same base $\overline{BC}$, and having equal altitudes.

To Prove: area of (ΔABC) = area of (ΔDBC)

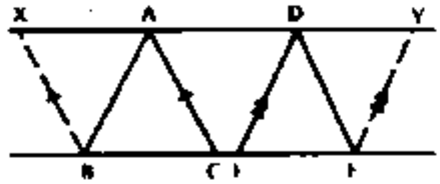
Construction: Draw $\overline{BM}$ ∥ to $\overline{CA}$, $\overline{CN}$ ∥ to $\overline{BD}$ $\overline{AD}$ meeting produced in M, N.

Proof:

Statements	*Reasons*
ΔABC and ΔBCD are between the same ∥s	Their altitudes are equal
Hence MADN is parallel to $\overline{BC}$	
∴ Area (∥gm BCAM) = Area (∥gm BCND) (i)	These ∥gm are on the same base $\overline{BC}$ and between the same ∥s
But $\Delta ABC = \frac{1}{2}$(∥gm BCAM) (ii)	Each diagonal of a ∥gm bisects it into two congruent triangles
And $\Delta DBC = \frac{1}{2}$(∥gm BCND) (iii)	
Hence Area (ΔABC) = Area (ΔDBC)	From (i), (ii) and (iii)

Theorem

Triangle on equal bases and of equal altitudes are equal in area.



Given: Δs ABC, DEF on the same base $\overline{BC}$, $\overline{EF}$ and having altitudes equal.

To Prove: Area of (Δ ABC) = Area of (Δ DEF)

Construction:

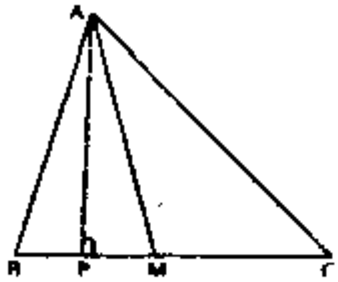
Place the Δ^s ABC and DEF so that their equal bases $\overline{BC}$ and $\overline{EF}$ are in the same straight line BCEF and their vertices on the same side of it. Draw BX || to CA and FY || to ED meeting AD produced in X, Y respectively.

Proof:

Statements	Reasons
Δ ABC and Δ DEF are between the same parallels	Their altitudes are equal (given)
$\therefore$ XADY is || to BCEF	
$\therefore$ Area (||gm BCAX) = Area (||gm EFYD) (i)	These ||gm are on equal bases and between the same parallels
But $\Delta ABC = \frac{1}{2}$ (||gm BCAX) (ii)	Diagonal of a ||gm bisects it
And $\Delta DEF = \frac{1}{2}$ (||gm EFYD) (iii)	
Hence Area (ΔABC) = Area (ΔDEF)	From (i), (ii) and (iii)

EXERCISE 16.2

Q.1 ***Show that a median of a triangle divides it into two triangles of equal area.***



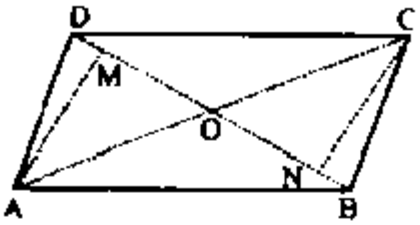
Given: In ΔABC; $\overline{AM}$ is a median i.e. $m\overline{BM} = m\overline{MC}$

Required: Area Δ AMC = Area Δ ABM

Proof:

Statements	***Reasons***
Area of $\Delta ABM = \dfrac{m\overline{AP} \times m\overline{BM}}{2}$ (i)	
Area of $\Delta AMC = \dfrac{m\overline{AP} \times m\overline{MC}}{2}$	
$= \dfrac{m\overline{AP} \times m\overline{BM}}{2}$ (ii)	$m\overline{BM} = m\overline{MC}$ (given)
$\therefore$ Area of ΔABM=Area of ΔAMC	
Similarly it can be proved that **every median** of a triangle divides it **into two** triangles equal in area.	

Q.2 Prove that a parallelogram is divided by its diagonals into four triangles of equal area.



Given: (i) ABCD is a $\|^{m}$

(ii) $\overline{AC}$ and $\overline{BD}$ are its diagonals that intersect each other at O.

Required: $\Delta OBC = \Delta OCD = \Delta OAD = \Delta OAB$

Construction:

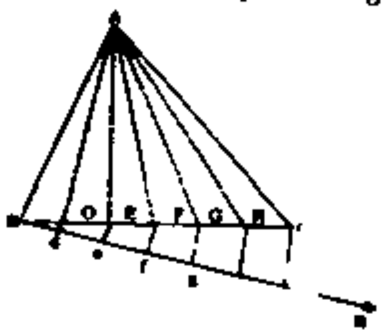
(i) Drop perpendicular from C to $\overline{DB}$ i.e. $\overline{CN}$

(ii) Drop perpendicular from A to $\overline{DB}$ i.e. $\overline{AM}$

Proof:

Statements	*Reasons*
$m\overline{OB} = m\overline{OD}$ $m\overline{OA} = m\overline{OC}$	Diagonals of a $\|$m bisect each other
$\Delta OBC = \dfrac{m\overline{OB} \times m\overline{CN}}{2}$ (i)	
And $\Delta OAD = \dfrac{m\overline{OD} \times m\overline{CN}}{2}$	
$= \dfrac{m\overline{OB} \times m\overline{CN}}{2}$ (ii)	
$\therefore$ $\Delta OBC = \Delta OAD$	
Similarly $\Delta OAD = \Delta OAD$	
$\Delta OBC = \Delta OCD = \Delta OAD = \Delta OAB$	From (i) and (ii)

Q.3 *Divide a triangle into six equal triangular parts.*



Given: ABC is a triangle.

Required: To divide ΔABC ... ual triangular parts (triangles)

Construction:

(i) Take $\overrightarrow{BP}$ any ray ... acute angle with $\overline{BC}$

(ii) Draw six arcs of the ... us on $\overrightarrow{BP}$
i.e. mBd = mde = mef ... mgh = mhc

(iii) Join c To C and parallel lir ... ments as
$\overline{cC} \parallel \overline{hH} \parallel \overline{gG} \parallel \overline{fF} \parallel \overline{eE} \parallel \overline{do}$

(iv) Join A to O, E, F, Q, R

Proof:

Base $\overline{BC}$ of ΔABC has been divided to equal parts six in number.

We get six triangles having equal base and same altitude

∴ Their area is equal.

Hence ΔBOA = ΔOEA = ΔEFA = ΔFGA = ΔGHA = ΔHCA

Review Exercise 16

Q.1 Which of the following are true and which are false?

(i) Area of a figure means region enclosed by bounding lines of closed figure.

(ii) Similar figures have same area.

(iii) Congruent figures have same area.

(iv) A diagonal of a parallelogram divides it into two non-congruent triangles.

(v) Altitude of a triangle means perpendicular from vertex to the opposite side (base).

(vi) Area of a parallelogram is equal to the product of base and height.

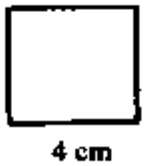
Answers:

(i)	T	*(ii)*	F	*(iii)*	T	*(iv)*	F	*(v)*	T
(vi)	T								

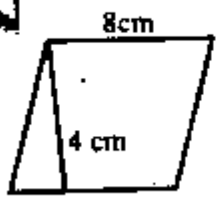
Q.2 Find the area of the following figures.

(i)

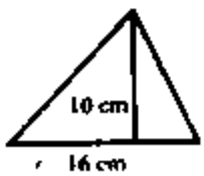
(ii)



(iii)



(iv)



Answers:

2(i) Length of the rectangle = 6 cm

Width of the rectangle = 3 cm

Area of the rectangle = 6×3

$= 18$ sq. cm

2(ii) Length of side of square $= 4$ cm

Area $= (\text{side})^2$

$= (4)^2$

$= 16$ sq. cm

2(iii) Length of side of a $\|^{m}$ $= 8$ cm

Act of the $\|^{m}$ $= 4$

Area $= 8 \times 4$

$= 32$ sq. cm

2(iv) Base of the triangle $= 16$ cm

Alt $= 10$ cm

Area of the Δ $= \dfrac{16 \times 10}{2}$

$= 80$ sq. cm

Q.3 Define the following

(i) Area of a figure (ii) Triangular Region

(iii) Rectangular Region

(iv) Altitude or Height of a triangle

Solution:

3(i) Area of a figure

Area of a figure means region enclosed by the boundary lines of a closed figure.

3(ii) Triangular Region

A triangular region means the union of triangle and its interior.

3(iii) Rectangular Region

A rectangular region is the union of a rectangle and its interior.

3(iv) Altitude or Height of a triangle

Altitude or height of a triangle means perpendicular distance to base to base from its opposite vertex.

Unit 17 PRACTICAL GEOMETRY

EXERCISE 17.1

Q.1 Construct a ΔABC in which

(i) $m\overline{AB} = 3.2$ cm, $m\overline{BC} = 4.2$ cm, $m\overline{CA} = 5.2$ cm

(ii) $m\overline{AB} = 4.2$ cm, $m\overline{BC} = 3.9$ cm, $m\overline{CA} = 3.6$ cm

(iii) $m\overline{AB} = 4.8$ cm, $m\overline{BC} = 3.7$ cm, $m\angle B = 60^\circ$

(iv) $m\overline{AB} = 3$ cm, $m\overline{AC} = 3.2$ cm, $m\angle A = 45^\circ$

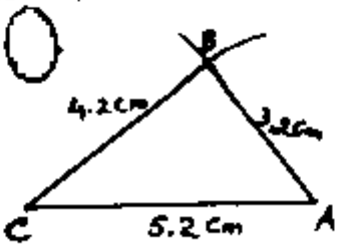
(v) $m\overline{BC} = 4.2$ cm, $m\overline{CA} = 3.5$ cm, $m\angle C = 75^\circ$

(vi) $m\overline{AB} = 2.5$ cm, $m\angle A = 30^\circ$, $m\angle B = 105^\circ$

(vii) $m\overline{AB} = 3.6$ cm, $m\angle A = 75^\circ$, $m\angle B = 45^\circ$

1(i) Construct a ΔABC in which

$m\overline{AB} = 3.2$ cm, $m\overline{BC} = 4.2$ cm, $m\overline{CA} = 5.2$ cm



Given: $m\overline{AB} = 3.2$ cm

$m\overline{BC} = 4.2$ cm

$m\overline{CA} = 5.2$ cm

Required: Construct a ΔABC

Step of Construction:

(i) Take a line segment $\overline{CA} = 5.2$ cm.

(ii) Taking C as centre and draw an arc of radius 4.2 cm.

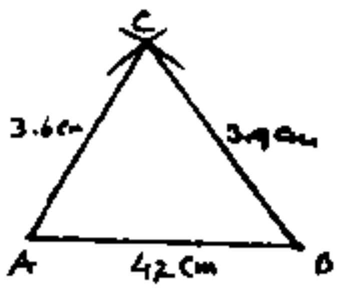
(iii) Taking A as centre and draw another arc of radius 3.2 cm. It intersect the first arc at B.

(iv) Join C to A and B.

Result: ABC is the required triangle.

I(ii) Construct a ΔABC in which

$m\overline{AB} = 4.2$ cm, $m\overline{BC} = 3.9$ cm, $m\overline{CA} = 3.6$ cm



Given: $m\overline{AB} = 4.2$ cm

$m\overline{BC} = 3.9$ cm

$m\overline{CA} = 3.6$ cm

Required: Construct a ΔABC

Steps of Construction:

(i) Take a line segment $\overline{AB}$ of length 4.2 cm.

(ii) Take A as centre and drawn arc of 3.6 cm radius.

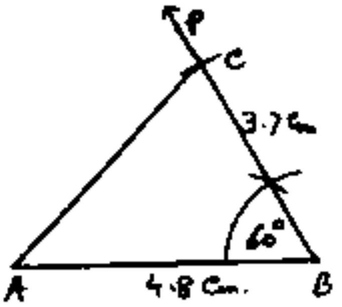
(iii) Take B as centre and draw an arc of 3.9 cm radius. This cuts the first arc at C.

(iv) Join C to A , B.

Result: ABC is the required triangle.

1(iii) Construct a ΔABC in which

$m\overline{AB} = 4.8$ cm, $m\overline{BC} = 3.7$ cm, $m\angle B = 60^\circ$



Given: $m\overline{AB} = 4.8$ cm

$m\overline{BC} = 3.7$ cm

$m\angle B = 60^\circ$

Required: Construct triangle ABC

Steps of Construction:

(i) Take a line segment $\overline{AB} = 4.8$ cm.

(ii) Make an angle of 60° at B.

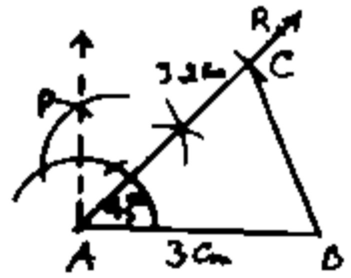
(iii) Cut off $\overline{BC} = 3.7$ cm from $\overrightarrow{BP}$.

(iv) Join C to A.

Result: ABC is the required triangle.

1(iv) Construct a triangle ABC in which

$m\overline{AB} = 3$ cm, $m\overline{AC} = 3.2$ cm, $m\angle A = 45°$



Given: $m\overline{AB} = 3$ cm

$m\overline{AC} = 3.2$ cm

$m\angle A = 45°$

Required: Construct a triangle ABC

Steps of Construction:

(i) Take a line segment $\overline{AB} = 3$ cm.

(ii) Make an angle of 45° at A.

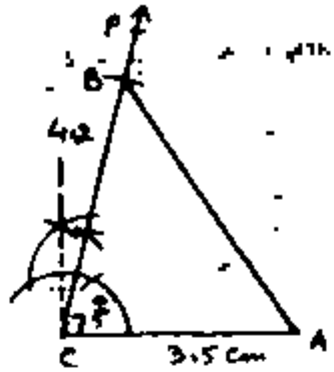
(iii) Take A as centre and cut off $\overline{AC} = 3.2$ cm on $\overrightarrow{AR}$.

(iv) Join C to B.

Result: ABC is the required triangle.

1(v) Construct a ΔABC in which

$m\overline{BC} = 4.2$ cm, $m\overline{CA} = 3.5$ cm, $m\angle C = 75°$



Given: $m\overline{BC} = 4.2$ cm

$m\overline{CA} = 3.5$ cm

$m\angle C = 75°$

Required: Construct a ΔABC

Steps of Construction:

(i) Take a line segment $\overline{CA} = 3.5$ cm.

(ii) Make an angle of 75° at C.

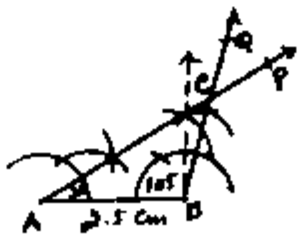
(iii) Cut off $\overline{CB} = 4.2$ cm from $\overrightarrow{CP}$.

(iv) Join B to A.

Result: ABC is the required triangle.

1(vi) **Construct a ΔABC in which**

$m\overline{AB} = 2.5$ cm, $m\angle A = 30°$, $m\angle B = 105°$



Given: $m\overline{AB} = 2.5$ cm

$m\angle A = 30°$

$m\angle B = 105°$

Required: Construct a triangle ABC

Steps of Construction:

(i) Take a line segment $\overline{AB} = 2.5$ cm.

(ii) Make an angle of 45° at A.

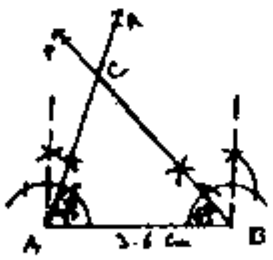
(iii) Make an angle of 105° at B.

(iv) $\overrightarrow{AP}$, $\overrightarrow{BQ}$ intersect at C.

Result: ABC is the required triangle.

I(vii) Construct a ΔABC in which

$m\overline{AB} = 3.6$ cm, $m\angle A = 75°$, $m\angle B = 45°$



Given: $m\overline{AB} = 3.6$ cm

$m\angle A = 75°$

$m\angle B = 45°$

Required: Construct a triangle ABC

Steps of Construction:

(i) Take a line segment $\overline{AB} = 3.6$ cm.

(ii) Make an angle of 75° at A.

(iii) Make an angle of 45° at B.

(iv) $\overline{AR}$ and $\overline{BP}$ intersect at C.

Result: ABC is the required triangle.

Q.2 Construct a ΔXYZ in which.

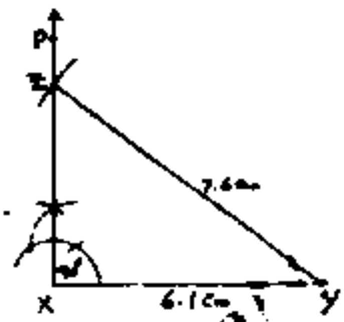
(i) $m\overline{YZ} = 7.6$ cm, $m\overline{XY} = 6.1$ cm, $m\angle X = 90^\circ$

(ii) $m\overline{ZX} = 6.4$ cm, $m\overline{YZ} = 2.4$ cm, $m\angle Y = 90^\circ$

(iii) $m\overline{XY} = 5.5$ cm, $m\overline{ZX} = 4.5$ cm, $m\angle Z = 90^\circ$

2(i) Construct a ΔXYZ in which.

$m\overline{YZ} = 7.6$ cm, $m\overline{XY} = 6.1$ cm, $m\angle X = 90^\circ$



Given: $m\overline{YZ}$ 7.6 cm

$m\overline{XY}$ 6.1 cm

$m\angle X$ 90°

Required: Construct a triangle XYZ

Steps of Construction:

(i) Take a line segment $\overline{XY}$ 6.1 cm.

(ii) Make an angle of XYP of 90° at X.

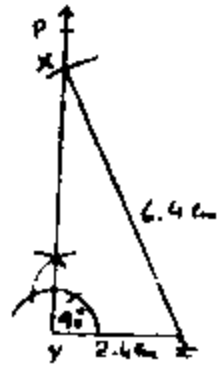
(iii) Take Y as centre and draw an arc of radius of 7.6 cm. This arc intersect $\overrightarrow{XP}$ at Z.

(iv) Join Z to Y.

Result: XYZ is the required triangle.

2(ii) Construct a ΔXYZ in which.

$m\overline{ZX} = 6.4$ cm, $m\overline{YZ} = 2.4$ cm, $m\angle Y = 90^\circ$



Given: $m\overline{ZX} = 6.4$ cm

$m\overline{YZ} = 2.4$ cm

$m\angle Y = 90^\circ$

Required: To construct a ΔXYZ

Steps of Construction:

(i) Take a line segment $\overline{YZ} = 2.4$ cm.

(ii) Make an angle of 90° at Y.

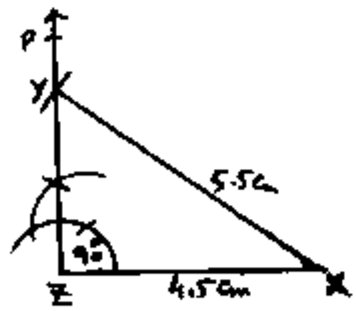
(iii) Take Z as centre and draw an arc of radius 6.4 cm which cuts $\overrightarrow{YP}$ at X.

(iv) Join X to Z.

Result: XYZ is the required triangle.

2(iii) Construct a ΔXYZ in which.

$m\overline{XY} = 5.5$ cm, $m\overline{ZX} = 4.5$ cm, $m\angle Z = 90°$



Given: $m\overline{XY} = 5.5$ cm

$m\overline{ZX} = 4.5$ cm

$m\angle Z = 90°$

Required: Construct a ΔXYZ

Steps of Construction:

(i) Take a line segment $\overline{ZX} = 4.5$ cm.

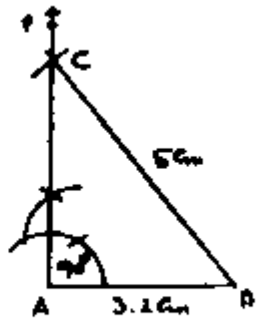
(ii) Draw an angle $\angle XZP$ of Z of 90°.

(iii) Take X as centre and draw an arc of 5.5 cm radius, which cuts $\overline{ZP}$ at Y.

(iv) Join Y to X.

Result: XYZ is the required triangle.

Q.3 Construct a right angled triangle measure of whose hypotenuse is 5 cm and one side is 3.2 cm.



Given: Length of hypotenuse = 5 cm
One side = 3.2 cm

Required: To construct a right angled triangle.

Steps of Construction:

(i) Take a line segment of 3.2 cm.

(ii) Make an angle $\angle BAP = 90^\circ$ at A.

(ii) Take B as centre and draw an arc of radius 5 cm which cuts $\overrightarrow{AP}$ cuts at C.

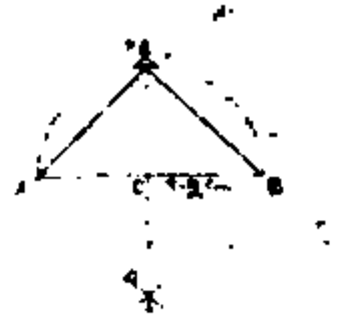
(iii) Join C to A.

Result: ABC is the required triangle.

Q.4 Construct a right angled isosceles triangle whose hypotenuse is

(i)	**5.2 cm**	**(ii)**	**4.8 cm**
(iii)	**6.2 cm**	**(iv)**	**5.4 cm long.**

4(i) ***5.3 cm***



Given: Length of hypotenuse = 5.2 cm

Required: To construct a right angled isosceles triangle

Steps of Construction:

(i) Take a line segment AB = 5.2 cm

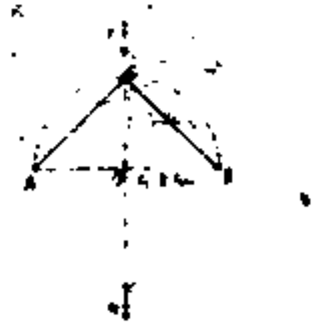
(ii) Take O as midpoint by drawing $\overleftrightarrow{PQ}$ as perpendicular bisector of $\overline{AB}$

(iii) Take O as centre and draw a semi-circle of radius OA or OB which cuts $\overleftrightarrow{PQ}$ at C

(iv) Join C to A and B

Result: $\triangle ABC$ is the required isosceles right-angled triangle at C

4(ii) ***4.8 cm***



Given: Length of hypotenuse = 4.8 cm

Required: To construct a right angled isosceles triangle.

Steps of Construction:

(i) Take a line segment $\overline{AB}$ = 4.8 cm.

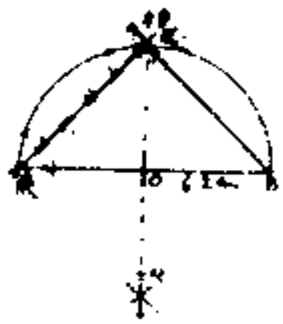
(ii) Draw $\overleftrightarrow{PQ}$ right bisector of $\overline{AB}$ which cuts $\overline{AB}$ at O.

(iii) Take O as centre and draw a semi-circle of radius $\overline{OA}$ or $\overline{OB}$ which cuts $\overleftrightarrow{PQ}$ at C.

(iv) Join C to A and B.

Result: ABC is the required triangle which is right-angled at C and $m\overline{AC} = m\overline{BC}$

4(iii) ***6.2 cm***



Given: Length of hypotenuse = 6.2 cm

Required: To construct a right angled isosceles triangle with hypotenuse = 6.2 cm

Steps of Construction:

(i) Take a line segment $\overline{AB}$ = 6.2 cm.

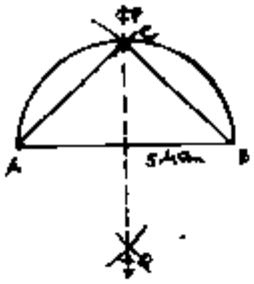
(ii) Draw $\overleftrightarrow{PQ}$ right bisector of $\overline{AB}$ which cuts $\overline{AB}$ at O

(iii) Take O as centre and draw a semi-circle of radius $\overline{OA}$ or $\overline{OB}$ which cuts $\overline{PQ}$ at C.

(iv) Join C to A and B.

Result: ABC is the required triangle which right-angled at C and $m\overline{AC} = m\overline{BC}$

4(iv) 5.4 cm



Given: Length of hypotenuse = 5.4 cm

Required: To construct a right angled isosceles triangle.

Steps of Construction:

(i) Take a line segment $\overline{AB}$ = 5.4 cm.

(ii) Draw $\overline{PQ}$ right bisector of $\overline{AB}$ which cuts it at point O.

(iii) Take O as centre and draw a semi-circle of radius $\overline{OA}$ or $\overline{OB}$ which cuts $\overline{PQ}$ at C.

(iv) Join C to A and B.

Result: ABC is the required triangle which is right-angled at C and $m\overline{AC} = m\overline{BC}$

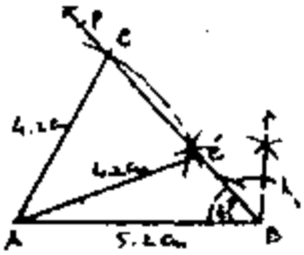
Q.5 (Ambiguous Case) Construct a ΔABC in which

(i) $m\overline{AC} = 4.2$ cm, $m\overline{AB} = 5.2$ cm, $m\angle B = 45°$

(ii) $m\overline{BC} = 2.5$ cm, $m\overline{AB} = 5.0$ cm, $m\angle A = 30°$

(iii) $m\overline{BC} = 5$ cm, $m\overline{AC} = 3.5$ cm, $m\angle B = 60°$

5(i) $m\overline{AC} = 4.2$ cm, $m\overline{AB}$, $m\angle B = 45°$



Given: $m\overline{AC} = 4.2$ cm

$m\overline{AB} = 5.2$ cm

$m\angle B = 45°$

Required: To construct triangle ABC.

Steps of Construction:

(i) Take a line segment $\overline{AB} = 5.2$ cm.

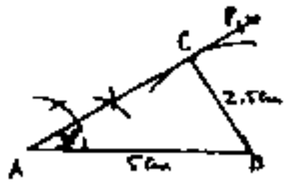
(ii) Make an angle $\angle ABP = 45°$ at point B.

(iii) Take point A as centre and draw an arc of 4.2 cm radius. It cuts $\overrightarrow{BP}$ at two points C, C'.

(iv) Join C, C' to A.

Result: Two Δ^s ABC, ABC' fulfill the requirements.

5(ii) $m\overline{BC} = 2.5$ cm, $m\overline{AB} = 5.0$ cm, $m\angle A = 30°$



Given: $m\overline{BC} = 2.5$ cm

$m\overline{AB} = 5.0$ cm

$m\angle A = 30^\circ$

Required: Construct a $\triangle ABC$.

Steps of Construction:

(i) Take a line segment $\overline{AB} = 5$ cm.

(ii) Make an angle $\angle BAP = 30^\circ$ at A.

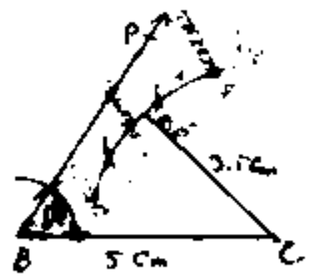
(iii) Take B as centre and draw an arc of 2.5 cm radius. This arc touches $\overrightarrow{AP}$ at C

(iv) Join C to A.

Result: ABC is the required triangle.

5(iii) Construct a triangle ABC in which

$m\overline{BC} = 5$ cm, $m\overline{AC} = 3.5$ cm, $m\angle B = 60^\circ$



Given: $m\overline{BC} = 5$ cm

$m\overline{AC} = 3.5$ cm

$m\angle B = 60^\circ$

Required: Construct a triangle ABC.

Steps of Construction:

(i) Take a line segment $\overline{BC} = 5$ cm

(ii) Make an angle $\angle CBR$ of 60° at B.

(iii) Take C as centre and draw an arc of 3.5 cm radius. This arc does touch and cut $\overrightarrow{BR}$ at any point.

Result: Triangle ABC cannot be constructed with the given sides and angle.

Remember:

(i) The internal bisectors of the angles of a triangle meet at a point called the incentre of the triangle.

(ii) The point of concurrency of the three perpendicular bisectors of the sides of a Δ is called the circumcentre of the Δ.

(iii) The point of concurrency of the three altitudes of a Δ is called its orthocenter.

(iv) The point where the three medians of a Δ meet is called the centroid of the triangle.

EXERCISE 17.2

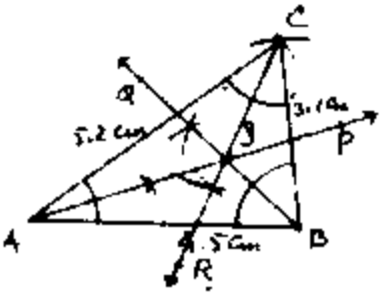
Q.1 Construct the following Δ' ABC. Draw bisectors of their angles and verify their concurrency.

(i) $m\overline{AB} = 4.5$ cm, $m\overline{BC} = 3.1$cm and $m\overline{CA} = 5.2$cm

(ii) $m\overline{AB} = 4.2$ cm, $m\overline{BC} = 6$ cm and $m\overline{CA} = 5.2$cm

(iii) $m\overline{AB} = 3.6$ cm, $m\overline{BC} = 4.2$ cm and $m\angle B = 75°$

I(i) $m\overline{AB} = 4.5$ cm, $m\overline{BC} = 3.1$cm and $m\overline{CA} = 5.2$cm



Given: $m\overline{AB}$ 4.5 cm

$m\overline{BC} = 3.1cm$

$m\overline{CA} = 5.2cm$

Required: Construct ΔABC and see the concurrency of the bisectors of its angles.

Steps of Construct:

(i) Take a line segment $\overline{AB} = 4.5$ cm

(ii) Take A and B as centers and draw arcs of 5.2, 3.1cm radii respectively. These arc intersect each other at point C.

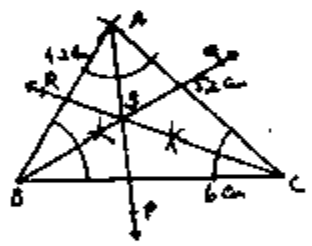
(iii) Join C to A and B. ABC is the required triangle.

(iv) Take $\overrightarrow{AP}$, $\overrightarrow{BQ}$, $\overrightarrow{CR}$ bisectors of angles A, B, C respectively.

Result:

We see that bisectors $\overrightarrow{AP}$, $\overrightarrow{BQ}$, $\overrightarrow{CR}$ are concurrent at point I.

1(ii) $m\overline{AB} = 4.2$ **cm,** $m\overline{BC} = 6$ **cm and** $m\overline{CA} = 5.2$**cm**



Given: $m\overline{AB} = 4.2$ cm

$m\overline{BC} = 6$ cm

$m\overline{CA} = 5.2$ cm

Required: Construct ΔABC and verify the concurrency of the bisectors of its angles.

Steps of Construction:

(i) Take a line segment $\overline{BC} = 6$ cm

(ii) Take B as center and draw an arcs of 4.2 cm radius.

(iii) Take C as centre and draw an arc of 5.2 cm radius that cuts the first arc at point A.

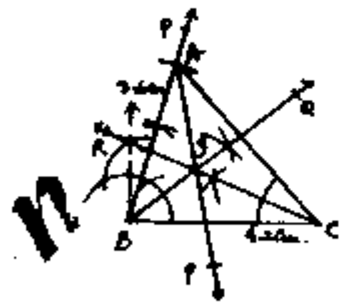
(iv) Join A to B and C.
ΔABC is the required triangle.

(v) Take $\overrightarrow{AP}$, $\overrightarrow{BQ}$ and $\overrightarrow{CR}$ bisectors of angle A, B and C respectively.

Result:

$\overrightarrow{AP}$, $\overrightarrow{BQ}$, $\overrightarrow{CR}$ are concurrent at point I.

1(iii) $m\overline{AB} = 3.6$ cm, $m\overline{BC} = 4.2$ cm and $m\angle B = 75°$



Given: $m\overline{AB} = 3.6$ cm

$m\overline{BC} = 4.2$ cm

$m\angle B = 75°$

Required: Construct ΔABC and see that bisectors of angles are concurrent.

Steps of Construct:

(i) Take a line segment $\overline{BC} = 4.2$ cm

(ii) Make an angle $\angle CBP = 75^\circ$.

(iii) Take B as centre and draw an arc of radius 3.6 cm which cuts $\overrightarrow{BP}$ at A.

(iv) Join A to C.
ABC is the required triangle.

(v) Take $\overline{AP}$, $\overline{BQ}$ and $\overline{CR}$ bisectors of angles.

Result:

$\overrightarrow{AP}$, $\overrightarrow{BQ}$ and $\overline{CR}$ are concurrent at point I.

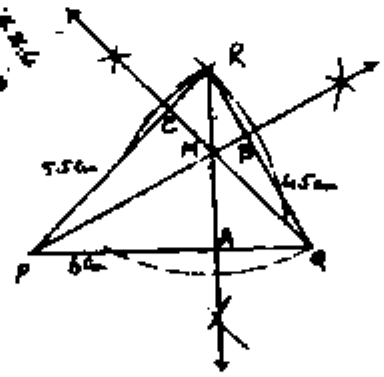
Q.2 Construct the following Δ' PQR, draw their altitudes and show that they are concurrent.

(i) $m\overline{PQ} = 6$ **cm**, $m\overline{QR} = 4.5$ **cm**, $m\overline{PR} = 5.5$ **cm**

(ii) $m\overline{PQ} = 4.5$ **cm**, $m\overline{QR} = 3.9$ **cm**, $m\angle R = 45^\circ$

(iii) $m\overline{RP} = 3.6$ **cm**, $m\angle Q = 30^\circ$, $m\angle P = 105^\circ$

2(i) $m\overline{PQ} = 6$ **cm**, $m\overline{QR} = 4.5$ **cm**, $m\overline{PR} = 5.5$ **cm**



Given: $m\overline{PQ} = 6$ cm

$m\overline{QR} = 4.5$ cm

$m\overline{PR} = 5.5$ cm

Required: Construct $\triangle PQR$ and take their altitudes.

Steps of Construction:

(i) Take a line segment $\overline{PQ} = 6$ cm

(ii) Take P as centre and draw an arc of 5.5 radius.

(iii) Take Q as centre and draw an arc of 4.5 radius.

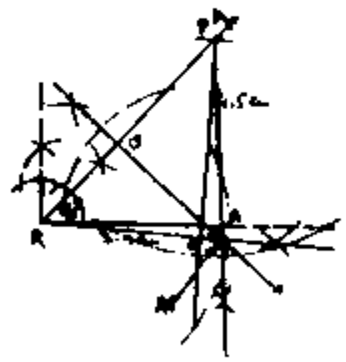
(iv) Join R to P and Q.
PQR is the required triangle.

(v) Drop $\overline{RA}$, $\overline{PB}$ and $\overline{QC}$ perpendiculars to $\overline{PQ}$, $\overline{QR}$, $\overline{RP}$ respectively.

Result:

$\overline{RA}$, $\overline{PB}$ and $\overline{QC}$ are concurrent at M.

2(ii) $m\overline{PQ} = 4.5$ cm, $m\overline{QR} = 3.9$ cm, $m\angle R = 45°$



Given: $m\overline{PQ} = 4.5$ cm

$m\overline{QR} = 3.9$ cm

$m\angle R = 45°$

Required: Construct $\triangle PQR$ and draw its altitude.

Steps of Construction:

(i) Take $\overline{RQ} = 3.9$ cm.

(ii) Make an angle $\angle QRA = 45°$.

(iii) Take Q as centre and draw an arc of 4.5 cm radius. which cuts $\overrightarrow{RA}$ at P.

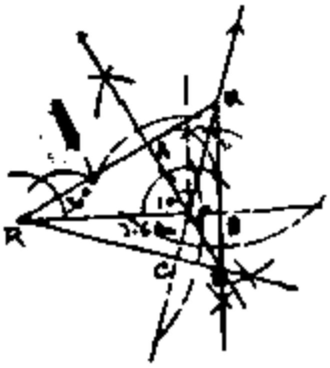
(iv) Join P to Q.
PQR is the required triangle.

(v) Drop $\overline{QM}$, $\overline{RN}$ and $\overline{PW}$ perpendiculars on $\overline{RP}$, $\overline{PQ}$ and $\overline{RQ}$ respectively.

Result:

$\overline{QM}$, $\overline{RN}$ and $\overline{PW}$ are concurrent at O.

2(iii) $m\overline{RP} = 3.6$ **cm,** $m\angle R = 30°$**,** $m\angle P = 105°$



Given: $m\overline{RP} = 3.6$ cm
$m\angle R = 30°$
$m\angle P = 105°$

Required: Construct ΔPQR and verify that its altitudes are concurrent.

Steps of Construction:

(i) Take a line segment $\overline{RP} = 3.6$ cm

(ii) Draw an angle $\angle PRQ = 30^\circ$.

(iii) Draw an angle $\angle RPQ = 105^\circ$

$\overline{RM}$ and $\overline{PW}$ intersect at point Q.
PQR in the required triangle.

(iv) Drop $\overline{RB}$, $\overline{PA}$ and $\overline{QC}$ perpendicular on $\overline{QP}$, $\overline{PA}$, $\overline{RP}$ respectively.

Result:

$\overline{RB}$, $\overline{PA}$, $\overline{QC}$ are concurrent at O.

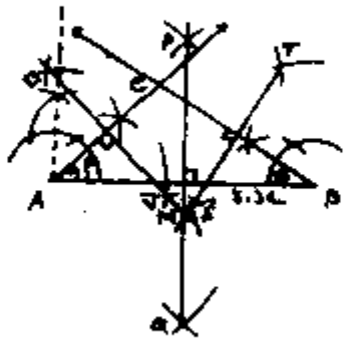
Q.3 Construct the following triangles ABC. Draw the perpendicular bisector of their sides and verify their concurrency. Do they meet inside the triangle.

(i) $m\overline{AB} = 5.3$ cm, $m\angle A = 45^\circ$, $m\angle B = 30^\circ$

(ii) $m\overline{BC} = 2.9$ cm, $m\angle A = 30^\circ$, $m\angle B = 60^\circ$

(iii) $m\overline{AB} = 2.4$ cm, $m\overline{AC} = 3.2$ cm, $m\angle A = 120^\circ$

3(i) $m\overline{AB} = 5.3$ cm, $m\angle A = 45^\circ$, $m\angle B = 30^\circ$



Given: $m\overline{AB} = 5.3$ cm

$m\angle A = 45^\circ$

$m\angle B = 30^\circ$

Required: Construct ΔABC. Draw perpendicular bisectors of its sides and verify their concurrency.

Steps of Construction:

(i) Take a line segment $\overline{AB} = 5.3$ cm

(ii) Make an angle $\angle BAW = 45^\circ$.

(iii) Make an angle $\angle ABJ = 30^\circ$

$\overrightarrow{AW}$, $\overrightarrow{BJ}$ intersect at point C.
ABC is the required triangle.

(iv) Take $\overleftrightarrow{PQ}$, $\overleftrightarrow{TS}$, $\overleftrightarrow{UV}$ right bisectors of $\overline{AB}$, $\overline{BC}$, $\overline{CA}$ respectively.

Result:

$\overleftrightarrow{PQ}$, $\overleftrightarrow{TS}$, $\overleftrightarrow{UV}$ are concurrent at point M.

3(ii) $m\overline{BC} = 2.9$ cm, $m\angle A = 30^\circ$, $m\angle B = 60^\circ$

Given: $m\overline{BC} = 2.9$ cm
$m\angle A = 30^\circ$
$m\angle B = 60^\circ$

Required: Construct triangle ABC. Draw perpendicular bisector of its sides and verify their concurrency.

Steps of Construction:

(i) Take a line segment $\overline{BC} = 2.9$ cm

(ii) Draw an angle $\angle CBW = 60^\circ$.
Place of A is not known, we find $m\angle C$
$m\angle C = 180 - m\angle B - m\angle A$
$m\angle C = 180 - 60 - 30 = 90^\circ$

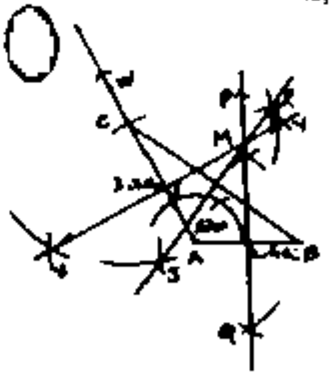
(iii) Draw an angle $\angle BCN = 90^\circ$
$\overrightarrow{PW}$, $\overrightarrow{CN}$ intersect at point A.
ABC is the required triangle.

(iv) Take $\overline{PQ}$, $\overline{RS}$, $\overline{UV}$ the right bisectors of $\overline{BC}$, $\overline{CA}$, $\overline{AB}$ sides.

Result:

$\overline{PQ}$, $\overline{RS}$, $\overline{UV}$ are concurrent at point M.

3(iii) $m\overline{AB} = 2.4$ **cm,** $m\overline{AC} = 3.2$ **cm,** $m\angle A = 120^\circ$



Given: $m\overline{AB} = 2.4$ cm
$m\overline{AC} = 3.2$ cm
$m\angle A = 120^\circ$

Required: Construct a ΔABC and draw perpendiculars of its sides and verify their concurrency.

Steps of Construction:

(i) Take a line segment $\overline{AB} = 2.4$ cm.

(ii) Make an angle $\angle BAW = 120^\circ$ at A.

(iii) Cut off $m\overline{AB}$ - 3.2 cm from $\overrightarrow{AW}$.

(iv) Join C to B.
ABC is the required triangle.

(v) Take $\overline{PQ}$, $\overline{RS}$, $\overline{UV}$ right bisectors of $\overline{AB}$, $\overline{BC}$, $\overline{CA}$ sides.

Result:

$\overline{PQ}, \overline{RS}, \overline{UV}$ are concurrent at point M.

(out side the Δ)

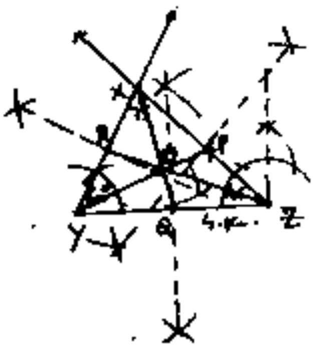
Q.4 Construct the following Δs XYZ. Draw their three medians and show that they are concurrent.

(i) $m\overline{YZ} = 4.1$ cm, $m\angle Y = 60^\circ$ and $m\angle X = 75^\circ$

(ii) $m\overline{XY} = 4.5$ cm, $m\overline{YZ} = 3.4$ cm, $m\overline{ZX} = 5.6$ cm

(iii) $m\overline{ZX} = 4.3$ cm, $m\angle X = 75^\circ$, $m\angle Y = 45^\circ$

4(i) $m\overline{YZ} = 4.1$ cm, $m\angle Y = 60^\circ$ and $m\angle X = 75^\circ$



Given: $m\overline{YZ} = 4.1$ cm

$m\angle Y = 60^\circ$

$m\angle X = 75^\circ$

Required: Construct ΔXYZ. Draw medians and verify their concurrency.

Steps of Construct:

(i) Take a line segment $\overline{YZ} = 4.1$ cm.

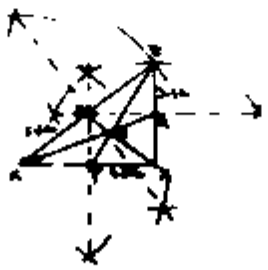
(ii) Make an angle $\angle ZYW = 60^\circ$.

(iii) Make an angle $\angle YZS = 45^\circ$.
$\overrightarrow{YW}$ and $\overline{ZS}$ intersect at X.
XYZ is the required triangle.

(iv) Find mid point of $\overline{YZ}$, $\overline{ZX}$, $\overline{XY}$ as Q, P, R.

(v) Join X to Q, Y to P, Z to R.
$\overline{XQ}$, $\overline{YP}$ and $\overline{ZR}$ are medians these are concurrent at O.

4(ii) $m\overline{XY} = 4.5$ cm, $m\overline{YZ} = 3.4$ cm, $m\overline{ZX} = 5.6$ cm



Given: $m\overline{XY} = 4.5$ cm

$m\overline{YZ} = 3.4$ cm

$m\overline{ZX} = 5.6$ cm

Required: Construct triangle XYZ. Draw medians and verify their concurrency.

Steps of Construction:

(i) Take a line segment $\overline{XY} = 4.5$ cm.

(ii) Take X as centre and draw an arc of radius 5.6 cm.

(iii) Take Y as centre and draw an arcs of radius 3.4 cm. These arcs intersect at point Z.

(iv) Join Z to X and Y.
XYZ is the required triangle.

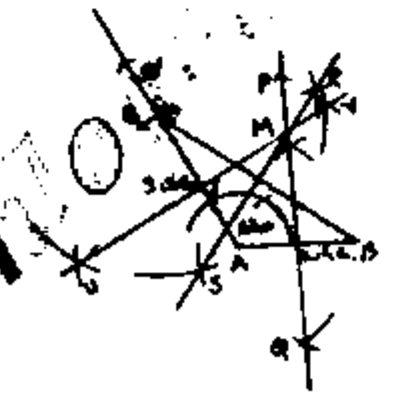
(v) Find mid points P, Q, R of $\overline{XY}$, $\overline{YZ}$, $\overline{ZX}$.

(vi) Draw $\overline{PZ}$, $\overline{QX}$, $\overline{RY}$ medians.

Result:

$\overline{PZ}$, $\overline{QX}$, $\overline{RY}$ are concurrent at O.

4(iii) $m\overline{ZX} = 4.3$ cm, $m\angle X = 75°$, $m\angle Y = 45°$



Given: $m\overline{ZX} = 4.3$ cm
$m\angle X = 75°$
$m\angle Y = 45°$

Required: Construct ΔXYZ and draw its medians and verify their concurrency.

Steps of Construction:

(i) Take a line segment $\overline{ZX} = 4.3$ cm.

(ii) Make an angle $\angle ZXM = 75°$.
Place of Y is not known, we find angle at Z.
$m\angle Z = 180° - m\angle Y - m\angle X$
$m\angle Z = 180° - 45° - 75° = 60°$

(iii) Draw an angle $\angle XZW = 60°$ at Z.
$\overrightarrow{ZW}, \overrightarrow{XM}$ intersect at Y.
XYZ is the required triangle.

(iv) Find P, Q, R mid points of $\overline{ZX}$, $\overline{XY}$, $\overline{ZY}$ as the mid points.
Join Y to P, Z to Q, X to R.

Result:

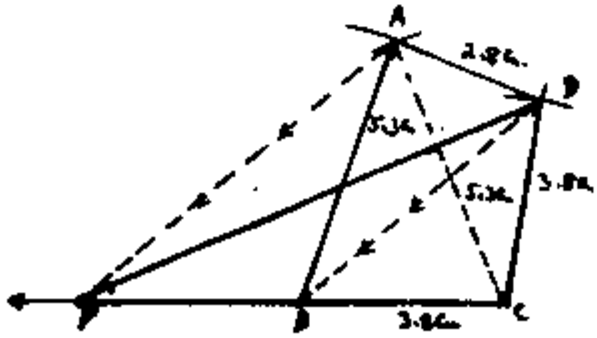
Medians $\overline{YP}$, $\overline{XR}$, $\overline{ZQ}$ are concurrent at point O.

EXERCISE 17.3

Q.1(i) Construct a quadrilateral ABCD, having
$m\overline{AB} = m\overline{AC} = 5.3$ cm. $m\overline{BC} = m\overline{CD} = 3.8$ cm
And $m\overline{AD} = 2.8$ cm.

(ii) On the side $\overline{BC}$ construct a Δ equal in area to the quadrilateral ABCD.

1(i) Construct a quadrilateral ABCD, having
$m\overline{AB} = m\overline{AC} = 5.3$ cm. $m\overline{BC} = m\overline{CD} = 3.8$ cm
And $m\overline{AD} = 2.8$ cm.



Given: $m\overline{AB} = m\overline{AC} = 5.3$ cm

$m\overline{BC} = m\overline{CD} = 3.8$ cm

$m\overline{AD} = 2.8$ cm

Required: Construct a quadrilateral ABCD

(ii) Construct a D equal in area to quad. ABCD.

Steps of Construction:

(i) Take a line segment $\overline{BC} = 3.8$ cm

(ii) Take B, C as centre and draw two arcs of 5.3 cm radius. These intersect each other at A. Join A to B and C.

(iii) Take A as centre and draw an arc of radius 2.8 cm.

(iv) Take C as centre and draw an arc of radius 3.8 cm. This cuts the first arc at **D.**

(v) Join D to A and C.
ABCD is the required **quadrilateral.**

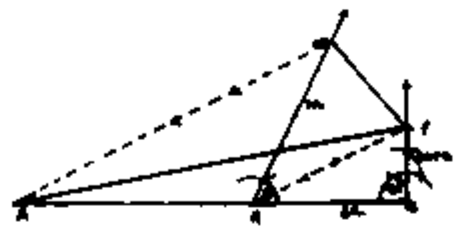
Part II

(vi) Join B to D.

(vii) Take $\overline{AP} \parallel \overline{BD}$ which meets extended $\overline{CB}$ at P.

(viii) Join P to D.
PCD is the required triangle which is equal in area to **quadrilateral.** ABCD.

Q.2 Construct a Δ equal in area to the quadrilateral. PQRS, having $m\overline{QR} = 7$cm. $m\overline{RS} = 6$cm, $m\overline{SP} = 2.75$, $m\angle QRS = 60°$ and $m\angle RSP = 90°$



Given: $m\overline{QR} = 7cm$, $m\overline{RS} = 6cm$, $m\overline{SP} = 2.75$, $m\angle QRS = 60°$ and $m\angle RSP = 90°$

Required: Construct a triangle equal in area to quadrilateral. PQRS of which measurements are given.

Steps of Construction:

(i) Take a line segment $\overline{RS} = 6cm$

(ii) Make an angle of 60° at R.

(iii) Make an angle of 90° at S.

(iv) Cut off $\overline{SP} = 2.75$ cm.

(v) Cut off $\overline{RQ} = 7$ cm

PQRS is the required quadrilateral.

Part II

(vi) Join P to R. $\overline{PR}$ is diagonal of the quadrilateral.

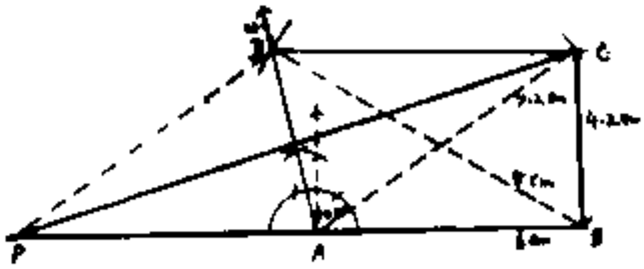
(vii) Take $\overline{QA} \parallel \overline{RP}$ which meets extended SR at A.

(viii) Join A to P

Result:

ASP is the required Δ of which area is equal to quadrilateral PQRS.

Q.3 Construct a Δ equal in area to the quadrilateral. ABCD having $m\overline{AB} = 6$ cm, $m\overline{BC} = 4$ cm, $m\overline{AC} = 7.2$ cm, $m\angle BAD = 105°$ and $\overline{BD} = 8$cm.



Given: $m\overline{AB} = 6$ cm, $m\overline{BC} = 4$ cm, $m\overline{AC} = 7.2$ cm, $m\angle BAD = 105°$ and $\overline{BD} = 8$cm.

Required: Construct a quadrilateral. ABCD and a Δ equal in area of ABCD.

Steps of Construction:

(i) Take a line segment $\overline{AB} = 6$ cm

(ii) Make an angle $\angle BAW = 105°$ at A.

(iii) Take B as centre and draw an arc of radius 8 cm which cuts $\overline{AW}$ at D.

(iv) Take A and B as centre and draw arcs of radius 7.2 and 4.5 cm respectively. These intersect at C.

(v) Join C to B and D.

∴ ABCD is the required quadrilateral.

Part II

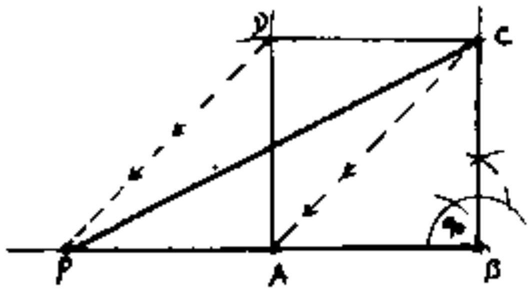
(vi) Join C to A.

(vii) Take $\overline{CP} \parallel \overline{CA}$ which meets $\overline{BA}$ extended at P.

Result:

CPB is the required Δ which is equal in area to quadrilateral ABCD.

Q.4 Construct a right angled triangle equal in area to a given square.



Given: ABCD is a given square.

Required: Construct a right angled triangle equal in area to square. ABCD.

Steps of Construction:

(i) Take $\overline{CA}$ the diagonal of the square. ABCD.

(ii) Extend $\overline{BA}$ towards A.

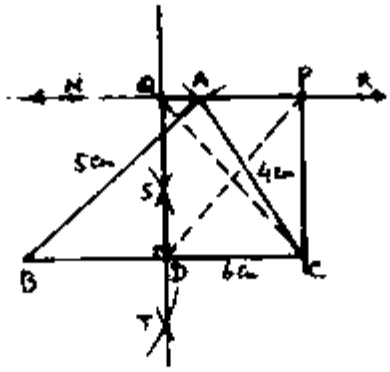
(iii) Take $\overline{DP} \parallel \overline{CA}$ which meets extended $\overline{BA}$ at P.

Result:

PBC is the required Δ equal in area to square. ABCD.

EXERCISE 17.4

Q.1 Construct a Δ with sides 4 cm, 5 cm and 6 cm and construct a rectangle having its area equal to that of the Δ. Measure its diagonals. Are they equal?



Given: $m\overline{BC} = 6cm$

$m\overline{AB} = 5cm$

$m\overline{AC} = 4$ cm

Required: To construct the triangle and a rectangle equal in area to the Δ.

Steps of Construction:

(i) Take line segment $\overline{BC} = 6$ cm.

(ii) Take B and C as centre and draw arc of radii 4, 5 cm respectively. These arcs intersect at A.

(iii) Join A to B, C

ABC is the required triangle.

(iv) Draw $\overline{NAP} \parallel$ to $\overline{BC}$ passing through A.

(v) Take right bisector $\overline{WS}$ which cuts $\overline{BC}$ at D and $\overleftrightarrow{NP}$ at Q.

(vi) Take Q as centre and draw an arc of radius $\overline{DC}$

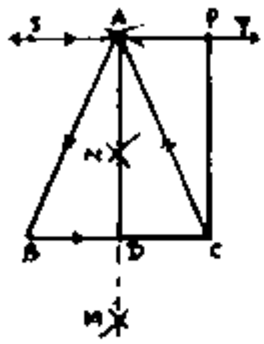
(vii) Take C as centre and draw an arc of radius $\overline{DQ}$, which cuts the first arc at P.

(viii) Join P to Q and C.

Result:

DCPQ is the required rectangle which is equal in area to ΔABC. $m\overline{DP} = m\overline{QC} = 4.5$ cm

Q.2 Transform an isosceles Δ into a react angle.



Given: ABC is an isosceles Δ in which $\overline{AB} \cong \overline{AC}$

Required: To construct a rectangle equal in area to the Δ.

Steps of Construction:

(i) Take $\overleftrightarrow{SAT} \parallel \overline{BC}$ passing through A.

(ii) Take right bisector of $\overline{BC}$ which cuts $\overline{BC}$ at D and passes through A.

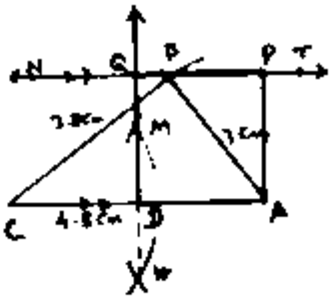
(iii) Take A as centre and draw an arc of radius $\overline{DC}$ which cuts $\overline{ST}$ at P.

(iv) Join P to C.

Result:

ADCP is the required rectangle which is equal in area to Δ ABC.

Q.3 **Construct a Δ ABC such that $m\overline{AB} = 3$ cm, $m\overline{BC} = 3.8$ cm, $m\overline{AC} = 4.8$ cm. Construct a rectangle equal in area to the Δ ABC, and measure its sides.**



Given: ABC is a triangle that

$m\overline{AB} = 3$ cm

$m\overline{BC} = 3.8$ cm

$m\overline{AC} = 4.8$ cm

Required: Construct the ΔABC and a rectangle equal in area to ΔABC

Steps of Construction:

(i) Take a line segment $\overline{CA} = 4.8$ cm.

(ii) Take C and A as centre and draw arcs of 3.8 cm, 3 cm radii respectively. These cut each other at B.

(iii) Join B to C and A.

ABC is the required Δ.

(iv) Take $\overleftrightarrow{NBT} \parallel \overline{CA}$ which passes through B.

(v) Take right bisector of $\overline{CA}$ which cuts $\overline{CA}$ at D and $\overleftrightarrow{NT}$ at Q.

(vi) Cut off $\overline{OP} = m\overline{OD}$ and P to A.

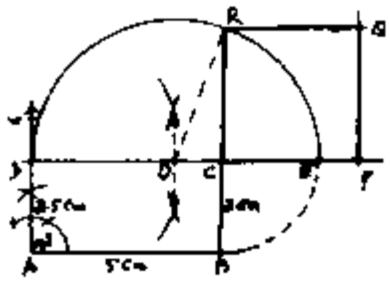
Result:

QDAP is the required rectangle equal in are to ΔABC.

$m\overline{DA} = 2.4$ cm, $m\overline{DQ} = 2.4$ cm

EXERCISE 17.5

Q.1 Construct a rectangle whose adjacent sides are 2.5 cm and 5 cm respectively. Construct a square having area equal to the given rectangle.



Given: Sides of a rectangle are 2.5 cm, 5 cm

Required: (i) Construct rectangle

(ii) Construct a square equal in area to the rectangle.

Steps of Construction:

(i) Take a line segment $\overline{AB} = 5$ cm

(ii) Make an angle of 90° at A.

(iii) Cut off 2.5 cm $\overline{AD}$ and $\overline{AW}$.

(iv) Draw arcs with centre at D, B respectively of radius 5 cm and 2.5 cm. These cut each other at C.

(v) Join C to D, B.

ABCD is the required rectangle.

Part II

(vi) Extend $\overline{DC}$ towards C.

(vii) Cut off $\overline{CE} = \overline{BC}$ i.e. 2.5 cm on $\overrightarrow{DN}$.

(viii) Find midpoint O of $\overline{DE}$.

(ix) Take O as centre and a semicircle on $\overline{DE}$.

(x) Extend side $\overline{BC}$ towards C which meets the semicircle at R.

(xi) $m\overline{RC}$ is the length of the required square.

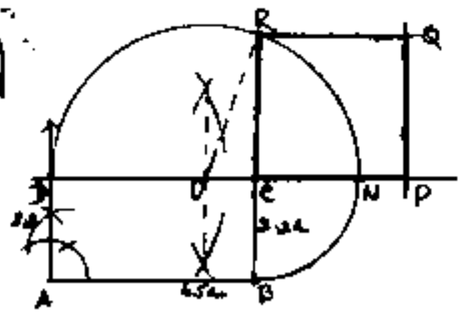
(xii) Cut off $\overline{CP} = \overline{CR}$ on $\overline{DE}$

(xiii) Take R and P as centre and draw two arcs that cut each other at Q.

(xiv) Join Q to R and P.

CPQR is the required square whose area is equal to the area of rectangle ABCD.

Q.2 Construct a square equal in area to a rectangle whose adjacent sides are 4.5 cm and 2.2 cm respectively. Measure the sides of the square and find its area and compare with the area of the rectangle.



Given: Sides of a rectangle are 2.5 cm, 5 cm.

Required: Construct the rectangle and construct a square equal in area to the rectangle.

Steps of Construction:

(i) Take a line segment $\overline{AB} = 4.5$ cm

(ii) Make an angle $\angle BAW = 90^\circ$ at A.

(iii) Cut off 2.2 cm from $\overline{AW}$ as $\overline{AD}$.

(iv) Take D and B as centre and draw of radius 4.5 and 2.2 cm respectively. There arcs cut each other at C.

(v) Join C to D and B.
ABCD is the required rectangle.

Part II

(vi) Extend $\overline{DC}$ towards C.

(vii) Cut of $\overline{CE} = \overline{CB} = 2.2$ cm on $\overline{DN}$

(viii) Find midpoint O of $\overline{DE}$.

(ix) Take O as centre and draw a semi-circle on $\overline{DE}$ taking $\overline{OD}$ as radius.

(x) Extend $\overline{BC}$ towards C which meets the semicircle at R. $m\overline{RC}$ is the length of side of the required square.

(xi) Cut off $\overline{CP} = m\overline{RC}$ on $\overline{DN}$.

(xii) Take R and P as centre and draw two arcs of radius $\overline{CR}$. These cut each other at Q.

(xiii) Join Q to R and P.

Result:

CPQR is the required square whose area is equal to area of rectangle ABCD.

Part II

Area of the rectangle $= 4.5 \times 2.2$

$= 9.9$ sq. cm

Side of the square $= 3.1$ cm

Area of the square $= 3.1 \times 3.1$

$= 9.6$ sq. cm

Area of the square is approximately equal to the area of the rectangle.

Q.3 In Q.2 above verify by measurement that the perimeter of the square is less than that of the rectangle.

Solution:

Perimeter of the rectangle $= 2(4.5 + 2.2)$

$= 2(6.7)$

$= 13.4$ cm

Side of the square $= 3.1$ cm

Perimeter of the square $= 4(3.1)$

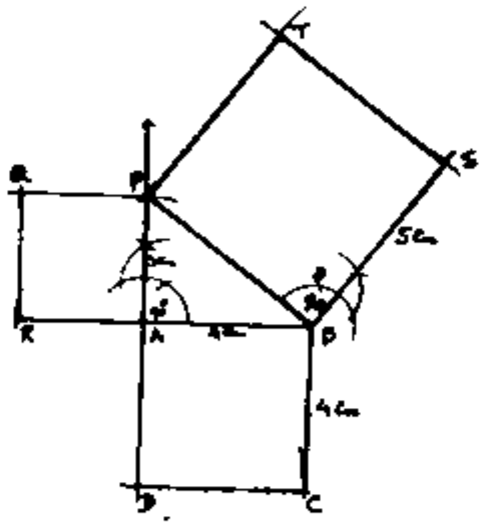
$= 12.4$ cm

Now $12.4 < 13.4$

Therefore:

Perimeter of the square is less than the perimeter of the rectangle.

Q.4 Construct a square equal in area to the sum of two squares having sides 3 cm and 4 cm respectively.



Given: Two square having 3 cm and 4 cm sides are given.

Required: To construction a square equal in area to the sum of the area of two squares.

Steps of construction:

(i) Take a line segment $m\overline{AB} = 4$ cm.

(ii) Complete square ADCB on $\overline{AB}$.

(iii) Extend $\overline{DA}$ towards A and cut off $\overline{AP} = 3$ cm. OR Make an angle of 90° at A and cut off $\overline{AP} =$ 3cm.

(iv) Construct a square ARQP on $\overline{AP}$

(v) Join B to P.
ABP is a right angled triangle of which one side is 4 cm and the other side is 3 cm.

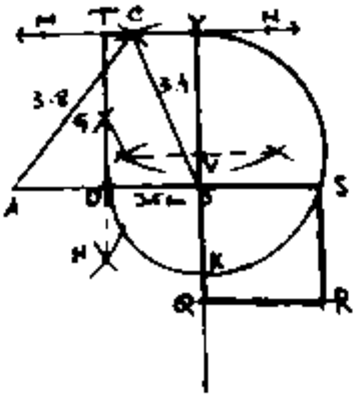
(vi) Construct a square on hypotenuse $\overline{BP}$

Result:

$(mPB)^2 = (mAB)^2 + m(AP)^2$

(Pythagorean theorem)

Q.5 Construct a Δ having base 3.5 cm and other two sides equal to 3.4 cm and 3.8 cm respectively. Transform it into an equal square.



Given: ΔBC is a triangle in which (suppose)

$m\overline{AB} = 3.5$ cm

$m\overline{AC} = 3.8$ cm

$m\overline{BC} = 3.4$ cm

Required: To construct a square equal in area to the triangle ABC.

Steps of Construction:

Part I

(i) Take a line segment $\overline{AB} = 3.5$ cm

(ii) Take A and B as centre and draw two arcs of 3.8 cm and 3.4 cm respectively. These arcs intersect at C.

(iii) Join C to A and B.

ABC is the required Δ.

Part II

(i) Take $\overline{MN} \parallel \overline{AB}$ passing through C.

(ii) Take right bisector $\overline{GH}$ of AB which bisects $\overline{AB}$ at O. and $\overleftrightarrow{MN}$ at T.

(iii) Sides line segments OT and OB are the sides of a rectangle.

(iv) Complete rectangle OBYT. Its area is equal to the area of ΔABC.

Part II

(i) Extend $\overline{YB}$ towards B.

(ii) Cut off $\overline{BK} = m\overline{BO}$ on $\overleftrightarrow{YW}$

(iii) Find midpoint V of $\overline{BK}$.

(iv) Take V as centre and draw a semicircle on $\overline{YK}$

(v) Extend $\overleftrightarrow{OB}$ towards B which meets the semicircle at S.

(vi) Complete square on $\overline{BS}$ square BQRS is equal in area of rectangle OBYT.

From Part I, II, III

Square BQRS is equal in area to triangle ABC.

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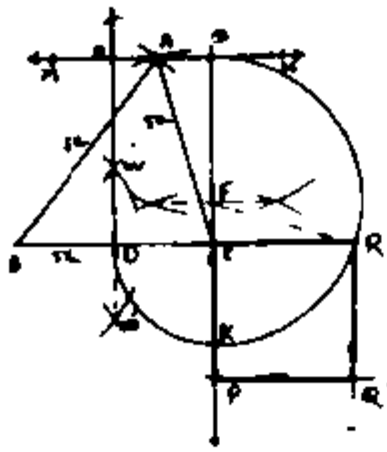
Q.6 Construct a Δ having base 5 cm and other sides equal to 5 cm and 6 cm. Construct square equal in area to given Δ.



Given: Let ABC be the D in which $m\overline{AC} - 5$ cm

$m\overline{AB} = 6$ cm

$m\overline{AC} = 5$ cm

Required: Construct a square equal in area to Δ ABC.

Steps of construction:

Part I

(i) Take a line segment $\overline{AB} = 5$ cm

(ii) Take B and C as centre and draw two arcs of radius 6 cm, 5 cm, respectively. These cut each other at A.

(iii) Join A to B and C.

ABC is the required Δ.

Part II

(i) Take $\overline{MN} \parallel \overline{BC}$ passing through A.

(ii) Take WW' right bisector of $\overline{BC}$ at O and cuts MN at E.

(iii) Complete the rectangle EOCD which is equal in area to ΔABC

Part III

(i) Extend $\overline{DC}$ side of rectangle towards C and take $m\overline{CK} = m\overline{CO}$

(ii) Find F the midpoint of $\overline{DK}$

(iii) Take F as centre and complete a semicircle on $\overline{DK}$.

(iv) Extend $\overline{OC}$ towards C, this cut the semicircle on R.

(v) Complete square CRQP and its area is equal to the area of rectangle EOCD

Result:

Area of square CRQP is equal to the area of rectangle EOCD which is equal to the area of the triangle ABC.

Review Exercise 17

1. **Fill in the following blanks to make the statement true:**

(i) The side of a right angled triangle opposite to 90° is called ___________

(ii) The line segment joining a vertex of a triangle to the mid-point of its opposite side is called a __________ .

(iii) A line drawn from a vertex of a triangle which is ___________ to its opposite side is called an altitude of the triangle.

(iv) The bisectors of the three angles of a triangle are ___________

(v) The point of concurrency of the right bisectors of the three sides of the triangle is from its vertices.

(vi) Two or more triangles are said to be similar if they are equiangular and measures of their corresponding sides are___________

(vii) The altitudes of a right triangle are concurrent at the ___________ of the right angle.

Answers:

(i)	hypotenuse	(ii)	median
(iii)	perpendicular	(iv)	concurrent
(v)	equidistant	(vi)	proportional
(vii)	vertex		

Q.2. **Multiple Choice Questions. Choose the correct answer.**

(i) A triangle having two sides congruent is called:

(a) scalene (b) right angled

(c) equilateral (d) isosceles

(ii) A quadrilateral having each angle equal to 90° is called:

(a) parallelogram (b) rectangle
(c) trapezium (d) rhombus

(iii) The right bisectors of the three sides of a triangle are:
(a) congruent (b) collinear
(c) concurrent (d) parallel

(iv) The ………… altitudes of an isosceles triangle are congruent:
(a) two (b) three
(c) four (d) none

(v) A point equidistant from the end points of a line segment is on its:
(a) bisector (b) right-bisector
(c) perpendicular (d) median

(vi) ……… congruent triangles can be made by joining the mid-point of the sides of a triangle:
(a) three (b) four
(c) five (d) two

(vii) The diagonals of a parallelogram ………. each other:
(a) bisect (b) trisect
(c) bisect at right angle (d) none of these

(viii) The medians of a triangle cut each other in the ratio:
(a) 4 : 1 (b) 3 : 1
(c) 2 : 1 (d) 1 : 1

(ix) One angle on the base of an isosceles triangle is 30°. What is the measure of its vertical angle:
(a) 30° (b) 60°
(c) 90° (d) 120°

(x) If the three altitudes of a triangle are congruent, then the triangle is:
(a) equilateral (b) right angled
(c) isosceles (d) acute angled

(xi) If two medians of a triangle are congruent then the triangle will be:

(a) isosceles (b) equilateral

(c) right angled (d) acute angled

Answers:

(i)	d	*(ii)*	b	*(iii)*	c	*(iv)*	a
(v)	b	*(vi)*	b	*(vii)*	a	*(viii)*	c
(ix)	d	*(x)*	a	*(xi)*	a		

Q.3. Define the following

(i) **In-centre** **(ii)** **Circum-centre**

(iii) **Ortho-centre** **(iv)** **Centroid**

(v) **Point of concurrency**

(i) **In-centre:** The point of concurrency of the bisectors of interior angles of a triangle is called its In-centre.

(ii) **Circum-centre:** The point of concurrency of the three right bisectors of sides of a triangle is called its circum-centre.

(iii) **Ortho-centre:** The point of concurrency of the three altitudes of a triangle is called its Ortho-centre.

(iv) **Centroid:** The point of concurrency of three medians of a triangle is called its Centroid.

(v) **Point of concurrency:** If three or more than three straight lines, rays, line-segments, intersect each other at a single point then that point is a point of concurrency of these straight lines, rays or line-segments.